SYNCHROTRON RADIATION IN DIRECTIONS CLOSE TO MAGNETIC-FIELD LINES

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Abstract (*Astrophys. Space Sci.*). It is characteristic of the radiation from a particle of mass m bearing a charge e moving with ultrarelativistic velocity $\boldsymbol{\beta}c$ in a magnetic field of induction $\mathbf{B}_0$ that the bulk of the emission is confined to a small cone of directions within $O(\xi)$ of the direction of motion $\boldsymbol{\tau}$ which makes a constant angle α with $\mathbf{B}_0$, and that for any direction $\mathbf{n}$ at an angle θ with $\mathbf{B}_0$ such emission is observed as harmonics of the fundamental frequency $f_{B_0}\xi/a$, where f_{B_0} is the nonrelativistic cyclotron frequency $eB_0/2\pi m$, $\xi=\sqrt{(1-\beta^2)}\ll 1$, and

$$a=1-\beta\cos\alpha\cos\theta.$$

It follows that the radiation from a distribution of particles in general appears as a series of lines, broadened with respect to the dependence of the distribution on both pitch angle and energy $E=mc^2/\xi$.

In directions away from the field lines, $\sin\theta=O(1)$, $a\simeq\sin^2\theta$, and the bulk of the emission is in the high-order harmonics $n=O(\xi^{-3})$. Although the broadening with respect to pitch angle is slight, the observed radiation is quasicontinuous.

In directions close to the field lines, for which $\sin\theta=O(\xi)$, $a\simeq\frac{1}{2}(\xi^2+\sin^2\alpha+\sin^2\theta)= =O(\xi^2)$, so that pitch-angle broadening is significant and there is a significant contribution to the radiation from low-order harmonics. Specifically, if $N(1/\xi)\,\mathrm{d}(1/\xi)$ is the number density of particles in the energy range $(E, E+\mathrm{d}E)$ and $2\pi\phi(\alpha)\sin\alpha\,\mathrm{d}\alpha$ is the fraction of these in the pitch-angle range $(\alpha, \alpha+\mathrm{d}\alpha)$, the emissivity tensor corresponding to the nth harmonic is, to a first approximation, distributed with respect to the frequency f according to the formulae

$$\boldsymbol{\eta}_n(\mathbf{n})=\int_0^{nf_{B_0}/\sin\theta}\boldsymbol{\eta}_{nf}(\mathbf{n})\,\mathrm{d}f,$$

$$\boldsymbol{\eta}_{nf}(\mathbf{n})=\frac{2\pi n^2y^3}{f_{B_0}}\int_{ny-\sqrt{(n^2y^2-\sin^2\theta)}}^{ny+\sqrt{(n^2y^2-\sin^2\theta)}}N\left(\frac{1}{\xi}\right)\phi(\alpha)\langle\mathbf{P}_n(\mathbf{n})\rangle\,\mathrm{d}\xi,$$

where $y=f_{B_0}/f$ and is of magnitude $O(\xi/n)$, and, in terms of base vectors $\mathbf{i}_1$ along the projection of $\mathbf{B}_0$ on the plane transverse to $\mathbf{n}=\mathbf{i}_3$ and $\mathbf{i}_2=\mathbf{i}_3\times\mathbf{i}_1$, the emission polarization tensor

$$\langle\mathbf{P}_n(\mathbf{n})\rangle=\frac{\mu e^2cf_{B_0}^2}{2n^2y^4\xi^2}\left[\left(\frac{ny\xi-\sin^2\theta}{\sin\theta}\right)^2J_n^2\left(\frac{\sin\alpha\,\sin\theta}{y\xi}\right)\mathbf{i}_1\mathbf{i}_1\mp\right.$$

Gordon Newkirk, Jr. (ed.), Coronal Disturbances, 295–297.

$$\mp i \frac{ny\xi - \sin^2\theta}{\sin\theta} \sin\alpha\, J_n\left(\frac{\sin\alpha\,\sin\theta}{y\xi}\right) J_n'\left(\frac{\sin\alpha\,\sin\theta}{y\xi}\right)(\mathbf{i}_1\mathbf{i}_2 - \mathbf{i}_2\mathbf{i}_1) +$$

$$+\sin^2\alpha\, J_n'^2\left(\frac{\sin\alpha\,\sin\theta}{y\xi}\right)\mathbf{i}_2\mathbf{i}_2\Bigg],$$

according as $\cos\theta \gtrless O$. The pitch angle α is given in terms of ξ and y by the relation

$$\sin\alpha = \sqrt{(2ny\xi - \xi^2 - \sin^2\theta)}.$$

Since the argument of the Bessel functions is $n \times O(1)$, it is not possible to make a further simplification without making further assumptions. For emission in directions close to the field lines it is appropriate to take $\sin\theta \ll \xi$ and to expand the formulae in powers of $(\sin\theta)/\xi$. Then, for $n=1$, the magnitude of the expression in square brackets is $O(\xi^2)$, and for each subsequent harmonic it is reduced by the factor

