Accretion Phenomena and Related Outflows, IAU Colloquium 163
ASP Conference Series, Vol. 121, 1997
D.T. Wickramasinghe, L. Ferrario, and G.V. Bicknell, eds.

Disc Accretion Onto Magnetic Stars: Slow Rotator And Propeller

Jianke Li and Dayal T. Wickramasinghe

ANUATC, and Department of Mathematics, The Australian National University, ACT 0200, Australia

Abstract. We study two magnetised accretion – outflow problems: the slow rotator and the propeller. In the former case, we show that the funnel flow does not result in the accretion of a significant amount of matter angular momentum, contrary to what is assumed in the standard Ghosh and Lamb model. Implications of this finding are investigated, particularly in relation to the fastness parameter. We also argue that magnetospheric mass ejection (the magnetospheric streamer model) is a viable mechanism for the ejection of mass and angular momentum from a magnetic star-disc system in the propeller regime.

1. Introduction

The rapid inward increase of a dipolar stellar magnetic field may become important to enable the field to balance the plasma matter pressure or ram pressure outside a star. When this balance is achieved, two regions are defined: the magnetosphere, which is magnetic field dominated, and the plasma region, which is matter dominated. A so-called magnetospheric radius or surface is introduced which characterises the boundary between these two regions. In some cases, the magnetospheric radius is named as "Alfvén surface" (e.g., Lipunov 1992; Longair 1994), but one must make a clear distinction between this radius, and the conventionally defined radius of the true Alfvén surface (e.g., Weber & Davis 1967; Mestel 1968).

The pressure balance between matter and magnetosphere is important for a basic understanding of the matter-magnetic field interaction, but does not on its own provide insight into the processes by which matter and angular momentum are transfered to the star. The matter in the accretion disc is not a perfect conductor, because of various instabilities (e.g, Wang & Robinson 1985), and the matter will mix with the magnetic fields so that in the accretion process there may be flow across field lines. In fact, it is this aspect of the accretion process that is of fundamental interest. The new equilibrium, if it exists, must therefore be a dynamic equilibrium. It is clear that a class of interesting problem would be the study of this dynamical equilibrium and its evolution with time.

There are difficulties in treating the full accretion problem in any detail because the physics is inherently complicated. Early models have therefore been based on simplifying assumptions to make the problem tractable, and often different boundary conditions are selected by different groups, resulting in a

https://doi.org/10.1017/S0252921100042706 Published online by Cambridge University Press

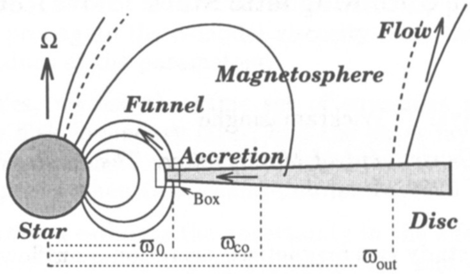

Figure 1. An ideal star - funnel - disc system.

wide range of models which are difficult to compare. It is fair to say that our understanding of the details of the process of accretion is at best rudimentary at the present time.

In the following discussions, we concentrate on one particular aspect of the problem, namely the angular momentum issue. Here we believe the physics is clear and leads to unambiguous predictions almost independently of uncertainties on other aspects of the problem. We first summarise our current understanding about the funnel flow and the stellar boundary condition, and then show that the earlier understanding about the funnel torque, as based on the standard model of Ghosh & Lamb (1979a,b; thereafter GL), is inconsistent with the physics of flow in the funnel, and also the stellar boundary condition. Therefore, we conclude that the rate of accretion of angular momentum (AM) due to funnel flow (the matter torque term) as given by the standard model is incorrect. We then consider some of the implications of this discovery and demonstrate some theoretical predictions. Finally we present a steady propeller model which allows us to estimate the magnetospheric mass and angular momentum ejection in a simple manner.

