IX THEORY

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PULSATIONS OF HYDROGEN DEFICIENT STARS

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ABSTRACT. A review is given on the properties of the pulsating hydrogen-deficient stars. Since they lie in a large area of the HR diagram, the properties of pulsations are greatly different among them. Pulsations tend to change from radial pulsations to non-radial pulsations as the surface gravity increases. Strong nonadiabaticity affects the dynamical properties of pulsations in high luminosity helium stars. The kappa and gamma mechanisms for helium ionization excite radial pulsations in relatively cool helium stars and nonradial pulsations in DB (helium atmosphere) white dwarfs, while the cyclical ionization of the K-shell electrons of carbon and oxygen seems to be responsible for the excitation of the nonradial pulsations in the GW Vir (PG1159-035) stars (very hot hydrogen-deficient pre-white dwarf stars). The excitation mechanism of the radial pulsations in a unique extreme helium star V652 Her (BD+13°3224) is not known. Period changes have been detected for many pulsating helium stars except the pulsating DB white dwarfs.

1. INTRODUCTION

The first periodic light variation in a hydrogen deficient star was discovered by Jacchia (1933, quoted by Alexander et al. 1972). Jacchia found RY Sgr, an R Coronae Borealis (R CrB) star, to show a semi-regular variation with an average periodicity of 39 days. The number of the presently known pulsating hydrogen deficient stars is about 15 or more. Some of the pulsating hydrogen deficient stars and their pulsation periods are shown in the HR diagram in Figure 1. R CrB and RY Sgr are the best studied pulsating R CrB stars. Pulsations of most of the other stars in this figure were discovered recently. The pulsations of the exteme helium stars, BD+1°4381, BD-1°3438, and BD-9°4395 were discovered by Jeffery and Malaney (1985), Jeffery, Hill, and Morrison (1986), and Jeffery et al. (1985), respectively. Landolt (1975) discovered the light variation of a unique hydrogen deficient star V652 Her (BD+13°3224) which is less luminous than the above stars by about one order of magnitude. Grauer and Bond (1984) found the central star of the planetary nebula Kohoutek 1-16 (kl-16) to be a pulsating variable. The spec-

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trum of K1-16 is similar to that of GW Vir (PG1159-035) whose variability was discovered by McGraw et al. (1979). Winget et al. (1982b) discovered that GD 358, a helium atmosphere (DB) white dwarf, is a pulsating variable star.

These pulsating hydrogen-deficient stars may be classified into the following groups; 1) pulsating high luminosity helium stars (pulsating R CrB stars and extreme helium stars), 2) V652 Her (BD+13°3224), 3) the GW Vir (PG1159-035) stars including the pulsating planetary nebulae nuclei, and 4) pulsating DB white dwarf stars. Each group represents different evolutionary phase. The pulsation properties for each of these groups are reviewed in the following sections.

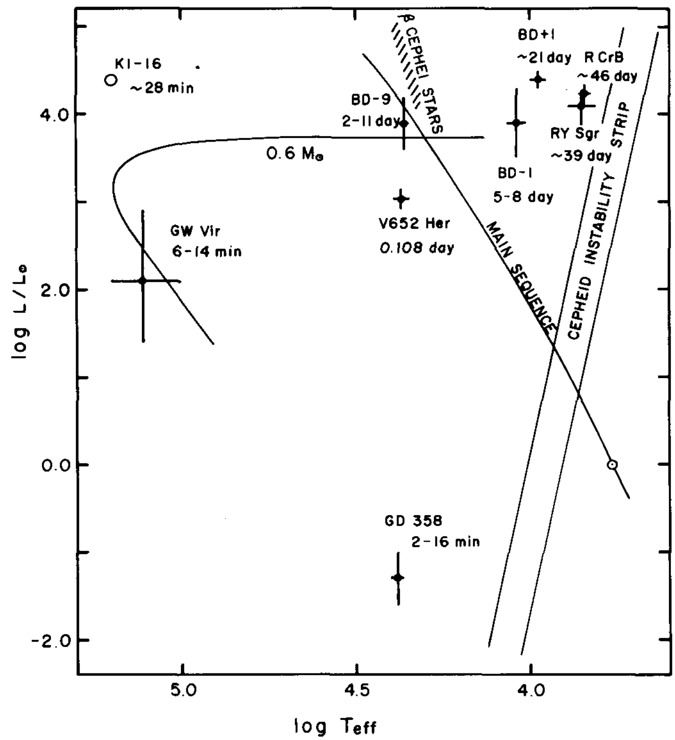

Figure 1. Location of pulsating hydrogen-deficient stars on the HR diagram. The periods of pulsations are shown below the names of the stars (BD+1=BD+1°4381, BD-1=BD-1°3438, BD-9=BD-9°4395). In addition, the evolutionary track of 0.6 $M_{\odot}$ pre-white dwarf star computed by Schönberner (1979), the Cepheid instability strip, the Beta Cephei instability zone, and the Pop.I main sequence are shown.

