1D, 2D AND 3D COLLAPSE OF INTERSTELLAR CLOUDS

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This review is concerned with recent theoretical investigations and numerical models of star formation with various symmetries. Observations strongly support the fact that stars condense out of cool ($\approx$10 K) and dense (10^3-10^4 atoms/cm^3) interstellar clouds due to gravitational instability and collapse. Bright, young stellar objects (O- and B-stars) are always found in the vicinity of cloud complexes.

In the past decade the dynamics of collapse have been theoretically studied in some detail. Extensive numerical calculations have been carried out including a great deal of realistic physical processes, e.g. thermodynamics, transport phenomena, dynamics of dust etc., in addition to hydrodynamics and self-gravitation reflecting the basic physics of cloud collapse. Nevertheless, there remain more open questions than answers on specific mechanisms of star formation.

I. INTRODUCTION, GENERAL RESULTS

Because of the essential non-linearities encountered in self-gravitating gas flows, numerical procedures are the only possibility to treat collapse phenomena in detail. (Semi-)analytical approaches generally suffer from overly restrictive assumptions such as self-similarity properties (Shu, 1977) in order to be tractable. Their applicability to the phenomenon of star formation in the real world is therefore rather doubtful. A glance at Table 1 immediately reveals the fact that there is an obvious correlation between a particular symmetry with which a calculation has been made and the amount of detailed physics that has been taken into account. Since computers have a finite size and computing time is expensive, it is not surprising that the more one reduces symmetry constraints the worse the spatial resolution will be. Also, more restrictive assumptions for the physics, e.g. energy transport, have to be made, but fortunately, for densities below 10^{-13} g/cm^3 the isothermality is well guaranteed. As a matter of fact, at present we can only simulate star formation in full detail in spherical symmetry (1D-models), i.e. the accumulation of single stars with zero

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angular momentum and vanishing magnetic field. Some progress has been made in the understanding of the dynamics of rotating, non-magnetic axially-symmetric cloud collapse (2D-models). Very recently Scott and Black (1980) have published results of calculations concerning the 2D-collapse of a non-rotating magnetic cloud. Though interesting by itself, these models cannot deal with the presumably most important effect: magnetic braking of a rotating, collapsing interstellar cloud. In order to describe this process properly 3D-models are required. There exist estimates and simplified calculations for static magnetic configurations (Gillis et al., 1974, 1979ab; Mouschovias, 1978; Nakano, 1979), but no dynamical models are available.

Table 1

symmetry	spherical	axial	general
physical input	1D	2D	3D
hydrodynamics + self-gravitation	yes	yes	yes
thermodynamics	realistic	realistic	isothermal adiabatic
energy transport: radiative	yes, variable Eddington-factor f	yes, f=1/3 or radiative conduction	no
convective	yes	no	no
magnetic field	-	yes^{+})	no
multi-component flow	yes gas-dust	no	no
thermonuclear burning	yes^{++})	no	no

covered by only one paper:
$^{+}$) Scott and Black, 1980; $^{++}$) Appenzeller and Tscharnuter, 1974

Aside from technical difficulties, such as numerical stability and accuracy, the physics of gravitational collapse and subsequent star formation causes almost insurmountable problems to any numerical scheme. In stellar formation - contrary to stellar oscillation - we are primarily interested in evolutionary time scales (typically being of the order of 10^5 yr), i.e. time intervals during which a forming star would accrete a substantial fraction of its current mass. The ratio of this "accretion" time scale and an oscillation period of the protostar can be as large as 10^9! For this very reason the enormous differences in relevant time scales demand rather specialized hydrodynamical codes (Tscharnuter and Winkler, 1979). For the 1D spherically symmetric case a fairly complete solution has become available just very recently (Winkler and Newman, 1980ab). Earlier attempts, starting with the pioneering paper by Larson (1969), usually make use of one or another additional physical assumption, e.g. the treatment of the accretion shock and the structure of the quasi-hydrostatic stellar core, and simplifying numerical approximations (Appenzeller and Tscharnuter, 1974, 1975). The qualitative agreement between the calculations is remarkably good.

