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TOPOLOGICAL METHODS IN EQUILIBRIUM ANALYSIS

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This thesis studies some purely mathematical problems that arise in the context of the axiomatic approach to equilibrium analysis initiated by Debreu [2]. Part I deals with fixed point theorems in topological vector spaces and Part II considers order-preserving real valued functions on topological preordered spaces.

Chapter 1 presents two new fixed point theorems in topological vector spaces. In the first theorem, a selection argument similar to Browder [1] is used to prove the existence of a fixed point for a multivalued map defined on a paracompact convex subset of a Hausdorff locally convex topological vector space. In the second theorem, a recent generalisation of the classical Knaster-Kuratowski-Mazurkiewicz theorem due to Fan [3] is used to prove the existence of a fixed point for a multivalued map defined on a convex subset of a Hausdorff topological vector space.

Two approaches to the proofs of equillibrium existence theorems have been used in Part I. According to the first approach one is given an ordered vector space X with a negative cone C, a subset D of the dual cone C^* and a multivalued map $E\colon D \to 2^X$ that satisfies certain conditions. A point p in D is said to be an equilibrium point if $E(p) \cap C \neq \phi$. Chapters 2, 3 and 5 contain some results on the existence of equilibria in Riesz spaces and ordered topological vector spaces. In particular, finite and infinite dimensional generalisations of the classical Gale-Nikaido-Debreu theorem are proved.

According to the second approach one considers some (possibly infinite) index set I. For each $i\epsilon I$ we define a subset X_i of some topological vector space E_i and two maps $P_i\colon X = \pi_{i\epsilon I} X_i \to 2^{X_i}$ and $A_i\colon X = \pi_{i\epsilon I} X_i \to 2^{X_i}$. A point $\overline{x}$ in X is said to be an equilibrium point if for each $i\epsilon I$, $\overline{x}_i \epsilon A_i(\overline{x})$ and $P_i(\overline{x}) \cap A_i(\overline{x}) = \phi$. The interpretation here is that $\overline{x}_i$ is a maximal element in $A_i(\overline{x})$ for each $i\epsilon I$. Chapters 4, 5 and 6 contain some abstract equilibrium existence theorems based on this approach. Finally, Chapter 7 contains some general theorems on the existence of maximal elements of multivalued maps in topological vector spaces.

Part II introduces a new approach to the problem of proving the existence of continuous order-preserving real valued functions on topological preordered spaces. This

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new approach is based on Nachbin's [4] generalisation to topological preordered spaces of the theorems of Urysohn and Weil in general topology. Some general theorems on the existence of continuous order-preserving real valued functions on topological preordered spaces are proved and generalisations are obtained of the classical theorems of Eilenberg and Debreu.

References

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