17. COMMISSION DU MOUVEMENT ET DE LA FIGURE DE LA LUNE

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Yale University Observatory, Dirk Brouwer, *November* 13, 1947

The discussions of observations of occultations have been published annually in the *Astronomical Journal*. The publication, dealing with the observations in the year 1942, which appeared in *A. J.* **51**, 144–5 (1945), concludes the series of annual discussions made at the Yale Observatory. Beginning with the observations in the year 1943, the British Nautical Almanac Office has assumed the responsibility for the reduction and discussion of occultations of stars by the Moon.

A discussion of the Moon's motion based upon about 5000 occultations in the eleven years, 1932–42, and about 1000 Washington meridian circle observations in the years 1933–44 was made by D. Brouwer and C. B. Watts, *A. J.* **52**, 169–76 (1947). Both series were reduced to the FK3 system and corrections for limb error were applied, based upon Hayn's charts. In the case of the occultations the tables by Watts, *A. J.* **48**, 170 (1940), were used for a partial elimination of the systematic limb effect from the lunation solutions. The meridian circle observations were corrected individually.

An analysis of each of the two series of observations separately was made by least squares. Terms having the periods of the tropical, nodical and anomalistic months were introduced in addition to a constant and, in the longitude only, a term varying linearly with the time. For the occultations the means by lunations were used, for the meridian data the individual observations. The substitution of the solutions yields residuals that show:

(1) Excellent agreement between meridian circle observations and occultations, especially in the longitude.

(2) The lack of uniformity of the Earth's rotation during the ten years 1933·0–1943·0 produced a maximum deviation of only 0″·3 in the Moon's observed mean longitude.

(3) The well-known annual term in the occultation results in longitude has practically disappeared from the final results by lunation; it appears that the three different periods were present in the original residuals.

A complete interpretation of the coefficients of the periodic terms found in the solutions cannot be made on the basis of the available data. Appreciable differences between the coefficients from east and west limb of the meridian observations and occultations indicate that, to some extent at least, the periodic terms must arise from residual limb effects.

U.S. Naval Observatory, C. B. Watts, *November* 17, 1947

At the U.S. Naval Observatory, Washington, C. B. Watts has undertaken to make a new survey of the Moon's marginal zone with the purpose of deriving an improved set of corrections for those classes of measures that involve the Moon's limb. The work will be based on photographs made at Washington and at the Yale Southern Station at Johannesburg. A photo-electric measuring engine is being constructed with which it is expected that the photographs can be measured automatically.

H.M. Nautical Almanac Office, London, D. H. Sadler, *December* 22, 1947

(*a*) Since the I.A.U. meeting in Stockholm the Nautical Almanac Office has continued with its routine programme of prediction of occultations for about sixty stations. Throughout the war predictions were sent to every station with which communication was possible.

The occultation reduction elements were omitted from the *Nautical Almanac*, 1942, but were issued in the U.S.A. through the courtesy of the American Section of the I.A.U. It was then decided that the number of observed occultations did not warrant the continued pre-computation of the occultation reduction elements and these have been consequently dropped from the *Nautical Almanac* from 1943 onwards. From that year the Nautical Almanac Office has undertaken to reduce observations of occultations of all stars included in the *N.Z.C.* From the same year the Office has also taken over from Prof. Brouwer the compilation and discussion of the occultation observations in relation to the position of the Moon. The discussion for 1943 is in process of publication in the *Astronomical Journal* and it is hoped that the wartime arrears of reduction will shortly be made up.

(*b*) I would like to suggest that if an opportunity arises the following subjects might be discussed by the Commission in Zürich:

(i) The limitations to be adopted for the prediction of occultations so as to ensure a balance between the amount of work involved and the restriction of observations to those which can be made only in good conditions.

(ii) The topography of the limb of the Moon, especially in regard to the work now in progress by Mr C. B. Watts in Washington.

(iii) The value of **k** (the ratio of the Moon's diameter to that of the Earth) to be adopted for occultations.

(*c*) We have no motions for submission to the General Assembly.