In the case of axially symmetric cloud collapse with local conservation of angular momentum the situation is much less favorable. Although extensive comparative studies with entirely different computer codes (Bodenheimer and Tscharnuter, 1979) show reasonable good agreement in the overall features of the collapse, basic results are still not generally accepted. In these models the most prominent feature at issue is the excitation of a ring-like density wave which, under a wide range of initial conditions, becomes gravitationally dominant or even unstable (Larson, 1972; Black and Bodenheimer, 1976; Nakazawa et al., 1976; Takahara et al., 1976). Tohline (1980) gave a sufficient condition for such a ring wave to be excited on the basis of an approximate physical model, but there is no stringent proof that these conditions would hold in a collapsing gas cloud. Tscharnuter (1975) and Fricke et al. (1976) did not find rings, but their originally used difference scheme turned out to be slightly unstable. However, apparently minor modifications of the scheme in order to increase stability at the cost of numerical accuracy, leaving the physics unchanged, did produce rings (Tscharnuter, 1978; Bodenheimer and Tscharnuter, 1979). No rings either were observed by Kamiya (1976) who used a pure Lagrangian scheme implying a strict local conservation of angular momentum by definition. Unfortunately, due to the phantastic deformation of any Lagrangian grid when applied to collapse problems, the accuracy is poor.

Very recently Norman (1980) finished a comprehensive study on this problem using a more sophisticated higher order scheme and found that ring formation is sensitive to the distribution of angular momentum. Norman (1980) and Norman et al. (1980) claim that a ring wave will be excited if angular momentum is transported toward the inner region of the cloud, e.g. by the artificial effect of numerical diffusion. By contrast, transport of angular momentum away from the center into the outer parts of the cloud can easily impede ring formation (see, e.g. Regev and Shaviv, 1980). This behavior leads to the conjecture that the particular initial conditions (homogeneous spherical cloud, rigid rotation; isothermality in space and time) chosen for these 2D-models define a set of "indeterminacy" in the sense that the topology of the final configuration (disk-like or torus-like) depends critically on the perturbations inferred by any discrete numerical scheme. In this sense the long-standing ring controversy does not exist any more.

Calculations with no essential symmetry assumptions (3D-models) are to serve as the basis for any fragmentation theory. There is an urgent need to explain the existence of stellar clusters, multiple stellar systems, binaries and the general mass spectrum of the fragments as a function of the physical conditions encountered in their parent cloud. 3D-calculations carried out by various investigators (Norman and Wilson, 1978; Cook and Harlow, 1978; Narita and Nakazawa, 1978; Bodenheimer et al., 1980; Boss and Bodenheimer, 1980; Boss, 1980; Różyczka et al., 1980ab) altogether suffer from poor resolution and a lack of essential protostellar physics (see Table 1). With regard to cloud fragmentation, these numerical experiments have at most led to a schematic picture of how mass and angular momentum could be reduced in a series of discrete fragmentation steps (Bodenheimer, 1978). However, a major progress in our understanding of the very early stages of cloud collapse and fragmentation will be achieved, only if much more refined numerical schemes are available that allow one to connect the four physical quantities, i.e. self-gravitation, thermal pressure, angular momentum, magnetic fields, in a consistent way according to the basic evolution equations.

In the following Sections we will give selected examples of 1D-, 2D-, and 3D-collapse models.

II. 1D-MODELS (SPHERICAL SYMMETRY)

There exist already excellent reviews on the spherically symmetric protostellar collapse (e.g. Woodward, 1978). Therefore it may be justified to concentrate on the most recent results. Winkler and Newman (1980ab) have re-calculated Larson's (1969) classic $1M_{\odot}$-cloud collapse using a highly improved numerical technique and, in addition, a much better physical description with respect to thermodynamics and radiative transfer. The standard approximation of radiative conductivity for the energy transport has been replaced by the two time-dependent moment equations for the radiative field (Castor, 1972) and a variable anisotropy- (Eddington-) factor which takes into account the spherical geometry of a protostar's extended envelope. Qualitatively, the results fully confirm Larson's picture exhibiting four main evolutionary stages:

1. A non-homologous isothermal collapse during the first free-fall-time (about 10^5 yr).
2. Formation of an opaque core, rising in temperature and containing only a few percent of the total mass initially.
3. A second collapse triggered by molecular hydrogen dissociation;

again a much denser hydrostatic inner core forms containing just a few tenths of a percent of the total mass. Time scale: about 100 yr.
4. Mass accretion onto this second (stellar) core from a freely falling envelope, matter passing through a strong accretion shock wave. Time scale: 2-3 initial free-fall-times.