2. Magnetised funnel flow

Consider a slow rotator whose inner disc is truncated by the stellar magnetosphere at a radius smaller than the corotation radius ϖ_{co}, as shown in Fig. 1. We introduce axisymmetry and assume that the matter which follows the field lines can be described by ideal MHD equations. We assume also that the accretion is quasi-steady as compared to the evolutionary time scale of the system. Using these assumptions, and by separating the velocity and magnetic field into poloidal(subscript p) and toroidal (subscript ϕ) components $\mathbf{v} = \mathbf{v}_p + \Omega\varpi\mathbf{e}_\phi$ and $\mathbf{B} = \mathbf{B}_p + B_\phi\mathbf{e}_\phi$, the funnel flow can be described by the integral of the MHD equations along a field line or a poloidal field line:

$$0 = \mathbf{v}_p \times \mathbf{B}_p, \qquad (1)$$

$$\eta = \frac{v_p\rho}{B_p}, \qquad (2)$$

https://doi.org/10.1017/S0252921100042706 Published online by Cambridge University Press

$$\alpha = \Omega - \frac{B_\phi v_p}{B_p \varpi}, \tag{3}$$

$$-\frac{\beta}{4\pi} = -\frac{B_\phi \varpi}{4\pi} + \eta \varpi^2 \Omega, \tag{4}$$

$$\mu = \frac{1}{2} v_p^2 + \int \frac{d\,P}{\rho} - \frac{GM}{r} + \frac{1}{2}\Omega^2 \varpi^2 - \alpha \Omega \varpi^2, \tag{5}$$

where Ω is the angular velocity and ϖ is the radial distance in the cylindrical coordinate system, η, α, β and μ are constants along a field line, and other symbols have their usual meanings (see Li 96, thereafter L96; also Mestel 1968; Ghosh, Lamb & Pethick 1977). The physical interpretation of these constants is (see L96): η represents the mass transport rate per unit magnetic flux, α is a constant with the dimension of angular velocity (referred to as the angular velocity of the flux tube), $-\beta/4\pi$ represents the AM outflow rate per unit magnetic flux, and μ is related to the total energy per unit mass in the rotating frame (with angular velocity α). In the accretion problem, as shown in Fig. 1, $\eta < 0$, and if the funnel carries a significant matter AM onto the star $-\beta/4\pi \simeq \eta \varpi_d^2 \Omega_d < 0$, where the subscript "$d$" denotes the funnel end at the disc.

The constant α obtained by integrating the induction equation must exactly equal the stellar rotation rate if the star is a perfect conductor (Li, Wickramasinghe and Rüdiger 1996, thereafter LWR), or $\Omega_s = \alpha$, since the electric field must be tangentially continuous on the stellar surface as the flow connects to the star.

The funnel flow links the disc and the stellar surface, so that the boundary conditions on the disc cannot be prescribed at will. The funnel flow is expected to be supersonic close to the stellar surface due to the dominant gravitational pull along the magnetic field lines. Thus we assume that a standing shock will form near the stellar surface, and then the shock sets up an upstream boundary for the funnel flow close to the star. The shock is expected to be thin and the postshock region ideal, so we conclude that the shock cannot be fastmagnetosonic because otherwise the flux tube corresponds to double Alfvénic points as defined by $-\beta/4\pi = \eta\alpha\varpi_A^2$. Here an Alfvén point is located at ϖ_A and means that the poloidal flow velocity equals the poloidal Alfvén velocity (cf L96; Ghosh et al. 1977). So naturally the initial funnel flow is everywhere sub-Alfvénic, and we therefore consider only a slow accretion shock (cf. LWR).