2. PULSATIONS OF HIGH LUMINOSITY HELIUM STARS

2.1. Introduction

Many R CrB stars show semi-regular light variations with periods of a few tens of days (Feast 1975, Rao, Ashok, and Kulkarni 1980). An intensive investigation by Alexander et al. (1972) on the variability of RY Sgr revealed that its semi-regular variations are due to stellar pulsations. Most of the extreme helium stars seem to have small amplitude light variations (e.g., Walker and Kilkenny 1980). However, periods are obtained only for three or four stars (other than V652 Her, which is discussed in §3). These stars have luminosities $L/L_\odot \sim 10^4$ similar to those of the R CrB stars, and the latter stars are believed to be the immediate progenitors of the former stars (Heber and Schonberner 1981). The rapid period decrease of RY Sgr ($d\ln P/dt \sim -10^{-11}$/s) obtained by Kilkenny (1982) and Marraco and Milesi (1982) supports this evolutionary connection. The period changes of other R CrB stars are, however, somewhat confusing: Kilkenny and Flanagan's (1983) analysis suggests that the period of UW Cen might be increasing. Furthermore, Kilkenny (1983) found that the period of pulsations of S Aps had changed from about 120 days to about 40 days and that the $\sim$120 day variations exhibited a period decrease while the $\sim$40 day period seemed to increase with time.

Some properties of well studied pulsating high luminosity helium stars are listed in Table 1, in which P and Q are, respectively, pulsation period and the pulsation constant ($Q=P(R/R_\odot)^{-1.5}(M/M_\odot)^{0.5}$). The luminosities and masses of R CrB, RY Sgr, and BD+1°4381 were obtained by incorporating the effective temperatures and the pulsation periods with theoretical linear pulsation periods calculated by Saio, Wheeler, and Cox (1984), and a mass luminosity relation

$$\log (L/L_\odot) = 3.2 \log (M/M_\odot) + 4.6, \qquad (1)$$

which is based on Schönberner's (1977) models for post-giant helium stars. Equation (1) is applicable for $\log(L/L_\odot) \gtrsim 4$. Since the mode identifications to the observed periods of BD-1°3438 and BD-9°4395 are uncertain, the luminosities of these stars were obtained from the surface gravities with the effective temperatures and equation (1). Note that the listed ranges of luminosity and mass for RY Sgr and R CrB are considerably narrower than those given in Saio and Wheeler (1983), because by using equation (1),it is assumed that the total mass is nearly equal to the CO core mass.

The theoretical investigation of the pulsations of the luminous helium stars was started by Trimble (1972). A good review of the earlier theoretical works on the pulsations of R CrB stars is given in King (1980).

2.2 Very nonadiabatic radial pulstations

Pulsations in high luminosity helium stars are extremely nonadiabatic (Trimble 1972, Wood 1976, Cox et al. 1980, and Saio, Wheeler, and Cox 1984). Since the thermal time scale is roughly proportional to the mass-luminosity ratio M/L, the nonadiabaticity of pulsations can be significant in a high L/M star ($L/M \sim 10^4 L_\odot/M_\odot$). When the nonadiabaticity is very strong, the dynamical properties of pulsations deviate significantly from what we know for classical pulsators like Cepheids and RR Lyrae variables. The periods of pulsations deviate from the corresponding adiabatic periods. Furthermore, one to one correspondence between adiabatic and nonadiabatic modes is destroyed by the appearance of new nonadiabatic modes (Wood 1976). It should be mentioned here that Shibahashi and Osaki (1981) found that the breakdown of the one to one correspondence occurs for nonradial pulsations, too, in $L/M \sim 10^4 L_\odot/M_\odot$ models.

The nonadiabatic period which is close to the adiabatic period of the k-th overtone in the low effective temperature models tends to approach the adiabatic period of the (k+1)-th overtone in the high effective temperature models. Saio, Wheeler, and Cox (1984) showed that this phenomenon occurs because the nonadiabaticity of a pulsation mode increases as the effective temperature of the model increases. They also showed higher overtone modes to be more nonadiabatic.

Since the thermal timescale is comparable to the dynamical timescale, the growth or damping time can be comparable to the pulsation period. Moreover, the periods of thermal-dynamical damping oscillations which are called "strange modes" enter in the same range of the periods of dynamical (ordinary) pulsations. The properties of the strange modes are discussed in detail in Saio and Wheeler (1982), and Saio, Wheeler, and Cox (1984). When an ordinary pulsation mode has a period close to that of a strange mode, the ordinary mode tends to be stablized. This causes a "stable strip" in the instability region for the first overtone modes in the HR diagram (Saio, Wheeler, and Cox 1984).

