BOOK REVIEWS

Exercises in Mathematics, by J. Bass. Academic Press, 1966. xii + 459 pages. \$14.75.

This book is intended primarily for students of applied mathematics, physics and engineering. The problems considered are taken from such topics as sequences, series, definite integrals, uniform convergence, Fourier series and integrals, Fourier and Laplace transforms, line integrals and multiple integrals, complex variables, conformal mapping, special functions of applied mathematics, elliptic integrals, ordinary differential equations, integral equations, partial differential equations, boundary value problems, differential geometry, linear algebra, matrix calculus, vector and tensor analysis and the calculus of variations.

Unfortunately, it is basically due to the fact that such a large and diverse number of topics were considered that the book is of very little practical use. The examples chosen are in almost all cases of a very specialized nature and consequently do not successfully illustrate the underlying theory. Furthermore, the introductory statements for each section about the theory used in the problems are much too brief. A further disadvantage for students of applied mathematics is the fact that very rarely do the problems involve actual physical or engineering situations.

In the preface the author expresses hesitations about publishing a book of this nature. These hesitations are indeed well founded.

Charles Roth, McGill Uiversity

The theory of splines and their applications, by J.H. Ahlberg, E.N. Nilson and J.L. Walsh. Academic Press, New York, 1967. viii + 284 pages. \$13.50.

During the last decade a large number of articles were published dealing with either the theoretic or the applied aspects of the spline function theory. The spline function in its simplest form (cubic polynomial spline) is a twice continuously differentiable function on an interval [a, b] with the property that for some subdivision of [a, b], $\mathbf{x}_0 = \mathbf{a} < \mathbf{x}_1 < \ldots < \mathbf{x}_n = \mathbf{b}$ the function reduces to a cubic polynomial between any two of the "junction" points \mathbf{x}_k , \mathbf{x}_{k+1} . The amazing fact (first proved by Holladay) about cubic splines is that given $\mathbf{a} = \mathbf{x}_0 < \ldots < \mathbf{x}_n = \mathbf{b}$ and reals $\mathbf{y}_1, \ldots, \mathbf{y}_n$ the minimum of $\int_{\mathbf{a}}^{\mathbf{b}} |\mathbf{f}''(\mathbf{x})|^2 d\mathbf{x}$ among all twice continuously differentiable functions f satisfying $f(\mathbf{x}_j) = \mathbf{y}_j$ (j = 0, 1, ..., n) is attained by the cubic spline passing through the points $(\mathbf{x}_i, \mathbf{y}_i)$ and having vanishing second

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derivatives at a and b.

The fact that splines (in the simplest cases, cubic splines) interpolating to a given function provide a very good approximation to that function and its derivatives makes splines useful for the applied mathematician and numerical analyst.

After the introduction (Chapter I) the authors devote Chapters II and III to an intensive development of the theory of cubic splines. Both the theoretical and numerical aspects are stressed.

Chapters IV and V deal with polynomial splines of higher degrees, their minimization and convergence properties. In Chapter VI generalized splines are investigated. Instead of "piecewise polynomials" the generalized splines are "piecewise solutions" of certain differential equations. Chapters VII and VIII carry over certain results of polynomial splines to the two dimensional case.

The book does not (and the subject being a living one, could not) include all the results and byproducts of the spline function theory.

For the further development of the subject the authors have carried out an important task of collecting the main and basic ideas and results and made them available in a concise and readable form.

The Bibliography seems to be a weak point of the book. It does not include all the relevant and available material and, unusually, it includes a large number of references to abstracts in the Notices of A.M.S. not yet published.

A. Meir, University of Alberta

<u>Plane geometry and its groups</u>, by Heinrich W. Guggenheimer. Holden-Day Inc., 1967. x + 288 pages. \$9.35.

In the past ten years the study of Geometric transformations has enjoyed a modest revival in interest. Geometry courses involving transformations are becoming more prevalent in university programmes, and there is strong interest in introducing some of these concepts into the high school curriculum (this has in fact been done in many places). Several books on geometry from a transformation poing of view have appeared in recent years. This book is one of them, and it is particularly directed to future high school teachers.

Specifically, the book is an introduction to Euclidean plane geometry, using transformations. The restriction is not to real Euclidean Geometry, but is rather to Euclidean Geometry over an ordered field which contains the square roots of all its positive elements (i.e. a geometry in which all ruler and compass constructions can be made). Thus no continuity axiom appears in the text.

The development rests on thirteen axioms, outlined in the first chapter. Subsequent chapters deal with isometries and groups of isometries (chapters 2 and 3), circles (chapter 4), metric geometry (chapter 5),

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