By (3) to (5), the AM flux can be expressed as (LWR)

$$-\frac{\beta}{4\pi} = -[\frac{\varpi_d^2 B_p(\varpi_d)}{4\pi|v_1|}](\Omega_d - \Omega_1) + [\varpi_d^2]\ \eta\ (\frac{\varpi_1^2}{\varpi_d^2})\Omega_1 - [\varpi_d^2 S_1]\ \eta\Omega_d. \tag{6}$$

where subscript "1" denotes upstream shock boundary and "2" downstream shock boundary (Figure. 1), D is the square of the Alfvén Mach number, and $S \simeq v_d/v$. We will demonstrate later on that Ω_1 does not differ significantly from the stellar rotation Ω_s (or α), and accordingly we may write (LWR)

$$\Omega_d \simeq (1 - D_d)^{-1}\Omega_s \simeq (1 - D_d)^{-1}\alpha. \tag{7}$$

Since the funnel base is typically far away from the stellar surface we would usually find that only the first term in (6) is significant. Considering only this term, it is clear that a large v_1 reduces the AM flux. The toroidal magnetic field at the shock surface is given by:

$$-\frac{\beta}{4\pi} \simeq -\frac{B_{\phi_2}\varpi_2}{4\pi} \qquad (8)$$

For standard parameters for both neutron stars and white dwarfs, we find $S_2 D_d^{-1} \sim 1$ in the following

$$-\frac{\beta}{4\pi} \simeq S_2 D_d^{-1}(|\Delta\Omega|/\Omega_s)(\eta\varpi^2\Omega_s), \qquad (9)$$

where $S_2 \simeq v_d/v_2$ and $\Delta\Omega = \Omega_2 - \alpha$. It is thus apparent that a significant AM flux can result from the funnel flow only if

$$\Delta\Omega \sim \Omega_s \qquad (10)$$

3. The shock angular velocity

The funnel flow constrains the AM flux, but the AM flux is ultimately fixed by the value of the relative velocity of the shock with respect to the star, if such a velocity difference is physically permitted. We therefore need to examine the postshock region and to investigate the details of the physics at the stellar boundary. In what follows we reiterate the arguments given by LWR to conclude that there can in fact be no slippage between the shock and the star.

The postshock flow must be decelerated before hitting the stellar surface. If the funnel flow supplies a significant AM flux to spin up the star, one must assume a toroidal magnetic field in the postshock region and finally on the stellar surface. This requires that $\Omega < \alpha$. The rotation of the stellar surface must necessarily be a boundary for the fluid so that $\Omega_s = \alpha$ since v must decelerate to the value zero. Therefore, a toroidal magnetic field requires the existence of a rotational gradient or $d\Omega/d\varpi < 0$, where the sign is selected for producing a spin-up torque. The angular velocity shear must be associated with a current sheet in the fluid, causing a toroidal Lorenz force. This force, however, will not be balanced by a possible viscous force in the boundary layer corresponding to the current sheet. On the contrary, the viscous force has the same direction as the Lorentz force so that both of them tend to diminish the shear. Thus the stable case is one in which the shock must be synchronised with the star, or $\Omega_2 = \Omega_1 = \alpha$ and $B_\phi = 0$ in the shock. We show the situation for Ω for the stellar case in Fig. 2. The result we obtained is in sharp contrast with the classic Couette flow (cf. Roberts 1967) where a continuous shearing can occur near a conducting wall because the viscous force is opposite to the Lorentz force. The principle reason is that for a $B_\phi > 0$ in the stellar case, the induction equation requires $\Omega < \Omega_s = \alpha$, whereas in the Couette flow a similar $B_\phi > 0$ can occur only if $\Omega > \Omega_s$.

4. Zero matter torque

In section 2 we showed that the large terminal velocity expected for the funnel flow leads to a severe reduction of the toroidal magnetic field. Accordingly

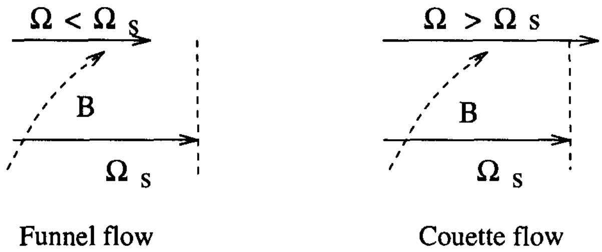

Figure 2. The required angular velocity for a $B_\phi > 0$

a significant amount of AM can be transferred to the star only if there is a large rotational drift of the shock relative to the star. However, in section 3 we demonstrated that any such drift, however small, violates the stellar boundary condition. Thus we concluded that the AM flux carried by the funnel flow must be almost zero or $-\beta/4\pi \simeq 0$. Thus the matter angular momentum term associated with the funnel flow, as proposed in the Ghosh and Lamb model cannot be significant.

