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Pulsar Electrodynamics — Pulsars and Puzzlers —

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Abstract. A gedanken experiment presented here provides basic understanding of how the pulsar magnetosphere operates. We discuss current issues about the electric-field acceleration along magnetic field lines and subsequent pair creation, and also about the pulsar wind problem. It is stressed that any local model, such as the inner gap model, the outer gap model and the pulsar wind model, must have free parameters to link it to other part of the magnetosphere.

1. Quiet Model

Jackson's gedanken experiment (Jackson 1976) is a good starting point to study the pulsar magnetosphere. Let us consider what would happen when a magnetized conducting sphere in vacuum is initially at rest and then gradually spins up. For definiteness of sign of charge, let us assume that the magnetic moment $\boldsymbol{\mu}_{\rm m}$ and the angular velocity $\boldsymbol{\Omega}_*$ are aligned (not counter-aligned), $\boldsymbol{\mu}_{\rm m} \cdot \boldsymbol{\Omega}_* > 0$.

The first order (in Ω_*) effect of the rotation is induction of the electric field, which is a quadrupolar field. The electric field outside of the star has a field-aligned component, $E_{||} = \mathbf{E} \cdot \mathbf{B}/|\mathbf{B}|$, and especially for the axisymmetric case, the electric field in the polar regions is nearly parallel to the magnetic field.

The second order (in Ω_*) effect is extraction of charged particles by the electric field. Fully developed extraction results in static clouds of the extracted charged particles (Rylov 1966, Krause-Polstorf & Michel 1985). Negative clouds purely of electrons form above the poles, and a positive cloud of ions forms an equatorial disc. Formation of the charged clouds reduces the electrostatic energy of the system. The condition $E_{||} = 0$ applies inside the clouds, and outside the clouds there is a vacuum with $E_{||} \neq 0$. The boundary, on which $E_{||} = 0$, is called the force-free surface. The net charge of the whole system is a free parameter of this model and is assumed positive.

This electrostatic model continues to apply even when the freedom in the distribution function, i.e., finite temperature, is taken into account (Neukirch, 1993), and when obliquity is taken into account (Thielheim & Wolfsteller 1994). It is the unique solution for the pulsar problem if the whole system is isolated in a vacuum with a positive net charge, provided that pair creation is absent. This model is 'quiet' from the viewpoint of pulsar physics.

2. Active System

There is a big potential difference between the positively and negatively charged clouds in the electrostatic model. The model becomes relevant to pulsars when a way is opened for electric current to flow across the potential drop.

One way to open the current circuit is by reducing the net charge to zero (Rylov 1979, Mestel et al. 1985). Another way is to have an MHD accelerator (the pulsar wind) through which the current flows (Shibata 1991).

In the former case, as the net charge is reduced to zero, the corotating electronic cloud expands to the light cylinder, where the particle velocity reaches the light speed c owing to the perpendicular component $\mathbf{E}_\perp$, which approaches the magnetic field strength $|\mathbf{B}|$ (the relativistic sling shot effect). The electrons achieve relativistic energies and lose power through emission of curvature gamma-photons. Thereby, the relativistic electrons with $\gamma \sim 10^7$ move across the magnetic field lines due to inertial and radiation reaction drifts. The associated dissipation of angular momentum beyond the light cylinder ensures satisfaction of torque-balance, which is discussed in §4. Eventually, the closed current circuit is developed. In this case, the rotation power is mostly emitted in the form of gamma-rays. Therefore, the dead pulsars, which are believed to be unable to create pairs by themselves, can become gamma-ray emitters.

In order to consider the latter wind case, let us continue the gedanken experiment. Suppose we put a cloud of dense pairs in the gap between the positive and negative clouds (fig. 1). Then the pair cloud becomes polarized with positive surface charge on the side of the polar cloud and with negative surface charge on the side of the equatorial cloud, so disrupting the force-free surface: the electric field between the polar cloud and the pair cloud and between the pair cloud and the equatorial cloud strengthens. On the other hand, when current flows in the pair plasma, $\mathbf{j} \times \mathbf{B}_\mathrm{p}$(poloidal field) drives the pairs to rotate, and $\mathbf{j} \times \mathbf{B}_\mathrm{t}$(toroidal field) drives the pairs out. If the strengthened electric field between the clouds accelerates particles with energy enough to create secondary pairs, then pairs are continuously supplied, and the $\mathbf{j} \times \mathbf{B}$-force acceleration of the pairs continues to form the pulsar wind. The circulating current I times the available voltage $V_\mathrm{L} \approx \mu_\mathrm{m}\Omega_*^2/c$ determines the pulsar power.