CRACOW UNIVERSITY OBSERVATORY, *Director:* T. BANACHIEWICZ

According to the *Engelh. Obs. Public.* **21**, 4 (1939) the determination of the libration constants from 200 heliometer observations is a huge computational problem; e.g. the solution by least squares of 400 equations with eight unknowns forms but one-fiftieth of the whole work. T. Banachiewicz examined the problem both from the fundamental and from the technical standpoint, and a new method which resulted from this study was applied—with some further developments—by K. Kozieł to Hartwig's Dorpat observations of 1884–85. Owing to certain observational particularities this Dorpat set was especially difficult to deal with. The new method of reduction uses throughout the Cracovian calculus. Some points of it are here reported on.

One of the first points to clear away was the puzzling libration term in the Moon's radius. Yakovkin found in 1934 from the discussion of his Kasan 1916–26 heliometer observations that the radius R of the Moon, corrected for the limb's irregularities after Hayn, shows an increase $b=+0''\cdot048$ ($\pm 0''\cdot006$ m.e.) for every degree of the libration in latitude. To test this result T. Banachiewicz investigated the great series of 235 observations of Hartwig, as reduced recently by Naumann, and found $b=+0''\cdot054$ ($\pm 0''\cdot006$) and $b=+0''\cdot029$ ($\pm 0''\cdot005$), respectively without and with Hayn's inequalities, in this way confirming qualitatively the existence of the term. The observed latitudes of Mösting A do not show, however, any corresponding changes, so that, contrary to some opinions, the mountains near the Moon's south pole taken alone could not explain the effect. The explanation of it may be as follows. In the usual (defective) method of determining R from the heliometer observations, R is based principally on the high-latitude regions of the Moon. Hence the apparent changes of R are a function of the particular slopes of these regions. Unfortunately, Hayn's photographs, owing to their lack of an absolute scale, give only relative heights which are self-influenced by the marginal elevations of a regular character, and cannot lead therefore to the true dimensions of the Moon. A confirmation of this view is afforded by the investigations of K. Kozieł. When treating the Dorpat observations in the ordinary way, he obtained $b=+0''\cdot040$ ($\pm 0''\cdot014$), like Yakovkin, but the new method gave him $b=-0''\cdot011$ ($\pm 0''\cdot010$), and the puzzle disappeared.

The heliometric observations of the Moon's libration lead to the values of λ, β, h (3 co-ordinates of Mösting A), I (inclination of the Moon's equator) the mechanical ellipticity f and the Moon's mean angular radius R. Every measured distance of

Mösting A brings in reality an observation-equation for these six unknowns. Instead of solving these equations directly, which was too long for the Gaussian algorithm of least squares, the usual method consisted of determining from every full observation-set three intermediary quantities, dL, dB and dR. Then dL and dB were expressed in terms of $d\lambda$, $d\beta$, dh, dI and df, and gave the observation-equations for these five unknowns. A discussion was needed for determining the relative weights of the dL and dB equations. These equations, however, obtained from the same measured distances, are not independent, and the ordinary rules of least squares, when applied to them, cannot give results of unambiguous mathematical significance. Moreover, by the distinct solution of reduced normal equations for dR, important relations between R and other unknowns, as yielded by the intercomparison of the east and west limb's distances, were lost, to the great detriment of the value of the whole solution. The correct procedure is very simple. Form, as usually, the normal equations for dL, dB and dR; consider the corresponding Cracovian square-root equations; put into them the expressions of dL and dB as functions of the five unknowns. Solve then the equations with six unknowns obtained in such a way, considering them as independent observation-equations of weight unity. The result is shown to be an exact least-squares solution of the whole material. This method gives also directly the *a posteriori* mean error of every measured distance, the important quantity which remained unknown in the usual method of solution.

To investigate the Yakovkin term, K. Kozieł introduced yet a seventh unknown b, and obtained in this way, as already said, b equal to zero. The doubtlessly positive value of b in the former method of reduction would indicate—if the trend of the marginal lunar zones might be extrapolated—that the axis of the Moon's elongation be directed not towards the centre, but to the southern hemisphere of the Earth.