Our result is unexpected, and was certainly not anticipated in previous investigations which attempted to look at the problem solely from the point of view of the disc surface where the funnel flow originates. Instead, we have shown that the funnel flow imposes a stellar boundary condition for the star-disc magnetic interaction, and it is this connection that has been completely ignored in the standard approach, resulting in what appears to be an erroneous conclusion.

It is interesting to compare our results with those of (Popham & Narayan 1991) who studied the angular momentum flux in the context of non-magnetic white dwarfs which accrete from a disc when the star is near rotational break-up. These authors demonstrated that if the star rotates very close to break-up, the star can continue to accrete matter but no angular momentum. The angular momentum is transported outwards by viscous torques through the disc. Our result can thus be seen to be a magnetic analogue of their result, with magnetic torques playing the role of viscous torques in initially transporting away the angular momentum from the accreting star. A clear difference is that in the magnetic case, accretion could continue without the addition of angular momentum to the star, even if the star is rotating at rate that is slower than break-up.

5. Implications

Our result suggests that a fundamental change may be required in our view of the boundary condition between the inner disc and the funnel flow. In the following we concentrate on two consequences of this new boundary condition as it applies to the angular momentum transport problem and the fastness parameter.

https://doi.org/10.1017/S0252921100042706 Published online by Cambridge University Press

5.1. The angular momentum transport mechanism

The disc must be magnetically stressed at the foot points of the funnel flow, and this in turn means that the surface perpendicular to the disc surface must also be stressed, because otherwise the inner part of the disc would be spun up. The disc magnetosphere interaction region is shown schematically as a box in Fig. 1.

If the accretion funnel has a certain width, we can argue that the mean field must not be force free in the toroidal direction, because a force free field in the toroidal direction would mean B_ϕ is zero along a poloidal field and this contradicts the disc boundary condition. One way to resolve this problem is to assume that turbulent magnetic stresses operate so that the mean field is not force free in the toroidal direction. The turbulent magnetic stress then acts to remove the AM outwards, so the total magnetic force in the toroidal direction is still force free. Here we completely neglect the viscous force, since given the sign of the gradient in Ω, it is difficult to see how one can produce the required angular velocity shear to transfer AM outwards through viscous torques (see Fig. 3).

In the literature, one can find theoretical studies which may support the view that AM can be transported via turbulent magnetic stresses. Lerche (1970) first proposed the possibility of a turbulent force free field by considering $\nabla \times \mathbf{B} = (\alpha^* + \delta\alpha^*)\mathbf{B}$, and later Hu (1982, 1983) considered a detailed model. Hu considered the time-dependent fluctuations, and the mean quantity equations for force and magnetic induction, and obtained a consistent relation for mean momentum balance:

$$(\nabla \times \mathbf{B}_0 + \mathbf{K} + \mathbf{L}) \times \mathbf{B}_0 = 4\pi(\nabla p_0 + \rho_0 \nabla \Phi), \tag{15}$$

where $\mathbf{K}$ represents the contribution from the magnetic boundary turbulent field and $\mathbf{L}$ from the magnetic internal turbulent field, and other quantities have their usual meanings (subscript "0" denotes mean). For our case in viewing toroidal quantities the right hand side is simply zero. The relation shows that a mean field which links the star may not be force free in the toroidal direction, due to the turbulent magnetic stress. This is exactly what is required for the balance of AM in the inner disc: the mean field supplies AM into the box (Fig. 1) but the turbulent stress removes it out so that a steady state is possible. Although their result is based on small amplitude fluctuations of turbulence as compared to the mean quantities, there is no reason why the model cannot work when the turbulent stress is comparable to the mean stress, similar to the mean field dynamo theory. The origin of the turbulence is not considered in this approach, but it may be caused by various instabilities in the inner disc. Thus, turbulent magnetic stress may be a possible mechanism for transporting AM in the inner part of the disc.