Table 1. Properties of pulsating extreme helium stars and GW Vir

name	log L	log T_{eff}	period (day)	$M/M_\odot$	Q (day)	ΔM_V (mag)	ΔV (km/s)	ref.
R CrB	4.3±0.1	3.845±.015	46±5	0.8±.1	.044	.1-.2	∿4	1,2,3
RY Sgr	4.1±0.2	3.850±.037	39	0.8±.1	.046	.5	40	4,5,6
BD+1°4381	4.4±0.1	3.977±.018	22	0.85±.05	.045	.06		7,8,9
BD-1°3438	3.9±0.4	4.037±.024	5-8	(0.6)	(.03-.05)	.07		7,9,10
BD-9°4395	3.9±0.3	4.362±.013	2-11	(0.6)	(.1-.6)	.03	∿10	7,11,12
V652 Her (BD+13°3224)	3.0±0.1	4.370±.024	.108	$0.75^{+.3}_{-.2}$	.034	.06	70	13,14

1) Cottrell and Lambert (1982)
2) Fernie (1982)
3) Fernie, Sherwood, and DuPuy (1972)
4) Schönberner (1975)
5) Alexander et al. (1972)
6) Lawson (1985)
7) Drilling et al. (1984)
8) Jeffery and Malaney (1985)
9) Jeffery, Hill, Morrison (1986)
10) Schönberner (1978)
11) Jeffery et al. (1985)
12) Kaufmann and Schönberner (1977)
13) Hill et al. (1981)
14) Lynas-Gray et al. (1984)

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The strong nonadiabaticity also has a destabilizing effect on pulsations and modifies the instability boundary in the HR diagram significantly. Figure 2 shows the location of the blue edges of the fundamental mode instability region in the HR diagram for helium star models with 90% helium and 10% carbon by mass (Saio, Wheeler,and Cox 1984). (The fundamental mode and a k-th overtone mode are conventionally defined in this paper as the modes which approach the adiabatic fundamental mode and the k-th overtone mode, respectively, in the limit of low effective temperature.) Below a critical luminosity the blue edge of the instability region due to the helium ionization is very similar to that for the Cepheids but becomes nearly horizontal in the HR diagram above the critical luminosity. The critical luminosity is higher for a larger mass, because the effect of nonadiabaticity is stronger in a higher L/M star. This extension of the pulsationally unstable region is interpreted as follows: The vibrational stability is roughly determined by the balance between the driving in the helium ionization zone and the radiative damping in the region interior to the ionization zone. When the luminosity is sufficiently high, the effect of the radiative damping decreases because the local thermal timescale becomes very short there. (The luminosity perturbations are "frozen in" in space [Cox 1974].) Then the excitation due to the ionization zone exceeds the radiative damping even in the models hotter than the bule edge of the classical instability strip (Saio, Wheeler, and Cox 1984). The decrease of the effect of the radiative damping occurs also in high luminosity models with hydrogen rich envelopes (Fadeyev and Fokin 1985, Zalewski 1985).

Several hydrogen deficient stars are also plotted in Figure 2, where the symbols •, Δ,and o are for (semi-)periodic variables, suspected variables, and non-variables, respectively. [It should be noted here that only BD+10°2179 has been confirmed to have no light variation (Hill Lynas-Gray, and Kilkenny 1984, Grauer, Drilling and Schönberner 1984).] The blueward excursion of the blue edge of the instability region accounts for the existence of pulsations in R CrB stars and BD+1°4381, which are fundamental mode pulsators. BD-1°3438 seems to reside outside the instability region for the fundamental mode, but inside the instability region for the first overtone mode. The instability region for the first overtone mode extends into less luminous region than the fundamental mode instability boundary (Saio, Wheeler, and Cox 1984). Actually, the period of BD-1°3438 seems to be consistent with the period of the first overtone mode. Nonradial g-mode pulsations seem to be excited in BD-9°4395 (Jeffery et al. 1985) and V2076 Oph (HD160641, Lynas-Gray et al. 1986). [The latter star (T_{eff}=31900±1500; Drilling et al. 1984) is not shown in Fig. 2 because its surface gravity is not available.] No theoretical investigation of the nonradial pulsations in such stars has not, to the authers' knowledge, been published. We need extensive theoretical investigations of radial and nonradial pulsations in hot (10^4K $\lesssim T_{eff} \lesssim 3\times10^4$K) hydrogen deficient stars.

2.3 Nonlinear pulsations

The nonlinear hydrodynamic calculations for the luminous helium stars had posed a serious difficulty. Pulsations of 1 $M_\odot$ models with T_{eff}

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$\sim$6000 K and $L \sim 10^4 L_\odot$ are so violent that the amplitude seems to grow until the expansion velocity attains the escape velocity (Trimble 1972, Wood 1976, King et al. 1980). Pulsations of lower L/M models are less violent. King et al.(1980) showed that for models with T_{eff}=6300 K, $L=1.13 \times 10^4 L_\odot$ and $M \geq 1.6\ M_\odot$ pulsations attained stable limit cycles, of which light and velocity curves were quite regular. According to the result, they suggested that the mass of a pulsating R CrB star should be larger than $\sim$1.4 $M_\odot$. However, this conclusion contradicts the result of Schönberner (1977) who obtained the typical mass of R CrB stars to be about 0.7 $M_\odot$ by using the spectroscopically determined surface gravities and evolutionary models with a CO core and a thin helium envelope.