Similar pictures apply for oblique cases (fig. 2). Although this gedanken experiment is very much simplified, one can naturally expect two kinds of $E_\parallel$ accelerator associated with the out-going current and in-going current (the inner and outer accelerators), and the outflow of the pairs (the pulsar wind). These are all thought of as the essential ingredients of the magnetosphere. It must be noticed that the magnetosphere is a sophisticated system in the sense that a part of the electromotive force (e.m.f.) of the star is used to create pairs, and then the created pairs are accelerated by using the rest part of the e.m.f.

Let us go on to discuss local dynamics and then global dynamics.

3. Field-aligned Accelerator with Pairs

There is still no universally accepted model for the field-aligned accelerator which creates pairs. We need a dynamical model to explain how the field-aligned electric field is maintained when pairs are produced. The following is a summary of

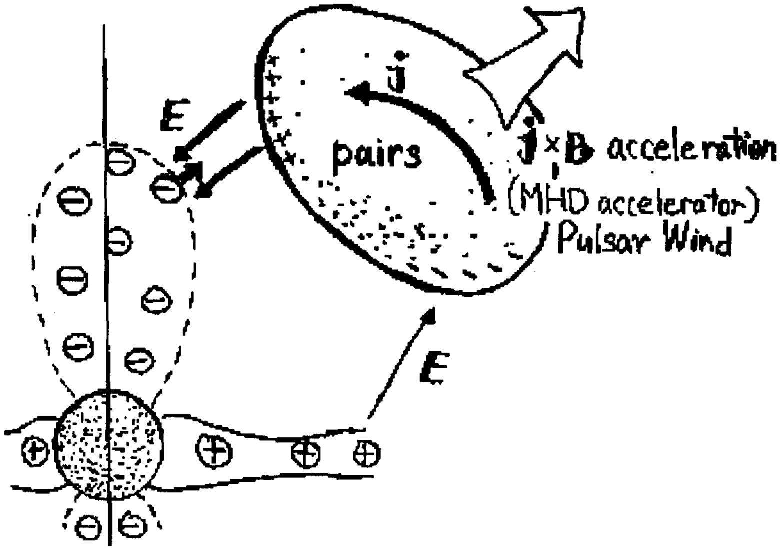

Figure 1. A gedanken experiment.

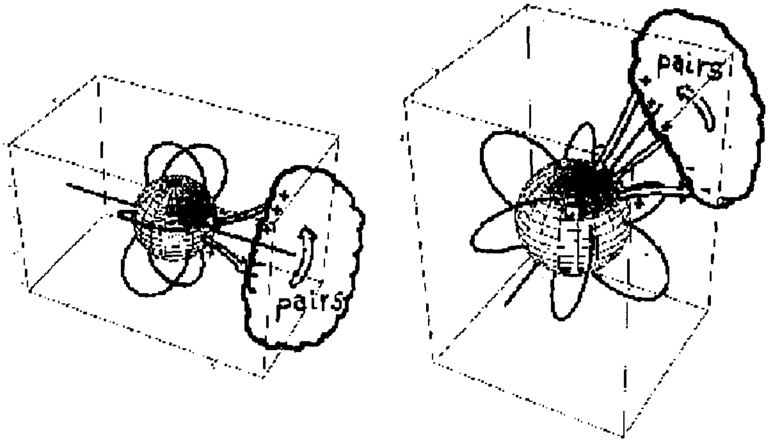

Figure 2. Oblique cases (90°, 30°).

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the basic processes to be taken into account.

(1) The basic electric field accelerating the particles is due to *the applied electromotive force* in the global current circuit, as seen in figs. 1 and 2, and not due to the local charge density.