Starting out with a slightly Jeans-unstable mass of $1 M_\odot$, a temperature T=10K and density $\rho = 1.2\times10^{-19}$ g/cm^3, Winkler and Newman (1980ab) found the final radius and luminosity of the protostar to be $\approx 2\ R_\odot$ and $\approx 1 L_\odot$, respectively. In the HRD the star would appear at the bottom of the Hayashi track. By contrast, a much denser initial cloud ($\rho = 7\times10^{-9}$ g/cm^3) would collapse within a couple of years to a very luminous pre-main sequence star at the top of the Hayashi-track (Narita et al., 1970). There is, however, no observational support favoring such rapidly evolving protostars.

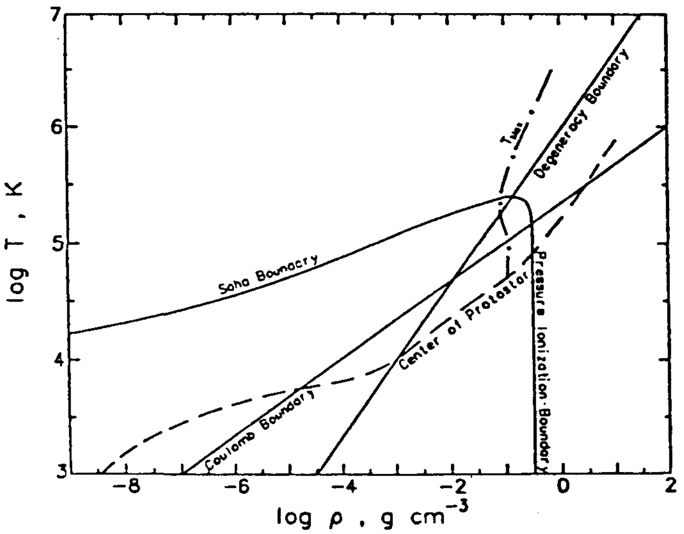

Fig. 1. logρ - log T diagram. Evolutionary track of the protostar's center (dashed line) and the maximum temperature layer off center (dashed-dotted line). Hydrogen is completely ionized above the Saha boundary and to the right of the pressure ionization boundary. Coulomb effects are important to the right and below the Coulomb boundary; electron degeneracy effects are important for a free electron gas below and to the right of the degeneracy boundary (from Winkler and Newman, 1980b).

The innermost 0.03 $M_\odot$ of the protostar exhibits an unexpected thermodynamic behavior of matter which is more reminiscent of a planetary interior rather than to the internal structure of a solar-type pre-main sequence star. Since a great amount of energy is radiated away during the collapse due to the long time scale available, matter becomes dense at relatively low temperatures. As a result, during the accretion phase the temperature maximum develops in a spherical shell containing matter that has not been compressed yet. Figure 1 displays the protostar's evolutionary path in the (central density-central temperature)- diagram. It crosses regions where Coulomb interaction, pressure ionization and degeneracy dominate the thermodynamic properties of protostellar core matter. Deuterium will ignite off center at the point of highest temperature and presumably burn in an extended shell broadened by convective energy transport; it might also be interesting to see in which way hydrogen burning itself will commence after the slow pre-main- sequence contraction has increased the temperatures above 10^7K.

For more massive ($\gtrsim 3M_\odot$) and more luminous (10^4-10^5 $L_\odot$) protostars the interaction of the outgoing radiation with the incoming flow of dust and gas gives rise to the so-called cocoon phenomenon (Kahn, 1974; Yorke and Krügel, 1977; Yorke, 1979) and to a double peaked spectrum emitted by such objects (Yorke, 1977ab, 1980; Bertout and Yorke, 1978).