The mass discrepancy of the pulsating R CrB stars was solved by Saio and Wheeler (1985) who showed that the nonlinear pulsation properties are sensitive to the effective temperature of the models. In the previous nonlinear calculations models with $T_{eff} \sim$6000 K are adopted, while the spectroscopic analyses by Schönberner (1975) and Cottrell and Lambert (1982) show the effective temperatures of R CrB and RY Sgr to be

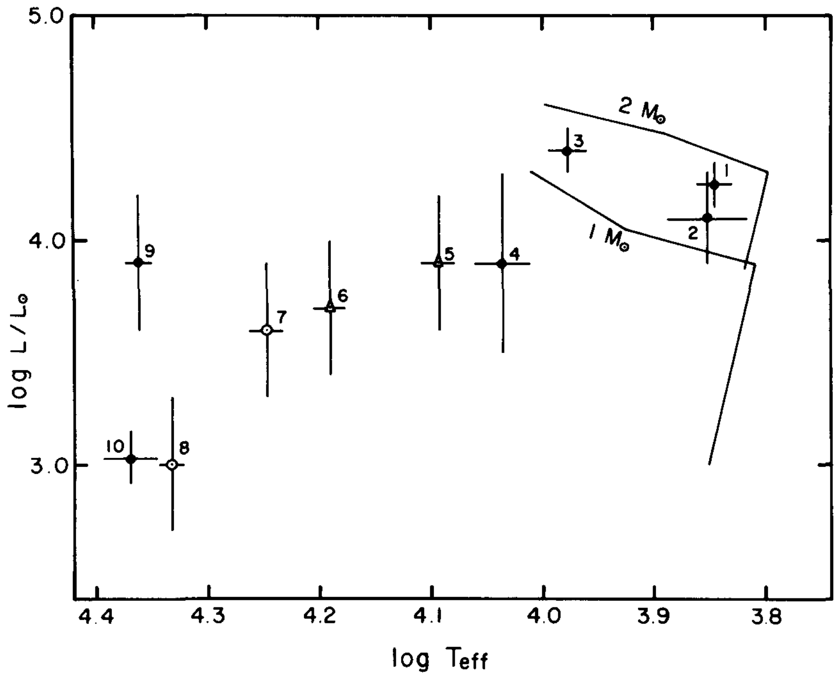

Figure 2. The blue edges of the fundamental mode instability region as a function of mass for helium star models with 90% helium and 10% carbon by mass, and location of extreme helium stars on the HR diagram. The symbols, •, Δ, and o indicate (semi-)periodic variables, suspected variables and non-variables, respectively. The names of the plotted stars are 1=R CrB, 2=RY Sgr, 3=BD+1°4381, 4=BD-1°3438, 5=PV Tel (HD168476), 6=HD124448, 7=BD+10°2179, 8=HD144941, 9=BD-9°4395, and 10=V652 Her (BD+1°3224).

about 7000 K. Saio and Wheeler (1985) showed that if $T_{eff} \sim 7000$ K was adopted,the amplitude of pulsations remained bounded even in models with $M < 1\ M_{\odot}$. The light and the velocity curves are irregular for low mass models ($M \lesssim 1.4\ M_{\odot}$), while pulsations in the 2 $M_{\odot}$ model are quite regular. Thus, pulsations in low mass models are consistent with the observed irregular light and velocity curves of pulsating R CrB stars. The growth of pulsation amplitude in a low mass model with $T_{eff} \sim 7000$ K is restrained by a strong shock wave which dissipates the kinetic energy of pulsations. Such a strong shock wave does not appear in the models with $T_{eff} \sim 6000$ K.

Figure 3 shows the light and velocity curves obtained by Lawson (1985) for RY Sgr and the theoretical ones for the model with $L=1.7 \times 10^4 L_{\odot}$, $T_{eff}=7200$ K, $M=0.9\ M_{\odot}$, and a chemical composition of 90% helium and 10% carbon by mass (Saio and Wheeler 1985). The observed velocity amplitude of RY Sgr is similar to the theoretical one. Although the bolometric correction is not applied in this figure, the observed amplitude of the light variations seems to be considerably smaller than the theoretical one, and the theoretical phase relation between light and velocity curves seems not to agree with the observed one.

