Invex functions and their application to mathematical programming

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A number of generalisations of convexity have previously been proposed in order to establish the weakest conditions required for optimality results and duality theorems in nonlinear programming. This thesis is concerned with invex functions, which are those differentiable functions $f : C \to \mathbb{R}$, C an open set in $\mathbb{R}^n$, for which there exists some function $\eta : C \times C \to \mathbb{R}^n$ such that

$$f(x) - f(u) \geqslant \eta(x, u)^T \nabla f(u) \quad \text{for all} \quad x, u \in C.$$

Such functions have the property that all stationary points are global minimisers. Chapter Two is a discussion of characterisations of invexity and conditions for several functions to be invex with respect to a common η or kernel. Comparisons with other forms of generalised convexity are made, and there are sections on the continuity of the kernel and the extension of invexity to nondifferentiable functions.

In Chapter Three, the role of invexity in the sufficiency of the Kuhn-Tucker conditions, and in Wolfe, Mond-Weir, and symmetric duality, is demonstrated. This is followed by an application to game theory: a constrained game is shown to be equivalent to a pair of symmetric dual programs under pseudo-invexity assumptions, and the results are adapted to constrained ratio games.

The next chapter is devoted to the extension of invexity to complex space, duality theorems for complex programs with polyhedral cone constraints, and equivalent real programs. The concept of invexity is carried on for functionals in Chapter Five, and sufficiency and duality results are established for variational and control problems. In the final chapter, invexity is used to obtain duality theorems for a class of finite and infinite dimensional optimisation problems with a nondifferentiable penalty term in the objective function.

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