III. 2D-MODELS (AXIAL SYMMETRY)

A general discussion related to the problems and controversies of the 2D-collapse of rotating clouds, assuming local conservation of angular momentum, has already been given in Section I. Now a typical example of the evolution of a rapidly rotating, gravitationally stable cloud, but far from equilibrium, is briefly outlined. We refer to calculations carried out by Bodenheimer and Tscharnuter (1979). The initial cloud is assumed to be a homogeneous sphere of $1M_\odot$ and rigidly rotating. The parameters α and β denoting the ratios of |thermal-/gravitational-| and |rotational-/gravitational-| energy, respectively, are chosen to be $\alpha = 0.46$ and $\beta = 0.32$. Figure 2 shows the central density as a function of time (solid line). Shortly after the maximum density has been reached a ring-like density wave running outward in the equatorial plane (dashed line) is excited. The ring structure never becomes dense enough to be gravitationally significant. No collapse of the ring onto itself takes place. After a few oscillations the configuration settles into a complete equilibrium (Figure 3). These results depend on the boundary conditions (constant volume). For a constant outer pressure no ring structure would develop (Norman, 1980). Examples of ring collapse for smaller initial β are given, e.g. by Black and Bodenheimer (1976).

Tscharnuter (1978) calculated models with very small β in the range of 10^{-4} - 10^{-2}. In these cases optically thick central regions

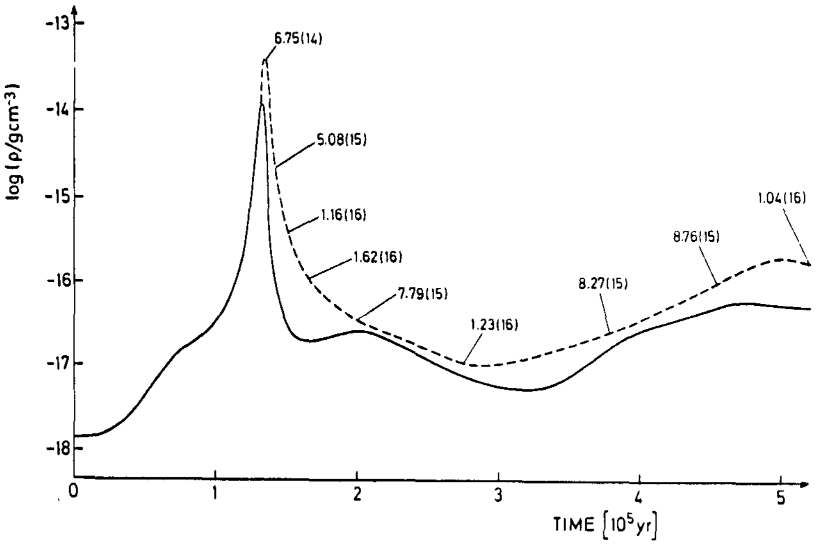

Fig. 2. The evolution of the central density (solid line) and the maximum (ring) density (dashed line) for an initially rapidly rotating isothermal cloud. The distance (in cm) from the rotation axis to the point of maximum density is indicated at various points along the dashed curve (from Bodenheimer and Tscharnuter, 1979).

formed. Nevertheless, after centrifugal forces had eventually caught up with gravity in the core, a toroidal density distribution developed there. According to (unpublished) test calculations β has to be as low as 10^{-5} in order to suppress the ring mode (see also Safronov and Ruzmaikina, 1978). The final outcome is a rapidly rotating hot (≳2000 K) spheroidal core. To summarize these results there is good evidence that the direct formation of a single star with planets around it (like our solar system) out of a rotating interstellar cloud is very unlikely, unless the assumption of local conservation of angular momentum is abandoned.

Indeed, the highly non-equilibrated distribution of mass and angular momentum in our solar system leads to the conjecture that in the primitive solar nebula some mechanism was operating which caused this separation. Transport of angular momentum to the outer regions of the nebula, e.g. by turbulent friction, would immediately imply the desired mass flow toward the center (Lüst, 1952; Lynden-Bell and Pringle, 1974) eventually forming a single central star (the sun). This idea is a very promising possibility to solve the angular momentum problem in cosmogony. Magnetic fields are not very likely to play an important role in an advanced stage of the collapse, because matter is expected to be neutral to a high degree. Simple estimates (Regev and Shaviv, 1980) and preliminary calculations (Tscharnuter, 1980) including turbulent friction - a simple viscosity parameter is $\eta = \xi \cdot c_s \cdot l$,