Wood (1976) calculated the nonlinear first-overtone pulsations for a model with $L=10^4 L_{\odot}$, $M=1\ M_{\odot}$, and $\log T_{eff}=4.0$. The period of the pulsations is 5 days, which is comparable to the period of BD-1°3438. He obtained the amplitudes, $\Delta M_{bol} \sim 1$ mag and $\Delta V \sim 40$ km/s. Again, the theoretical amplitude in luminosity seems too large compared to the observed

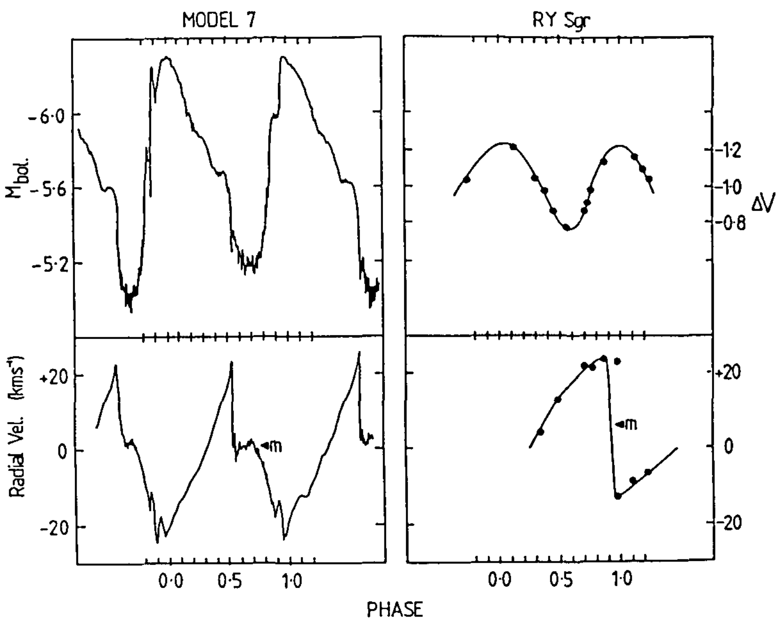

Figure 3. The light and velocity curves of Model 7 ($M=0.9M_{\odot}$, $T_{eff}=7200$ K, $L=1.7 \times 10^4 L_{\odot}$) of Saio and Wheeler (1985) and observed ones by Lawson (1985). From Figure 7 of Lawson (1985).

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amplitude of BD-1°3438. Accurate bolometric correction as a function of the pulsation phase is needed for detailed comparison between theoretical and observational light curves. Moreover, it is also needed to improve theoretical nonlinear pulsation models. For example, the effect of mass loss may affect the amplitude of pulsations.

3. PULSATIONS OF V652 HER (BD+13°3224)

The hot extreme helium star V652 Her is different from other extreme helium stars in many respects. The hydrogen abundance, $n_H=0.01\ n_{He}$ (Hill et al. 1981), and the surface gravity, log g = 3.7 (Lynas-Gray et al. 1984) of V652 Her are higher than those of the extreme helium stars discussed in §2. Besides, its luminosity ($L/L_\odot \sim 10^3$: Lynas-Gray et al. 1984) is lower than the extreme helium stars by about one order of magnitude. Lynas-Gray et al. (1984) indicated that V652 Her is comparable with HD144941 which has $n_H= 0.07\ n_{He}$ and log g = 3.5 (Hunger and Kaufmann 1973). Figure 2 shows that both stars are closely located in the HR diagram, although no periodic variation has been detected for HD144941. V652 Her shows regular variations in light and radial velocity (e.g., Lynas-Gray et al. 1984, Jeffery and Hill 1986) with a period of 0.108 day. The amplitudes in visual magnitude and radial velocity are listed in Table 1 with other quantities. The radial velocity curve is very similar to those of the classical Cepheids (see e.g. Cox (1974) for a review of the pulsations of the classical Cepheids). Therefore, we can safely say that the regular variations of V652 Her are due to radial pulsations.

The observed luminosity and effective temperature of V652 Her can be reproduced by a "helium horizontal branch" model (Jeffery 1984), which consists of a core with a mass $\sim 0.45\ M_\odot$, hydrogen burning shell, and an envelope with the observed hydrogen abundance. Jeffery (1984) also showed that the rapid evolution of the model is consistent with the rate of period change ($d\ln P/dt \sim -10^{-11}$/s) obtained by Kilkenny and Lynas-Gray (1982, 1984). The mass of the models which can reproduce the observed luminosity and the effective temperature is in the range of 0.6 - 0.7 $M_\odot$, which agrees with $0.7^{+0.4}_{-0.3} M_\odot$ obtained by Lynas-Gray et al. (1984) from the surface gravity.

Another way to estimate the mass of this star is using the pulsation period. Linear nonadiabatic pulsation periods were obtained for the helium horizontal branch models and some envelope models with excess carbon abundance ($Y=0.9$, $X_C=0.1$; X_C=mass fraction of carbon). (All the models considered turned out stable to radial pulsations. The stability will be discussed later.) The results are plotted in the log Q - log (M/R) plane in Figure 4. The two sequences are for the fundamental (F) and the first overtone (1H) modes. Since the luminosity of the star is about $10^3 L_\odot$, the effect of nonadiabaticity on the dynamical properties of pulsations is small and the pulsation periods are very close to the corresponding adiabatic ones. The period and the mean radius of V652 Her give a relation between Q and M/R, which is shown by the solid curve in Figure 4. From this figure we obtain a mass of $0.75^{+0.3}_{-0.2} M_\odot$ if the fundamental mode is assumed (about 0.5 $M_\odot$ if the first overtone is assumed).