As regards the accuracy of the solution, K. Kozieł obtained from the thirty-six Dorpat observations the following mean errors of the unknowns: $d\lambda$ $\pm 11''\cdot 5$, $d\beta$ $\pm 17''\cdot 0$, dh $\pm 0''\cdot 56$, dI $+22''\cdot 6$, df $\pm 0\cdot 051$. These mean errors are remarkably small as compared with the mean errors of the unknowns yielded by the usual method; e.g. a (greater) set of fifty-one observations, made by an experienced observer with a similar heliometer, yielded, *Engelh. Obs. Publ.* **23**, 11 (1945), the mean errors of the same unknowns respectively $\pm 32''$, $\pm 27''$, $\pm 1''\cdot 1$, $\pm 37''$, $\pm 0\cdot 09$. It appears, therefore, that the mathematically exact method of determination of all unknowns at one sweep marks a decisive progress. The mean error of one measured distance is 'with Hayn' $\pm 0''\cdot 614$. K. Kozieł has solved all equations using two assumptions: 'without' Hayn and 'with' Hayn, and has obtained, for the first time in the literature, all unknowns agreeing with themselves in two systems within their mean errors. The above mean errors were obtained with the initial value $f = 0\cdot 73$. On my proposition K. Kozieł made the calculations also with $f = 0\cdot 50$, and discovered that there exists still another solution for all unknowns with the initial value f lying on the other side of the critical value $f = 0\cdot 662$. In this second solution $f = 0\cdot 60$ $\pm 0\cdot 05_5$; the mean error of one measured distance becomes $\pm 0''\cdot 575$, being therefore somewhat less than in the first solution $f = 0\cdot 71$. It remains for future investigations to decide on which side of the critical value $0\cdot 662$ lies the mechanical ellipticity f. The assumption often made by modern writers on the authority of Hayn that f lies in the vicinity of $0\cdot 7$ appears, according to K. Kozieł, to be deprived of a sufficient basis.

In his thorough dissertation (printed in *Acta Astronomica*) K. Kozieł has taken into account a new reduction of the measured position angles: for the convergency of hour circles. It consists of two parts, and amounts up to about 3′ in the Dorpat series, made in moderate declinations of the moon.

The great increase of the accuracy, equivalent to a three-fold or greater extension of the available observations, as well as clarifying of results obtained, when calculating the observations with the help of the Cracovian method, adds new arguments for the fresh reduction and investigation of the whole material of heliometer measures. One might then hope to obtain among other things a better insight into the problem of the arbitrary libration of the Moon, as well as to decide the new problem, which of the two solutions for f corresponds to the truth. These vast problems demand international co-operation, and Dr Kozieł would desire a resolution of the Committee in this respect, as well as a subvention of the I.A.U. for this work.

As regards the photographic observations of the figure of the lunar disk, provision must be taken to secure in the future not only the position angles, but also the exact value of the scale, in order to avoid systematic level-errors, like those producing the Yakovkin term.

Publications of the Libration Constants

Since the Stockholm meeting of the I.A.U. there have appeared two important publications: (i) 'Selenographische Koordinaten' by Hans Naumann (*Veröff. der Sternw. zu Leipzig*, Heft VII, 1939), and (ii) 'The Rotation and the Figure of the Moon', in two parts, by A. A. Yakovkin (*Publ. Obs. Engelh*, **21**, 1939, and **23**, 1945, in Russian). The results are summarized as follows:

	Leipzig publication Observer Hartwig, 1890–1915–(1922)	*Kasan publication* Observer Yakovkin, 1916–1931
λ	$-5°\,10'\,0'' \pm 11''$	$-5°\,10'\,25'' \pm 10''$
β	$-3\ 10\ 56 \pm 9$	$-3\ 10\ 26\ \pm 8$
h	$15\ 34{\cdot}2 \pm 0{\cdot}41$	$15\ 34{\cdot}06 \pm 0{\cdot}4$
I	$1\ 31\ 29 \pm 12$	$1\ 31\ 44 \pm 12$
f	$0{\cdot}71 \pm 0{\cdot}033$	$0{\cdot}68 \pm 0{\cdot}02$
	157 (from 235) observations, each consisting of 10–15 distances. Heliom. 184 mm. apert., 2·7 m. focal length.	251 observations, each consisting of 7–8 measured distances. Heliom. 106 mm. apert., 1·7 m. focal length.