(2) The charge density ρ_e must be, however, distributed properly so as to maintain the field-aligned electric field. The electric field is written in terms of the non-corotational electric potential Φ: $\mathbf{E} = -(\mathbf{\Omega}_* \times \mathbf{r}) \times \mathbf{B}/c - \nabla\Phi$, and it obeys the Poisson equation,

$$-\nabla^2\Phi = 4\pi(\rho_e - \rho_0), \tag{1}$$

where $\rho_0 \equiv -\mathbf{\Omega}_* \cdot \mathbf{B}/2\pi c$ is the *local* Goldreich-Julian charge density. Imposing the boundary condition $E_{||} = 0$ on the stellar surface, (1) implies that Φ increases above the stellar surface when $\rho_e - \rho_0 < 0$ (or $|\rho_e| > |\rho_0|$). This increase of Φ implies acceleration of electrons. The increase of Φ takes place spontaneously on field lines curving *away* from the rotation axis (Mestel 1981). If the velocity of the electronic flow is much less than the light velocity, the negative charge density can be much higher than the local Goldreich-Julian charge density. In this case, the increase of Φ also takes place on field lines curving *toward* the rotation axis.

(3) 3-dimensional effect* is equivalent to a negative charge, which drives the acceleration.

There are two effects to screen out the electric field.

(4) One effect is the polarization of the created pairs. The pairs are neutral at creation, but as the positrons are decelerated in the electric field, a positive net charge appears. This positive charge can screen out the electric field, and is particularly effective when the positrons become non-relativistic as they are reflected by $E_{||}$. My recent work suggests that this kind of screening is dynamically possible, but an uncomfortably large value of the pair creation rate is required (Shibata 1996). In this model, however, no trapped particles are assumed, so future modification may resolve this difficulty.

(5) The local Goldreich-Julian charge density ρ_0 provides a positive screening charge density if it is larger than the real charge density, $|\rho_0| > |\rho_e|$. This works on the 'toward'-field lines (Fawley, Arons & Scharlemann 1977, Scharlemann, Arons & Fawley 1978) †.

(6) Some of the positrons may be reflected backward, so that the outflow of pairs has an excess of electrons. This excess provides a screening charge density on 'toward' field lines (Arons & Scharlemann 1979).

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*(1) may be approximated by $-\nabla_{||}^2\Phi = -\Phi/D_\perp^2 + 4\pi(\rho_e - \rho_0)$, where $-\Phi/D_\perp^2$ (< 0) represents the 3-D effect. The potential distribution becomes a V-shaped potential like that in the terrestrial auroral arc.

†The driving terms for this case are provided by the negative charge on the stellar surface and 3-D term (i.e., $\Phi = 0$ on the walls surrounding the flow).

(7) The reflected positrons which flow toward the star will produce pairs somewhere between the accelerating region and the stellar surface. This is another pair creation front. The next generation of reflected electrons may flow upward. If the reflected flux is significant, the current density can be much larger than the Goldreich-Julian value. The positron flux onto the star can be detected due to the polar cap heating ‡.

Local dynamics including the above effects with adjustable free parameters such as the current intensity is an issue for the future.

4. Torque Balance Problem

An important fraction of the released energy at the field-aligned accelerator is radiated away in gamma-rays. We must notice here that the accelerator emits energy but emits virtually no angular momentum. To understand what this fact demands, let us consider the unipolar inductor model. In fig.3, the rotating disc in the magnetic field produces e.m.f, and lights the discharge tube. When the current flows, the wire feels force so as to corotate with the rotating disc. In order to extract the rotation energy of the disc continuously, the circuit must be fixed in space. This force is provided by emission of photons or relativistic particles with angular momentum of $h\nu\varpi/c$ or $mc\gamma\varpi$, respectively, where ϖ is the length of the torque lever arm. Because the angular momentum loss rate $\dot{\mathcal{L}}$ must be the energy loss rate $\dot{\mathcal{E}}$ divided by Ω_* ($\dot{\mathcal{E}} = \Im\Omega_*\dot{\Omega}_* = \Omega_*\dot{\mathcal{L}}$, where $\Im$ is the moment of inertia of the star), ϖ must be the light cylinder radius c/Ω_* on average. Therefore, emission within the light cylinder brings about a deficit in the loss of angular momentum, which must be compensated by an efficient loss beyond the light cylinder (Shibata 1994, 1995). We return to this point in §6. As a corollary, we see that typical size of the current circuit in figs.1 & 2 is the light radius c/Ω_*, and the available voltage is of order of $V_{\rm L} \approx \mu_{\rm m}\Omega_*^2/c$.

