Theory of approximation of functions of a real variable, by A. F. Timan. (English translation). Hindustan Publishing Corporation, Delhi 6, India. vii + 631 pages. U.S. $10.

The present book is an English translation of A.F. Timan's book (in Russian) on approximation of functions of a real variable. An earlier translation into English of the same book by the Pergamon Press has already been in the market since 1963. However, the present translation does not differ in content essentially from the Pergamon Press translation, while the price of the present translation is nearly half the earlier one.

The reviewer notes a number of wrong spellings of names of mathematicians in the text. Thus on pages 578 and 580, "Marcinkiewicz" is distorted out of shape and on page 578, "Tomich" should read "Tomic". However, the new translation is a welcome addition to the literature partly because it is more reasonably priced.

A. Sharma, University of Alberta

Approximations of functions, by G.G. Lorentz (Syracuse University). Holt, Rinehart and Winston, New York, 1966. ix + 188 pages. U.S. $5.95.

This is perhaps the second book by the author, who is known for his first book on "Bernstein Polynomials", which has stood in the field alone for several years. The present work differs from other books in the market on the subject (of which 21 are listed in the bibliography) in its emphasis on degree of approximation and in the inclusion of a detailed treatment of entropy and its applications.

The author's aim has been, in his own words, 'to write an easily accessible book on approximation of functions, that is simple and without unnecessary details and is also complete enough to include the main results of the theory, including some recent ones'. To a great extent the author has succeeded in this objective, but in avoiding unnecessary details, the book has become a little inaccessible to the reader.

The book is divided into 11 chapters. 1) Possibility of approximation; 2) Polynomials of best approximation; 3) Properties of polynomials and moduli of continuity; 4) The degree of approximation by trigonometric polynomials; 5) The degree of approximation by algebraic polynomials; 6) Approximation by rational functions; 7) Approximation by linear polynomial operators; 8) Approximation of classes of functions; 9) Widths; 10) Entropy; 11) Representation of functions of several variables by functions of one variable.

The book contains considerable interesting material usually not available in English books. The treatment is without unnecessary trappings and is often succinct. A welcome feature of the book is a detailed treatment of saturation of approximation methods. More details on Newman's result on rational approximation and its subsequent extensions by Szüs and Turán and Szabados would add to the value of the book. Also the sections entitled "Notes and Problems" in each chapter could be amplified for the benefit of the reader in the next edition.

There are a few minor printing errors. For example, page 21, line 2↑ 'A' should read 'A_o'; page 40, line 13↓, $\|T_n\|$ should read $n\|T_n\|$; page 41, line 6↑ 'ist' should read 'its'; page 52, line 7↓ needs correction. On the whole, the book is a welcome addition to the literature. A comprehensive bibliography particularly on Russian literature adds to the value of the book. It is to be hoped that the book

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will soon be in every college library. The printing is pleasing and the book is moderately priced.

A. Sharma, University of Alberta

Calculus of variations and partial differential equations of the first order. Part II : Calculus of variations, by C. Carathéodory. (Translated from the German by Robert B. Dean). Holden-Day Inc., San Francisco, 1967, xvi + 398 pages. U.S. $10.75.

The original (1935) German version of this work consists of two parts but was published as a single volume. The long-awaited translation follows the trend of a more recent - essentially unchanged - edition; the first translated volume, entitled "Partial Differential Equations of the First Order", appeared in 1966.

The underlying philosophy of the second volume can best be described by briefly summarizing Carathéodory's views on the overall trends in the development of the calculus of variations at the time of writing. Three distinct approaches are recognized : the first is the variational calculus, begun by Lagrange, which is now a part of the tensor calculus; the second is the theory of Tonelli, which reveals the more delicate relationships between the minimum problem and set theory; and thirdly, the trend originated by Euler, which depends on the close association between the calculus of variations on the one hand, and the theory of differential equations on the other, and which is accordingly oriented towards differential geometry and physical applications. It is the latter trend with which the book under review is chiefly concerned, one of its main objectives being the inclusion of the theory of Weierstrass within this context.

After an elementary, but extremely rigorous survey of the theory of extreme values of functions of several variables, in particular of quadratic forms subject to constraints, the fundamentals of the simplest problem in the calculus of variations are treated from a local point of view. Already at this stage the book departs significantly from the well-trodden path of its many predecessors : the treatment depends on Carathéodory's brilliant use of the concept of equivalent integrals, which not only leads directly to the so-called fundamental equations of the calculus of variations, but which also suggests the immediate introduction of appropriate canonical variables. (It is, perhaps, worth remarking that later writers have referred to this approach as "der Königsweg von Carathéodory".) A subsequent chapter deals with the corresponding theory for parameter-invariant problems (theory of Weierstrass); again particular stress is laid on an essentially new canonical formalism and a certain class of Hamiltonian functions. One of the most remarkable features of the book is the thoroughness with which many diverse and intrinisically important examples are treated : this is done in particular in the subsequent chapter on positive definite variational problems, in which also differential-geometric concepts such as the indicatrix, figuratrix, transversality and families of geodesically equidistant hypersurfaces are described. The next chapter is devoted to quadratic variational problems and consequently to the theory of the second variation and the accessory problem. Again, the corresponding canonical formalism plays a special role in the construction of the relevant fields of extremals. Naturally, this involves a detailed discussion of focal points, focal surfaces, conjugate points and the envelope theorems.

Up to this stage the book is primarily concerned with curves of which it is merely expected that sufficiently small subarcs display the required extremum properties. The general boundary value problem, according to which a curve possessing these attributes is required to pass through two given fixed points, is therefore now investigated in detail, both locally and globally. By means of

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