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The obtained mass (assuming the fundamental mode) is consistent with the results of other estimations (see above), which confirms the validity of the radius and luminosity obtained by Lynas-Gray et al. (1984).

Although the pulsation period is thus consistent with other theoretical and observational results, no radial pulsation is excited in the models with luminosities and effective temperatures similar to those of V652 Her. The effective temperature is too high for the helium-ionization driving mechanism to be effective. The epsilon mechanism due to the hydrogen burning shell is negligible because the fractional radius at the burning shell ($r/R \sim 0.1$) is too small. The chemical composition could change from the observed surface composition to a carbon/oxygen-rich mixture at a layer slightly deeper than the photosphere as the models considered by Starrfield, et al. (1984) for the GW Vir variables (§4). In this case we can expect some driving effect for pulsations due to the high order ionization of carbon and oxygen. However, such a model seems unlikely because there is no indication of carbon enrichment in the atmosphere of V652 Her (Lynas-Gray et al. 1984).

Another possibility is the excitation due to Stellingwerf's (1978) helium opacity bump which exists around $T=1.5\times10^5$K (Osaki 1982, Cox 1985). This bump is confined in a small range of temperature so that it

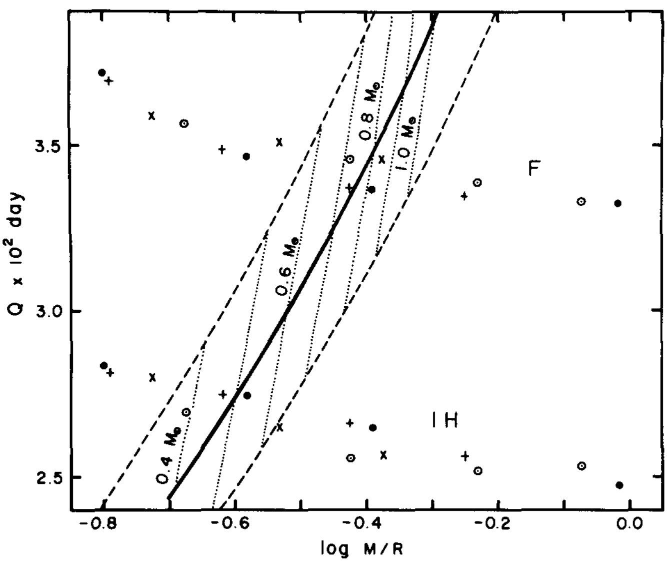

Figure 4. The pulsation constant Q for the fundamental mode (F) and the first overtone mode (1H) versus log M/R (M and R are in the solar units). The symbols • and + are for the 'helium horizontal branch' models ($M_c=0.475\ M_\odot$) with X=0.005 and X=0.002, respectively, and o and x are for the envelope models ($L=10^3L_\odot$, $1.5\times10^4K \leq T_{eff} \leq 3\times10^4K$; Y=0.9, $X_C=0.1$) with 1 $M_\odot$ and 0.5 $M_\odot$, respectively. The solid curve shows the relation which is satisfied by the period and the mean radius of V652 Her. The dashed curves indicate allowable region due to probable error in the mean radius. The dotted lines indicate the loci of constant masses.

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is hardly apparent in a coase grid opacity table. Therefore, the effect of the opacity bump was negligible in the models presented in the previous paragraphs. Stellingwerf (1978) showed that the opacity bump is most effective (but still unable to excite pulsations) in models with effective temperatures similar to those of the Beta Cephei stars. The effect of the opacity bump in models of V652 Her is worthy of studying in detail, because the effective temperature of V652 Her is similar to those of the Beta Cephei stars and it has a helium-rich envelope.

Very accurate light and velocity curves of V652 Her have been obtained by Lynas-Gray et al. (1984) and Jeffery and Hill (1986). In particular, Jeffery and Hill have detected high frequency structure in the radial velocity curve. It is unfortunate for us not to have a theoretical pulsation model which can be compared with these very interesting data.

4. GW VIR (PG1159-035) STARS

Since the discovery of light variations in GW Vir (PG1159-035) by McGraw et al. (1979), a few more GW Vir variables (very hot hydrogen deficient pre-white dwarf stars including the nuclei of planetary nebulae) have been discovered (Grauer and Bond 1984, Bond et al. 1984). The known GW Vir variables are listed by Van Horn (1984). The atmospheric parameters lie in the ranges of 8×10^4K $< T_{eff} < 1.6\times10^5$K and log g $\gtrsim 7$ for GW Vir (Wesemael, Green, and Liebert 1985), and $T_{eff} > 8\times10^4$ K and log $L/L_\odot > 2.5$ for the central star of the planetary nebula K1-16 (Kaler 1983). If we assume $M=0.6M_\odot$, $T_{eff}=(1.3\pm0.3)\times10^5$K and log g=7.5$\pm$0.5 for GW Vir, we obtain log $L/L_\odot$ = 2.1$\pm$0.8. The dominant period of the pulsations of K1-16 is 28.3 minutes (Grauer and Bond 1984), which is considerably longer than the periods of other GW Vir variables (6 - 14 minutes). This is consistent with a preliminary luminosity estimate of $L=2.5\times10^4L_\odot$ obtained by Kaler and Feibelman (1984, quoted by Starrfield et al. 1985) for K1-16. In the HR diagram these stars are located around the "knee" of the evolutionary tracks of the pre-white dwarf phase (e.g., Shönberner 1979, and Iben 1984).