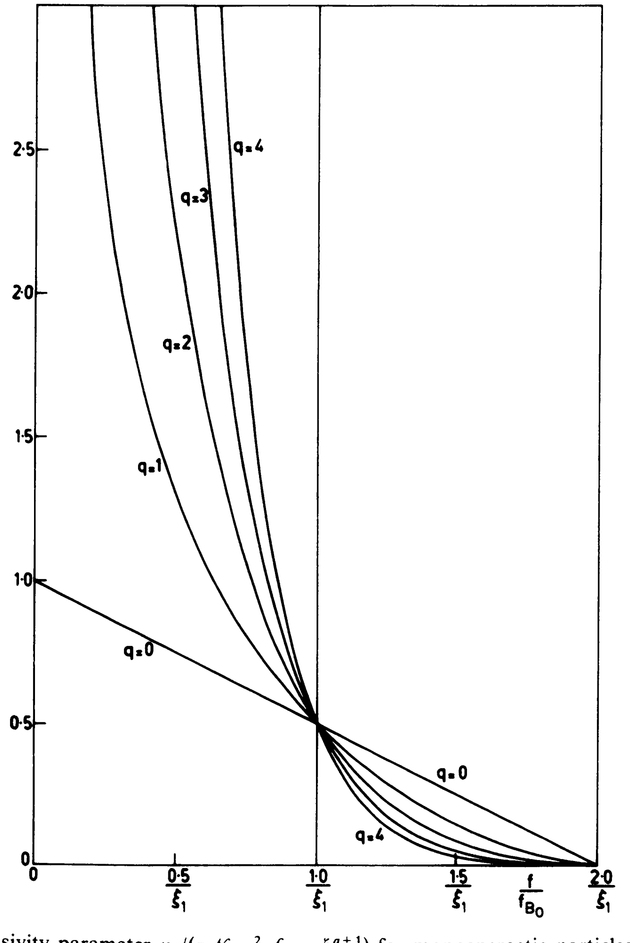

Fig. 1. The emissivity parameter $\eta_f/(\pi\mathcal{N}\mu e^2 c f_{B_0} a_q \xi_1^{q+1})$ for monoenergetic particles with pitch-angle distribution $\phi(\alpha) = a_q \sin^q\alpha$.

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$O((\sin^2\theta)/\xi^2)$. The approximations have been carried through for the first two harmonics, giving formulae for the total emissivity tensor

$$\boldsymbol{\eta}_f(\mathbf{n}) = \sum_n \boldsymbol{\eta}_{nf}(\mathbf{n})$$

of consistent accuracy and showing that, contrary to what has previously been reported, the degree of circular polarization is $1+O((\sin^4\theta)/\xi^4)$, RH for $\cos\theta > O$ and LH for $\cos\theta < O$.

In particular, for a monoenergetic distribution for which $N(1/\xi) = \mathcal{N}\delta(1/\xi - 1/\xi_1)$, we find as a first approximation to the emissivity

$$\eta_f(\mathbf{n}) = \pi\mathcal{N}\mu e^2 c f_{B_0}\, \phi(\alpha_1)\, \xi_1 \left(1 - \frac{\xi_1 f}{2f_{B_0}}\right),$$

where the pitch angle α_1 corresponding to the frequency f is given by

$$\frac{\sin\alpha_1}{\xi_1} = \left(\frac{2f_{B_0}}{\xi_1 f} - 1\right)^{1/2}.$$

A particular case of the dependence of emissivity on pitch-angle distribution is exhibited in Figure 1 where we have taken

$$\phi(\alpha) = a_q \sin^q \alpha,$$

a distribution which varies from isotropic with respect to velocity when $q=1$ to a concentration of flat helical trajectories about the value $\alpha = \frac{1}{2}\pi$ when q is increased, resulting in reduced emission close to $\theta = 0$ and π. The portion of the curves for which $O \leqslant f/f_{B_0} < 1/s\xi_1$ correspond to $(\sin\alpha_1)/\xi_1 > (2s-1)^{1/2}$, and should be ignored when s is so large that the requirement that $(\sin\alpha_1)/\xi_1$ should be $O(1)$ is breached. They should all fall sharply to zero at $f/f_{B_0} = \frac{1}{2}\xi_1$.