5.2. Critical fastness parameter

Beyond the funnel flow, there may be a region which is responsible for the star-disc magnetospheric interaction. For a slow rotator, we naturally assume that this interaction region tends to spin up the star, since one expects a rotational profile as shown in Fig. 3. The fastness parameter is defined at a radius ϖ_0, where the magnetospheric spin-up torque is expected to maintain a Keplerian

https://doi.org/10.1017/S0252921100042706 Published online by Cambridge University Press

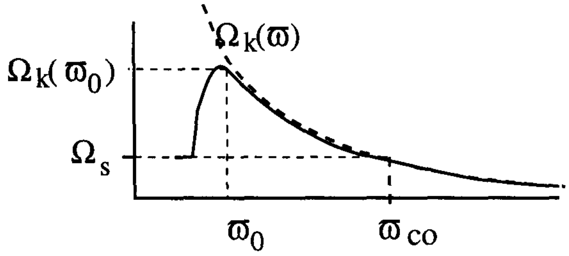

Figure 3. The expected angular velocity profile.

disc rotation (cf. Wang 1995):

$$\omega = \Omega_s/\Omega_K(\varpi_0) = (\varpi_0/\varpi_{co})^{3/2}. \tag{11}$$

When a rotational equilibrium is achieved with the disc, we call ω the critical fastness parameter, ω_c, which is an important parameter in the standard theory. Based on the standard picture, Wang (1995) obtained three estimates for the fastness parameter by adopting different assumptions for the nature of the resistivity: reconnection inside the disc, turbulent mixing and finally reconnection outside the disc. The three suggested values for the critical fastness parameter are $\omega_c = 0.949$; 0.87; 0.950.

However, our results in section 4, i.e., that the funnel flow does not carry matter angular momentum, suggests a modification of Wang's results. We have recalculated ω_c for Wang's three cases and find: $\omega_c = 0.47$; 0.500; 0.639, where we have ignored the magnetospheric contribution within $\varpi < \varpi_0$. Our results are significantly smaller than Wang's entirely because a larger magnetospheric spin-up region is necessarily required between ϖ_0 and ϖ_{co} to counter balance the spin-down torque beyond ϖ_{co}. If we define ϖ_0 at the Ω maximum in Fig. 3, as adopted by Li & Wang (1996), our values of ω_c increase somewhat but are still significantly smaller than their results (see Li & Wickramasinghe 1996).

6. The propeller

When the disc is truncated at a radius which is larger than the corotation radius, the disc-star system is regarded to be a propeller and so accretion is believed to be no longer possible because of the centrifugal barrier.

Let us consider a general propeller, and assume axisymmetry for simplicity, as shown in fig 4. We assume that the instabilities operate between the corotating magnetosphere and the disc, because otherwise we have Aly's (1980) model for a perfect conducting material separated by a corotating magnetosphere and no accretion would occur. The instabilities result in an amount of matter diffusing inwards, and this matter would necessarily be heated and absorb AM from the corotating magnetosphere. Because of the special field geometry due to the pinching effect of the disc, the diffused material may tend to flow along field

https://doi.org/10.1017/S0252921100042706 Published online by Cambridge University Press