5. Relativistic MHD Accelerator: Pulsar Wind

The extracted and created particles flow out due to the centrifugal slingshot effect. This is known as the pulsar wind. The power of the wind is shared between the Poynting flux $\dot{\mathcal{E}}_{\rm EM}$ and the kinetic energy flux of the particles $\dot{\mathcal{E}}_{\rm KE}$. The ratio is denoted by $\sigma = \dot{\mathcal{E}}_{\rm EM}/\dot{\mathcal{E}}_{\rm KE}$. Observations of the Crab Nebula suggest that the wind is asymptotically kinetic energy dominant, $\sigma \ll 1$.

The pulsar wind may be well-described by the MHD system of equations. For the axisymmetric case, it can be separated into two parts. One is a set of algebraic equations made up of three conservation laws along field lines and the isorotation law. These are called the field-aligned equations. The other equation represents force balance across the magnetic field lines to determine the poloidal field structure. This trans-field equation is a mixed elliptical-hyperbolic type of partial differential equation.

‡A measure of the polar cap heating is $V_{\rm l}(\mu_{\rm m}\Omega_*^2/c) = 4.3 \times 10^{30}\mu_{30}P^{-2}(V_{\rm l}/10^{12}{\rm V})$erg/sec, where $V_{\rm l}$ is the voltage of the accelerator.

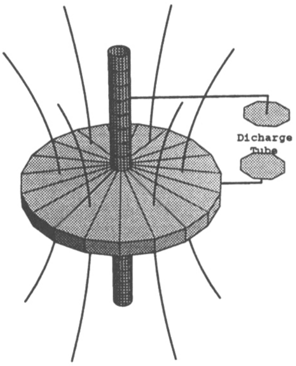

Figure 3. Unipolar inductor model.

The field-aligned equations can be solved if the poloidal field structure is given. The field-aligned equations have only one parameter concerning the poloidal field structure, which is $\hat{B} \equiv B_{\rm p}\varpi^2$, where $B_{\rm p}$ is the poloidal field strength, and ϖ is the the axial distance. For a radial field, $\hat{B}$ is constant along each field line. As shown by Takahashi (in this issue), when $\hat{B}$ decreases with the distance, the fast critical point appears near the light cylinder. Beyond the fast critical surface, the kinetic energy increases in accordance with the decrease of $\hat{B}$ (Holzer 1977, Camenzind 1989, Chiueh, Li, & Begelman 1991). Asymptotically, we have the relation $\sigma_\infty \approx \hat{B}_\infty/\hat{B}_{\rm fast}$ (Begelman & Li 1994). If $\hat{B} \to 0$, $\sigma \to 0$. It has been shown that in the asymptotic region there is a kinetic dominant solution with $\hat{B} \to 0$ of the trans-field equation (Chiueh, Li, & Begelman 1991, Begelman & Li 1994). How the asymptotic solution is connected to the inner region is the next problem.

Alternatively, if the field-aligned flow is given, then the trans-field equation can be solved. In practice, the Fourier transform with respect to z of the cylindrical coordinates is useful (Mestel & Price 1992). Then we obtain a second order ordinary differential equation for each Fourier component. Putting a dipole at the center, and imposing regularity on the Alfvén surface, where the equation is singular, we obtain the unique solution between the center and the Alfvén surface, and then extrapolate the solution to infinity. However, Mestel & Shibata (1994) are faced with break down of the solution within two light-cylinder radii (the Fourier components grow up exponentially). They introduce a dissipation layer, through which some energy and angular momentum is extracted, and thereby a solution extending to infinity is obtained.

For this solution, the geometrical factor $\hat{B}$ is found to decrease by factor of 0.4 as the flow goes from $\varpi = c/\Omega_*$ to $\varpi = 4(c/\Omega_*)$, which suggests that the Poynting-flux-dominant flow may be accelerated roughly up to equipartition. A further gradual decrease of $\hat{B}$, say decrease by $\sim 10^{-2}$ in 10^7 light-cylinder radii, may account for the kinetic-energy-dominant wind.