All the mean errors given in the Leipzig publication are here multiplied by 4/3, owing to an erroneous formula used there.

The agreement of both values of f does not warrant this quantity, because of the possibility of the second solution for f, both authors having departed from the value of f in the same region; on f depend (feebly) all unknowns.

Yakovkin makes an attempt to determine the arbitrary libration in longitude by harmonic analysis based on the assumed value of the period, corresponding to $f=0{\cdot}72$. He obtains the amplitude $31'' \pm 12''$.

Naumann utilised only 59% of the observations of Hayn, who observed till 1922. The incomplete observations that he left out as inaccessible to treatment by the usual method of reduction, could be utilized by using the Cracovian method. The great value of his publication would be, according to Dr Kozieł, still much enhanced if it contained more particulars, and especially the moments of individual observations, enabling the reader to introduce some delicate corrections. Experience having shown that the same sets of observations are being reduced again and again, it is highly desirable that the libration-observations be published as fully as possible, giving all observed moments, and even the habits of the observer, especially relative to the position of the measured crater in the field of the eye-piece.

Yakovkin reduced his 1916–22 observations to Brown's radius of the Moon by applying to Hansen's radius the correction $1''{\cdot}50$, this being the difference between the two radii on January 1, 1923. A variable correction, from $1''{\cdot}42$ for the horizontal parallax $54'$ till $1''{\cdot}64$ for the horizontal parallax $62'$ would fit better.

Mme Chandon published (*C.R. Paris*, juin 1941) a renewed investigation of forty plates already discussed in the memoir of Puiseux (*Ann. Obs. Paris, Mém.* **32**, 1925). She had regard to the rough error indicated in *Rocz. Astr. Obs. Krak.* **5**, 168 (1928), but the results are incomparable in accuracy to the heliometer measures. The memoir of Puiseux remains to be revised. Meanwhile four of these plates were used by Th. Weimer, *C.R. Paris*, **226**, 559 (1948), in an attempt to determine the elongation of the Moon from the magnitude of the parallactic displacements of the craters.

Prof. Sundman prepared a memoir 'The Motion of the Sun and the Moon at the Solar eclipse of July 9, 1945'.

Yakovkin gave a semigraphical, successive-approximations method for predicting occultations, *Russian Astr. Journ.* **24**, 223 (1947). The same author constructed a machine for this purpose, ibid., 228 (1947).

A. Strzałkowski calculated the Moon's limb irregularities (to be published in *Acta Astronomica*) for the solar eclipse of May 20, 1947. This eclipse was indicated in 1928 by T. Banachiewicz as suitable for the geodetic linking of Africa and South America, e.g. in the *C.R Comm. Géod. Balt.* (séance à Berlin), pp. 161–4.

Some contrivances for the photo-electronic, physical registration of the occultations, even of faint stars, are being devised now at the Cracow Observatory.

F. Koebcke gives, in *Poznań Reprint*, **11** (1947), Cracovian formulae adapted to the method of Comrie, as well as that of Davidson, for computing the selenographic co-ordinates P and D of the point of the Moon's limb where the occultation takes place.

The close constancy of the selenographic longitude of Mösting A during the last 100 years, as deduced from the heliometer observations, shows that the duration of one rotation of the Moon equals the length of one revolution around the Earth within about $\frac{1}{10}$ sec. This quantity amounts to only one-ninetieth of its upper limit deduced by Bessel from the unchanged appearance of the Moon from ancient to modern times.

T. BANACHIEWICZ
President of the Commission

For the report of the meetings, which was received with considerable delay, see page 430.