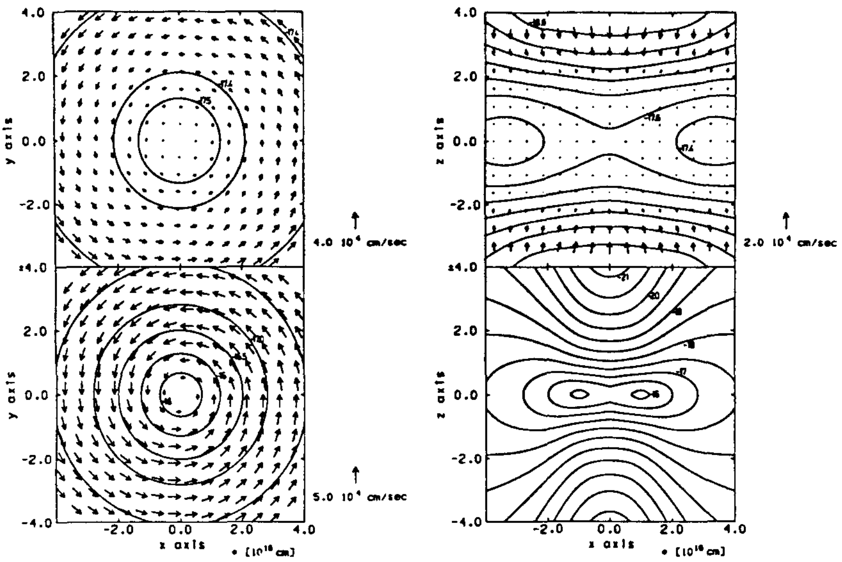

Fig. 3a-d. Equidensity contours and velocity field. Snapshots of the 2D-sequence indicated in Figure 2 at two different cross sections: a,b - at 13 initial free-fall times (7.2×10^5 yr) and c,d - at about 100 initial free-fall times, when final hydrostatic equilibrium was reached. a,c-cross sections in the equatorial plane; b,d - meridional cross sections (from Różyczka et al., 1980b). Numbers indicate the logarithms of the density for selected contours. The length of a velocity vector is proportional to the speed.

where ξ is an efficiency factor, c_s the local sound velocity and l a characteristic length scale, e.g. the thickness of the nebula - have shown that, starting out with a $3M_\odot$-cloud of about 8K and $\alpha \approx 1$, $\beta \approx 10^{-4}$, $\xi = 0.2$, one ends up with a rapidly rotating central "sun" of $0.5M_\odot$ surrounded by a "solar nebula" containing only a few 10^{-3} $M_\odot$ It extends from about 5×10^{12} cm to 2.4×10^{14} cm where the rotation is very nearly Keplerian. $2.5M_\odot$ are still farther outside in the collapsing outer envelope gradually feeding the Keplerian disk with matter. These calculations, details of which will be given elsewhere, indicate that

1. transport of angular momentum by turbulent friction could have been a very efficient mechanism for the formation of our solar system about 4.5 billion years ago,
2. models concerning the origin of the solar system which start out with the Sun surrounded by a low-mass nebula obeying a Keplerian rotation law may be considered as a reasonable working hypothesis.

IV. 3D-MODELS

In the past three years 3D-models of collapsing gas clouds with rotation and vanishing magnetic fields have become available. The aim of these first generation of 3D-models is to study fragmentation processes in interstellar clouds on a physical basis. Bodenheimer et al. (1980) explored the time evolution of "blobs" embedded in a homogeneous background to observe either damping or growth for a particularly chosen pair of the parameters α and β during the overall collapse. Another possibility is to begin with a perturbation in the initial velocity field (in general assumed be zero). Calculations made by Rózyczka et al. (1980a) starting out with a random velocity field indicate that fragments will grow, only if the original cloud contains several ten Jeans masses. Otherwise pressure effects would smooth out any initially developing non-homogeneity during the first free-fall-time.