Since the periods of these stars are much longer than their free-fall times (10-100 s), the observed pulsations must be higher order nonradial g-mode pulsations (for reviews of nonradial pulsations see e.g., Unno et al. 1979, Cox 1980). The stability of nonradial g-mode pulsations was examined for the GW Vir stars by Starrfield et al. (1984, 1985). The luminosity of the model for a given effective temperature was adopted from the evolutionary track of 0.6 $M_\odot$ model computed by Schönberner(1979). They found that if the matter in the temperature range from 2×10^5K to 3×10^6K consists of carbon and oxygen, say half carbon and half oxygen by mass, kappa and gamma mechanisms due to the cyclical ionization of the K-shell electrons of carbon and oxygen can excite high order g-mode pulsations ($k \sim 10 - 50$; k indicates the number of radial nodes of eigenfunctions). The periods of the unstable modes are $\sim$2- 20 min for models with $5.6 \leq L/L_\odot \leq 410$ and $\sim$15- 70 min for models with $1.3\times10^3 \leq L/L_\odot \leq 2.8\times10^3$. Although these period ranges of unstable g-modes are consistent with the observed periods, the number

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of the detected periods for a star (Winget et al. [1985] obtained eight periods for GW Vir.) is much smaller than the number of the unstable modes.

Moreover, Kawaler et al. (1986) showed that the helium-burning shell excite some g-mode oscillations whose periods range from 50s to 214s in evolutionary models of hydrogen-deficient planetary nebula nuclei ($2.0 \leq \log L/L_{\odot} \leq 3.2$). Such a short period variation has not been detected in K1-16. The possible interpretations for this discrepancy are; 1) there exists a mechanism which prevents the amplitudes of such unstable modes from growing large enough to be observable, 2) the basic characteristics of the evolutionary models are incorrect and the He-burning shell is extinguished in the real planetary nebula nuclei. Further investigation is needed to find the answer for the problem.

The possibility of detecting period change of the pulsations of GW Vir was first indicated in the discovery paper McGraw et al. (1979). Later Winget, Hansen, Van Horn (1983) have shown that the period change should be measurable on a time scale of 1-3 yr. Actually, Winget et al. (1985) measured a period change of $d\ln P/dt=(-2.3\pm0.2)\times10^{-14}\ s^{-1}$ for the 516 s period of GW Vir. The magnitude of the measured rate of period change is consistent with the theoretical estimation for high order g-modes in cooling pre-white dwarf models, but has the opposite sign (Kawalar, Hansen, and Winget 1985). This discrepancy prompted Kawalar, Winget, and Hansen (1985) to consider the effect of rotational spin-up, produced by gravitaional contraction, on the rate of period change. For a star rotating with a rotation frequency Ω , an observed pulsation frequency σ_{obs} is given by

$$\sigma_{obs}=\sigma_0-m\,\Omega\,(1-C), \qquad (2)$$

where σ_0 is the pulsation frequency obtained assuming the star not to rotate, and m is the azimuthal index of the spherical harmonic $Y_\ell^m(\theta,\phi)$, which represents the angular behavior of the pulsation eigenfunction. C is a positive number (< 1) which depends on the structure of the star and the pulsation eigenfunction. For a pulsation mode with $m < 0$, rotational spin-up has a tendency to decrease the rate of period change. Therefore, if m has a sufficiently large negative value, or the rotation speed is fast enough, the effect of rotational spin-up can exceed the rate of the period change due to evolutionary change of the structure. Kawalar, Winget, and Hansen (1985) found that the rate of period change for a pulsation mode with $m \lesssim -2$ can be consistent with the measured rate of period change of the 516 s period of GW Vir if the star rotates with a period of a few thousand seconds.

5. PULSATING DB WHITE DWARFS

Winget et al. (1982a) predicted that g-mode pulsations should be excited by the He-ionization mechanism in DB white dwarfs. Subsequently, Winget et al. (1982b) discovered the light variation of DB white dwarf GD 358. Three more variable DB white dwarfs have been found. The known variable DB white dwarfs are listed by Van Horn (1984). Winget et al. (1982b) found 28 pulsation modes in the variations of GD 358 with periods ranging from 142.3 s to 952 s. The period range is similar to that of the ZZ Ceti stars (e.g., Winget and Fontaine 1982, Van Horn 1984). Many of the pulsation frequencies of GD 358 fall into groups of 4 or 5 modes that are equally spaced. If the equal spacing is due to the rotational mode splitting (m-splitting; see eq. 2), the observed spacing between two adjacent frequencies implies a rotation period of about 90 min (Winget et al. 1982b), which corresponds to a equatorial rotational velocity of $\sim$10 km/s. Another variable DB white dwarf PG 1654+160 seems to have some equally spaced frequencies. The frequency spacing gives a rotation period of $\sim$150 min (Winget, et al. 1984). Thus, determining the periods of pulsations in DB white dwarfs can be a powerfull means to obtain their rotation rates.