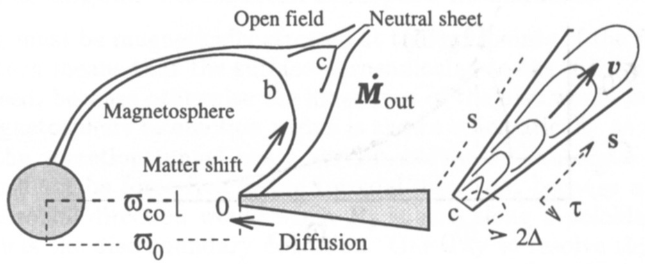

Figure 4. Propeller and magnetospheric streamer model.

lines and stay higher in the joint potential well, due to a large centrifugal force. Though time-dependent processes may result, we here assume a quasi-steady state. The accumulation of the matter in the magnetosphere would increase the density which would follow

$$\rho_b = \rho_0 \exp\{\Phi_0 - \Phi_b\} \tag{12}$$

where Φ is the joint potential which is a function of the position, and the subscripts denote different positions of a field line as shown in Fig. 4. Due to the nature of the corotating magnetosphere, the centrifugal potential increases with ϖ^2 and it will soon be much greater than the square of the sound speed. Thus the magnetic field would not be able to balance the thermal pressure and the field line must break, and the matter which has gained more specific angular momentum will move along the magnetic field lines. We associate the mass loss from this magnetosphere with the outflow in the propeller, and this outflow is sharply different from the outflow suggested by Lovelace et al. (1995). In the steady state approach, we may invoke a streamer model allowing the matter to flow along the current sheet, by an analogy with solar physics, where such a model is used to account for plasma flow from the closed field region. Due to the plasma resistivity, it is argued by Pneuman (1972) that an ideal current sheet is inadmissible since reconnection must occur, and the plasma must start to flow at the top of the streamer across the closed field line, as shown in Fig. 4.

In this model, the build-up of plasma in the magnetosphere is due to instabilities, and the mass loss from the magnetosphere is via the top of the magnetosphere where the magnetic confinement breaks down. Thus a steady propeller may be achieved by balancing the mass flux. When that is achieved, we must also expect a balance between the magnetic pressure and the plasma pressure in a direction perpendicular to the outflow. The balance condition can be simplified by assuming a cusp point (see Fig. 4) right below where the streamer outflow develops so that it is roughly the plasma pressure at the break point which balances the magnetic pressure in the open-field line region just beside the cusp point or

$$\frac{B_{open}^2}{8\pi} \simeq a_w^2 \rho_c, \tag{13}$$

where a_w is the isothermal sound speed. The equation of motion along the outflow (denoted by s) is

$$\rho v \frac{dv}{ds} = -\rho \nabla_s \Phi + \mathbf{j} \times \mathbf{B}/4\pi|_s, \tag{14}$$

where the last term is due to the the Lorenz body force caused by the magnetic dragging in the flow direction. Clearly the initial outflow must be subsonic and the balance of force between the thermal pressure and the centrifugal force and the Lorenz force must be achieved above the cusp point. Suppose the outflow is initiated with a width of 2Δ and the flow is confined within a sheet, following Pneuman (1972) and Priest & Smith (1972), the continuity equation indicates that the density gradient along the outflow is small compared to the density if the outflow velocity does not increase too quickly. So it may be reasonable to assume that the centrifugal force roughly balances the Lorentz force. This condition gives us a relation

$$\frac{B_\tau B_s}{4\pi \Delta \rho} \sim \varpi \Omega_s^2, \tag{15}$$

where subscript s and τ represent projection in the s direction and its perpendicular direction (see Fig. 4). Assuming $B_\tau \sim B_s \sim B_{open}$, we have

$$\Delta \sim \frac{2a_w^2}{\varpi \Omega^2}. \tag{16}$$

To estimate the total mass outflow from the magnetosphere, we further assume the sonic point is near the cusp point so that

$$\dot{M}_{out} = 8\pi \varpi \Delta \rho_c a_w \sim 2(\frac{a_w}{\Omega^2}) B_{open}^2. \tag{17}$$