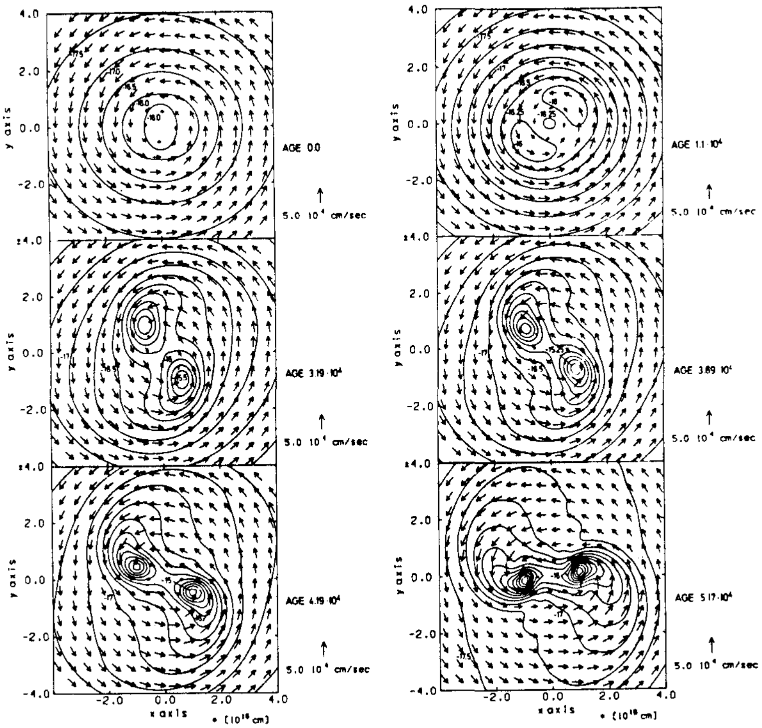

Fig. 4. Model from Fig. 3c,d with a 10% azimuthal (2ϕ-) mode density perturbation (at zero age), and its subsequent 3D-evolution. All figures are cross sections in the equatorial plane (from Różyczka et al., 1980b).

Figure 4 displays a nice example of how 3D-calculations may considerably change the results obtained by 2D-models. Różyczka et al. (1980b) have shown that the axially symmetric equilibrium (see Section III) found by Bodenheimer and Tscharnuter (1979) is unstable to an azimuthal (2ϕ-) perturbation. As a result, the stationary ring splits into two fragments orbiting each other. Such behavior of self-gravitating isothermal gaseous rings was first discovered by Norman and Wilson (1978). Bodenheimer (1978) compiled a fragmentation scheme, based on the transformation of the cloud's spin into orbital angular momentum of the fragments. When applied for each subsequent fragmentation step, this process yields a simultaneous reduction of mass and specific angular momentum. In this way it is easy to fragment a 10^4 $M_\odot$-cloud with density of 10^{-23} g/cm^3 and specific angular momentum of 10^{24} cm^2/s several orders of magnitude down to main-sequence values within 4-6 steps.

Keeping in mind that Bodenheimer's fragmentation scheme is just a far reaching extrapolation of rather crude 3D-models and, hence, must not be over-interpreted, it is nevertheless an important first contribution to a general fragmentation theory based on detailed hydrodynamical calculations.

V. CONCLUSIONS, REMAINING PROBLEMS

Spherically symmetric (1D) models of cloud collapse have become highly sophisticated during the past decade. This refers not only to the realistic physics taken into account but also to the numerical techniques developed for that purpose. The only weak points remaining concern the onset of thermonuclear burning during the accretion phase and the non-grey radiative transfer in the envelope, which ought to be consistent with the hydrodynamics. Non-grey radiative transfer has been included by Yorke (1977a, 1980) in order to explore the expected spectral appearance of massive protostars. Unfortunately, for solar-type protostars such calculations have not yet been carried out in a really consistent way (for an approximate solution see Bertout, 1976).

Axially symmetric (2D-) models exhibit difficulties with numerical accuracy and resolution. For this reason the dynamical ring formation, though found by many investigators using quite different numerical strategies, is still not a universally accepted process. Furthermore, the concept of turbulence in a collapsing rotating cloud is basically not worked out and remains physically obscure in the context of cosmogonic problems. At the same time it is evident that without transport of angular momentum by turbulent friction a planetary system like ours could hardly form.

Needless to say, resolution and accuracy problems are still more severe for 3D-models. At present there is no hope at all that we will be able to attack successfully the crucial question of how star clusters and the corresponding mass spectrum would look like from a theoretical point of view. Indeed, we are rather restricted to explore single processes such as the stability properties of blobs or rings in a rotating cloud as a function of the initial conditions. It should also be emphasized that there exist no 3D-models for collapsing magnetic clouds. In which way and to which extent magnetic fields will change our knowledge about cloud collapse is an open question.

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