Winget et al. (1983) showed that the effective temperature of the models at the blue edge of the instability region is extremely sensitive to the assumed efficiency of convection in the evolutionary models. In a star hotter than the blue edge, the helium ionization zone is located so close to the stellar surface that its driving effect is no longer strong enough to excite pulsations (e.g., Cox 1974). A larger efficiency of convection reduces super-adiabatic temperature gradient in the convective zone, and hence makes temperature variation less steep. So, the helium ionization zone in the convection zone is pushed to a deeper zone. Therefore, the effective temperature at the instability boundary in the HR diagram is increased by an increase of the efficiency of convection. Winget et al. (1983) found that in order for the theoretical instability blue edge to be consistent with the effective tempearature of GD 358 (24000$\pm$1000 K; Koester et al. 1985), the ratio of the mixing length to the pressure scale hight must be greater than 1.0. However, it should be noted that in the stability analysis the coupling between pulsation and convection is neglected, which may affect stability of nonradial g-modes significantly.

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DISCUSSION

HUNGER: I must say we are really very happy that you have found these small masses for the R CrB stars. For ten years or more Craig Wheeler was telling us that our analyses of helium stars were wrong because they led to masses of the order of one solar mass, where as he claimed 2 $M_{\odot}$.

FEAST: If the atmospheres of R CrB stars are very deep, one might expect the spectroscopically determined temperatures to be lower than the T_{eff} which is relevant to the pulsation calculations.

SAIO: Yes.

FEAST: The other point is related. If the atmospheres of R CRB stars are not in equilibrium, but are slowly expanding, what will be the effect on pulsation?

SAIO: During pulsation, a moving envelope is produced by mass loss. I guess the expanding atmosphere has a damping effect on pulsations.

FEAST: One should perhaps be cautious in adopting spectroscopically determined masses for R CrB stars since it is not entirely clear that these stars have atmospheres in hydrostatic equilibrium (the log g may not give the mass).

HUNGER: Do I understand you right? You think, if one has to include an inertial term, such as times r, in the hydrostatic equation then the effective gravity one derives may be different. However, the density is so low in the atmosphere that the inertial term is small compared to the other terms, which means that effective gravity and mass gravity will hardly differ.

LIEBERT: Concerning the PG 1159 stars, I wanted to make it clear that although models including carbon and oxygen have not yet been calculated, the application of helium/hydrogen models (Wesemael) and the analogy with cooler (80000 K) white dwarfs suggests that the atmospheric composition of the "PG 1159" star is helium-dominated and not likely to be 50% carbon/oxygen.

JEFFERY: What I really wanted to know is whether the blue edge of the instability strip, as it bends over, is a clear-cut thing or whether it is a fuzzy blue edge. Because we see that as we go to hotter temperatures the periods seem to be less regular and then we get non-radial pulsation at the very hottest temperatures.

SAIO: The irregularity is due to non-radial pulsations or maybe the appearance of shock waves in the atmosphere of the star. So far, the stability is determined by linear analysis. I think the stability boundary for radial fundamental pulsation is clear-cut and not fuzzy. The blue edge I showed is for radial fundamental mode. Therefore, it may be possible that other radial or non-radial models grow in the region bluer than the blue edge for radial fundamental pulsations.

JEFFERY: In your models for BD+13°3224 you used Y = 0.9, C = 0.1 composition. The observations suggest photospheric abundances which are extremely N-rich. Supposing the envelope layers to be homogeneous, how would the use of N-rich mixture affect the stability analysis?

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SAIO: I think that a N-rich mixture has an effect on stability similar to that a C-rich mixture has. If the ionization of carbon or nitrogen is responsible for the excitation of this pulsation, the abundance is most important in regions of 10^6 K or so. We do not see such a region, so we have a great choice as to any abundance in such a region.

MENDEZ: K1-16 sometimes pulsates and sometimes does not. Grauer and Bond have presented at least one case in which the star suddenly started pulsating at essentially full amplitude. To what extent is this a problem for the proposed pulsation mechanism?

SAIO: I think that phenomenon is beating. I mean, if there are two pulsation frequencies which are spaced very closely, then you get very long beating periods. So I think the quiet part is the duration of just a small amplitude part and I don't think the pulsation is stabilized in this period.

MENDEZ: When it starts the pulsation, it does it suddenly. It is not gradual.

LIEBERT: No, I don't think so. I think every time we looked at it, we did so for 10 months or more. There is considerable complication in the power spectrum of that object. There is a possibility of fundamental changes in modes between the time of the discovery observations by McGraw and the last year or two, although it is probably going to turn out to be some complicated mixing of the same modes. It would be premature to say which modes we have, but certainly the mode structure is very complicated.

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