In the Mestel-Shibata model, however, the shape of the Alfvén surface and the field-aligned flow are assumed. The Alfvén surface may adjust its shape, and simultaneously the parameters of the field-aligned flow may adjust themselves, so that a dissipation-free solution may be possible. However, our form of the trans-field equation does not explicitly suggests how the blowup of the Fourier components can be avoided. How the Alfvén and fast surfaces are determined, and how the flow behaves in between the two critical surfaces is a current issue.

Another future issue must be the magnetic neutral sheets and wind of oblique rotators, but I do not discuss about this now (see Melatos in this issue).

6. Global Treatment

Finally let us come back to the global model.

The deficit in the loss of angular momentum due to the field-aligned accelerators can be compensated by the wind. The mechanism is as follows.

The potential difference across the magnetic field lines in the wind domain, which originates in the e.m.f on the neutron star surface, is reduced because of the potential drop in the field-aligned accelerator. In the axisymmetric model, the voltage drop ΔV across the magnetic flux $\Delta\Psi$ is measured by the angular velocity of the field $\langle\Omega_{\rm F}\rangle = 2\pi c\Delta V/\Delta\Psi$, which is now reduced. As a result, the Alfvén radius ($\sim c/\Omega_F$) moves outward beyond the light cylinder, i.e., the lever arm gets longer, and then angular momentum is lost efficiently to compensate the deficit loss.

At the same time, the reduction of the applied voltage to the wind, causes two effects:

(1) the current circulating in the wind seems to be reduced correspondingly from the Goldreich-Julian value $\approx \mu_{\rm m}\Omega_*^2/c^2$ to $\approx \mu_{\rm m}\langle\Omega_{\rm F}^2\rangle/c^2$, and

(2) the magnetic flux opened by the wind seems to be reduced to leave some closed flux beyond the light cylinder.

If we make this inference, an accelerator with a field-aligned electric field must appear around the light cylinder[§]. According to this global model, we can derive the evolution of the pulsed luminosity and wind luminosity (Shibata 1995).

It should be emphasized that if there is an accelerator well within the light cylinder, then this inner accelerator must affect the dynamics of the outer magnetospheric accelerator and the wind, i.e., they are linked to each other. If one

[§]If the closed flux beyond the light cylinder is filled with plasmas and $E_\parallel = 0$, then the plasma is forced to corotate beyond the light cylinder, so violating special relativity.

constructs a local model, the model must have free parameters to adjust to other parts of the magnetosphere or to fit observational data.

References

Arons, J., Scharlemann, E.T., 1979, ApJ, 231, 854

Begelman, M. C., Li, Z.-Y., 1994, ApJ, 426, 269

Camenzind, M. 1989, in Accretion Disks and Magnetic Fields in Astrophysics, ed. G. Belvedere (Dordrecht, Kluwer), p 129

Chiueh, T., Li, Z.-Y., Begelman, M.C., 1991, ApJ, 377, 462

Fawley, W.M., Arons, J., Scharlemann, E.T., 1977, ApJ, 217, 227

Holzer, T.E., 1977, JGR, 82, 23

Jackson, E.A., 1976, ApJ, 206, 831

Krause-Polstorf, J., Michel, F.C., 1985, AA, 144, 72-80

Mestel, L., 1981, IAU Symposium 95, ed W. Sieber & R. Weilebinski (Dordrecht, Reidel), 9

Mestel, L., Robertson, J.A., Wang, Y.-M., Westfold, K.C. 1985. MNRAS, 217, 443

Mestel, L., Price, M.H.L., 1992, MNRAS, 254, 355

Mestel, L., Shibata, S., 1995, MNRAS, 271, 621

Neukirch, T., 1993, AA, 274, 319-329

Rylov, Y.A., 1966, AAS, 51, 59

Rylov, Y.A., 1979, AAS, 66, 401

Scharlemann, E.T., Arons, J., Fawley, W.M., 1978, ApJ, 222, 297

Shibata, S., 1991 ApJ., 378, 239

Shibata, S., 1994, MNRAS, 269, 191

Shibata, S., 1995, MNRAS, 276, 537

Shibata, S., 1996, *in preparation*

Thielheim, K.O., Wolfsteller, H., 1994, ApJ, 431, 718