This formulation is of interest since the mass loss rate could be predicted if we know the strength of the open magnetic field near the cusp point. Here the precise location of the cusp point need not be known but the balance condition (13) perpendicular to the streamer must be satisfied near the sonic point. This is the reason why the density at the cusp point does not appear in $\dot{M}_{out}$. If we assume that the outflowing matter contains a specific AM of order $\varpi^2 \Omega$, then the total AM loss rate from the magnetosphere is

$$\dot{J}_{out} \sim \dot{M}_{out}^2 \Omega \sim 2(\frac{a_w}{\Omega}) \varpi^2 B_{open}^2. \tag{18}$$

Assuming that AE Aquarii is a propeller (Meintjes & De Jager 1995), we find that its parameters would lead to $\dot{M}_{out} \sim 10^{16}$g s^{-1} and $\dot{J}_{out} \sim 10^{34}$g cm^2 s^{-2}, according to our model, which can be significant.

7. Conclusions

Our new theoretical understanding of angular momentum accretion in the slow rotators provides a well-determined boundary condition at the inner disc, severely constrains possible magnetic disk models, and changes our picture of spin equilibrium. An important conclusion of the present work is that the critical fastness

https://doi.org/10.1017/S0252921100042706 Published online by Cambridge University Press

parameter may be somewhat smaller than previously estimated, being closer to 0.5 rather than to unity. We have also shown that the boundary condition implies that an additional mechanism over and above viscous shear will have to be invoked to explain the transport of angular momentum in the funnel- disc interaction region. We have argued that turbulent magnetic stresses may play the desired role in agreement with general physical considerations. In the propeller region, we propose a new streamer model for carrying away mass and angular momentum from the magnetosphere. The streamer model is attractive because it provides a viable mechanism for magnetospheric-ejection of a steady mass outflow and predicts a mass loss rate that depends only on the magnetic field strength of the open field lines near field break-up points.

Acknowledgments. We are grateful to Lilia Ferrario for her help, and B. Warner for discussions.

References

Aly J.J., 1980, A&A, 86, 192

Meintjes, P. A., & De Jager O. C., 1995, Cape Workshop on Magnetic Cataclysmic Variables, ASP Conference Series, Vol., 85

Ghosh, P., Lamb, F. K. & Pethick C. J., 1977, ApJ, 217, 578

Ghosh, P., & Lamb, F. K., 1979a, ApJ, 232, 259

Ghosh, P., & Lamb, F. K., 1979b, ApJ, 234, 296

Hu, W-R, 1982, Ap&SS, 85, 351

Hu, W-R, 1983, Ap&SS, 97, 353

Lerche, I., 1970, ApJ, 162, 675

Li, J., 1996, ApJ, 456, 696

Li, J., Wickramasinghe D. T., & Rüdiger G., 1996, ApJ, in press

Li, J., & Wickramasinghe D. T., 1996, MNRAS, submitted

Li, X-D., & Wang Z-R, 1996, A&A, *letts*, 307, L5

Lipunov, V. M., 1992, Astrophysics of Neutron Stars, p89, Berlin Heidelberg: Springer-Verlag

Longair, M., 1994, High Energy Astrophysics, Vol. 2, P 157, Cambridge, Cambridge University Press

Lovelace, R.V.E., Romanova M.M., & Bisnovatyi-Kogan G.S., 1995, MNRAS, 275, 244

Mestel, L., 1968, MNRAS, 138, 395

Pneuman, G. W., 1972, *Solar Physics*, 23, 223

Popham R. & Narayan R., 1991, ApJ, 370, 604

Priest, E. R., & Smith D. F., 1972, *Astrophysical Letts*, 12, 25

Roberts P.H., 1967, An introduction to Magnetohydrodynamics, Longmans: London 1994, ApJ, 429, 781

Wang, Y-M, 1995, ApJ, 499, L153

Wang, Y-M, & Robinson, J. A., 1985, A&A, 151, 361

Weber E. J., & Davis L., 1967, ApJ, 148, 217