navigation. In cases, however, when risk of collision exists, they shall comply with these Rules.

(b). By night they shall carry, in addition to the lights prescribed for other vessels of their class and tonnage,—' some specified light such as the flashing amber light referred to by Lieutenant Commander J. H. Hardwick.

A Theoretical Note on System Malfunctioning

from J. B. Parker

THE problem of estimating equipment serviceability when the apparatus consists of a large number of individual components (possibly with standby features) is likely to be of increased interest now that navigational thought is being concentrated quite as much on the design of instruments the navigator uses as on the problems of finding position at sea or in the air. The resulting exercise in combining probabilities of component failure is a fairly straightforward one but in view of the accelerating interest in the matter (see, for example, Cluley's article in this *Journal*, 15, 387) a brief systematic treatment might be of interest to workers in these fields.

The basic concept is the failure rate of a component. It is in the specification and interpretation of this quantity that a possible pitfall lies. A rate implies time and one must guard against interpreting failure rates as if they were probabilities. A simple example will suffice. If a failure rate of say 10^{-3} per hour is specified, does it mean that after 500 hours the equipment has a 50 per cent survival chance? Is it bound to be inoperative after 1000 hours?

The answer is no and to interpret the failure rate it is of interest to consider the analogy of a radioactive substance. This consists of a large number of atoms each of which decays with known 'half life'. When the 'half life' is spent, half the material will have decayed but it is not true that after two 'half-lives' all the material will have been transformed.

Let us imagine an equipment with failure rate α (alternatively, a radioactive substance with decay rate α). At time t we imagine a large number of identical equipments (or atoms), $N(t)$. Then, after a short duration of time Δt, a proportion $\alpha\Delta t$ of the product will have become unserviceable. Thus

$$N(t+\Delta t) = N(t)\{1-\alpha\Delta t\}$$

As Δt shrinks to zero this equation leads to the differential equation

$$\frac{dN(t)}{dt} = -\alpha N(t)$$

with solution

$$N(t) = N(o) \exp -\alpha t$$

Thus however long the duration of time, there will always be some chance, $N(t)/N(o) = \exp -\alpha t$, of the equipment continuing to function correctly.

The conclusion is that in mathematical problems where failure rates are of

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concern, the actual probability of failure is not the product of the failure rate α and the duration t, but is related to it by

$$\begin{array}{c}\text{Prob. of failure}\\ \text{by time } t\end{array} = 1 - N(t)/N(o) = 1 - \exp\ -\alpha t$$

In many practical applications (e.g. Cluley's example) αt is so small that the exponential can be effectively replaced by $(1 - \alpha t)$, giving

$$\begin{array}{c}\text{Prob. of failure}\\ \text{by time } t\end{array} = 1 - (1 - \alpha t) = \alpha t$$

If now one has a whole host of components, n in number, with failure rates $\alpha_1, \alpha_2, \ldots \alpha_n$, then for the equipment to remain serviceable after time t, all n components must survive. The chance that the i^{th} component survives is

$$\exp\ -\alpha_i t$$

and the chance that all survive is the product of all such chances, i.e.

$$\Pi \exp\ -\alpha_i t = \exp\ -t\Sigma\alpha_i,$$

the corresponding failure probability being $1 - \exp\ -t\Sigma\alpha_i \simeq t\Sigma\alpha_i$ when $t\Sigma\alpha_i$ is very small. Note the second pitfall in the following argument:

'The chance of component failure is $1 - \exp\ -\alpha_i t$. Since there are n components the chance that the whole system fails is $\Pi\{1 - \exp\ -\alpha_i t\}$'.

This is wrong. The result is in fact the probability that *all* components fail; it is only necessary that *at least one* should fail for the equipment to be unsatisfactory.

We now pass on to a slightly more complicated example in the combination of probabilities. This is Cluley's second example where he considers the case of duplication of components. Consider first a single component with failure rate α_i. As we have seen, the probability of it being still serviceable after time t is $\exp - t\alpha_i$. If there is duplication the equipment can only fail if both duplicates are out of action by time t. The chance of this is obviously

$$(1 - \exp\ -t\alpha_i)^2$$

Suppose now there are n components. It is theoretically wrong (though in practice often quite accurate enough) to write, for the chance of overall failure

$$\sum_{i=1}^{n} (1 - \exp\ -t\alpha_i)^2 \sim t^2 \sum_{i=1}^{n} \alpha_i{}^2$$

since if t is large enough this will exceed unity, which is wrong. The exact chance is obtained as follows:

'For the i^{th} component, the chance that at least one of the duplicated pair survives is

$$1 - (1 - \exp\ -t\alpha_i)^2$$

For the equipment as a whole to be in service at time t, no duplicated pair can both fail, so that the chance of survival is

$$\prod_{i=1}^{n} \left[1 - (1 - \exp\ -t\alpha_i)^2\right]'.$$

When $nt \max_{(i)} \alpha_i$ is very small this takes the approximate form

$$\prod_{i=1}^{n} (1 - t^2\alpha_i{}^2) = 1 - nt^2\alpha^2$$

if all the α's are the same.

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This corresponds exactly to Cluley's case P_c (Appendix to his article). For the fault rates and times of interest in his application the approximations are amply adequate. But a formulation of the more general theory (which is of course well known) is not without interest and may very well turn out to be of practical value in other navigational problems.

The Sextant and Precision Celestial Navigation

from Captain C. H. Cotter

WHEREAS it is doubtless true that the two largest contributions to the errors of sights at sea are those referred to by Mr. Sadler in his Prefatory Note to Captain H. H. Shufeldt's paper on Precision Celestial Navigation Experiments[1], it cannot, I think, be gainsaid that the principal avoidable error in sights is due to faulty use of the sextant, especially in respect of collimation error.

It is important, when using a sextant for measuring the altitude of a star, to ensure that the direct ray from the point on the visible horizon vertically below the star, and the three parts of the doubly-reflected, or zig-zag, ray from the star, lie in a common vertical plane. If this is not the case when taking a sight, the plane of the instrument—that is the plane on which lies the sextant arc—will, in all likelihood, be held out of the vertical plane; and the measured angle will be larger than the required angle.

Three conditions are necessary for the three parts of the zig-zag ray from a star to lie in the same plane. First, the index mirror must be perpendicular to the plane of the instrument; second, the horizon glass must be perpendicular to the plane of the instrument; and third, the line of sight must be parallel to the plane of the instrument. If the first condition is not satisfied, the sextant possesses perpendicularity error: if the second condition is not satisfied, side error exists. These errors, if they are present, may readily be detected, and easily removed by making the first and second adjustments.

When adjusting a sextant; after making the first and second adjustments, the third adjustment—if necessary—is made. This involves slewing the horizon glass to bring it parallel to the index mirror, having first set the pointer on the index bar coincident with the zero mark on the sextant arc. The fourth adjustment, if necessary, should then be made to ensure that the axis of the telescope is parallel to the plane of the instrument. It is not until the four adjustments are made that the sextant is fit for use. Moreover, having made the adjustments once, it is not likely that they will have to be made again, provided that the sextant is handled in a way befitting of a scientific instrument of precision.

If the axis of the telescope is not parallel to the plane of the sextant, collimation error may result in all observed altitudes. It is commonly believed that collimation error is due solely to the axis of the sextant telescope not being parallel to the plane of the instrument. Collimation error may be due to one, or both, of two causes. One of these is faulty housing of the telescope; a cause

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