Cosmic Magnetic Field:
From Planets, to Stars and Galaxies.
Proceedings IAU Symposium No. 259, 2008
K.G. Strassmeier, A.G. Kosovichev & J.E. Beckman, eds.

doi:10.1017/S1743921309030270

Axisymmetric magneto-rotational instability in viscous accretion disks

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Abstract. Axisymmetric MRI in viscous accretion disks is investigated. The linear growth of the viscous MRI is characterized by the Reynolds number $R_{\rm MRI} \equiv v_A^2/\nu\Omega$, where v_A is the Alfvén velocity, ν is the kinematic viscosity, and Ω is the angular velocity of the disk. Although the linear growth of the MRI is suppressed as the Reynolds number decreases, its nonlinear behavior is found to be almost independent of $R_{\rm MRI}$. At the nonlinear stage, the channel flow grows and the Maxwell stress increases even though $R_{\rm MRI}$ is much smaller than unity. Nonlinear behavior of the MRI in the viscous regime can be explained by the characteristics of the linear dispersion relation. Applying our results to the case with both viscosity and resistivity, it is anticipated that the critical value of the Lundquist number $S_{\rm MRI} \equiv v_A^2/\eta\Omega$ for active turbulence would depend on the magnetic Prandtl number $Pm \equiv \nu/\eta$, where η is the magnetic diffusivity.

Keywords. Turbulence – accretion disks – methods: numerical

1. Introduction

The magnetic Prandtl number Pm takes a wide range of values in astrophysical disk systems. In protoplanetary disks, the magnetic Prandtl number is much smaller than unity because of their low ionization degree (Nakano 1984). In the disks of compact X-ray sources and AGNs, it ranges from $\simeq 10^{-3}$ to 10^3 depending on the disk radius (Balbus & Henri 2008). More systematic study on the MRI in the presence of both viscosity and resistivity is thus quite important for understanding the disk accretion.

One important unsettled matter is the role of the viscosity at the nonlinear stage of the MRI. In general, the viscosity as well as the resistivity can suppress the linear growth of the MRI. However the dependence of nonlinear outcome on the Prandtl number indicates that the role of the viscosity in MRI turbulence could be different from that of the resistivity (Lesur & Longaretti 2007; Fromang et al. 2007). Focusing on two non-dimensional parameters, Reynolds number $R_{\rm MRI}$ and Lundquist number $S_{\rm MRI}$, we clarify the difference in nonlinear features of the MRI between the viscous and resistive systems.

2. Results

The time- and volume-averaged $\alpha_{\rm tot}$ $[\equiv (\langle v_x \delta v_y\rangle - \langle B_x B_y\rangle/4\pi)/\langle P\rangle]$ at the nonlinear stage is depicted as a function of the initial $R_{\rm MRI}$ and $S_{\rm MRI}$ in Fig. 1. The diamonds show the results in the viscous fluid, and the crosses are those in the resistive one. The upward arrow over-plotted on the symbols denotes that the value is the lower limit, and the downward arrow stands for decaying models and thus the stress is the upper limit.

The stresses at the nonlinear stage are almost the same when $R_{\rm MRI} \gtrsim 1$ or $S_{\rm MRI} \gtrsim 1$. However, a huge difference can be seen in the highly diffusive regime. In the presence of the ohmic dissipation, the stress rapidly decreases with decreasing $S_{\rm MRI}$. For the models

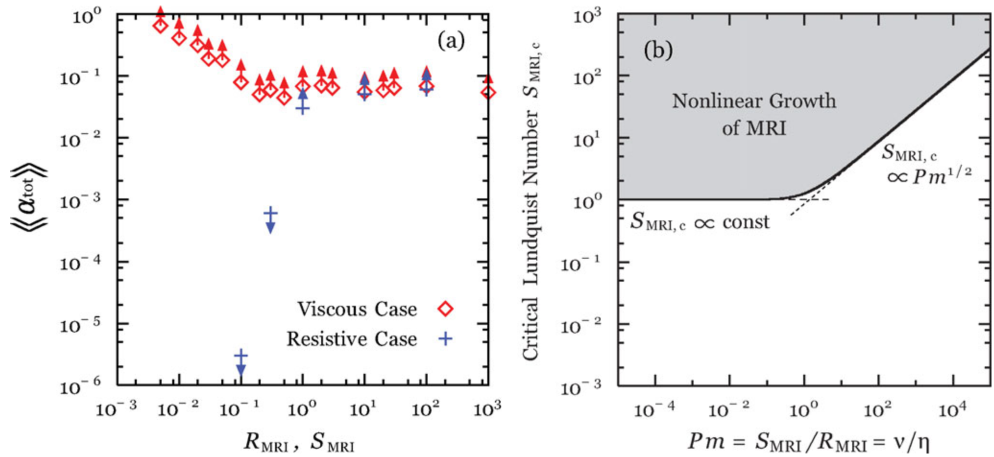

Figure 1. (a) Time- and volume-averaged α_{tot} at the nonlinear stage as a function of the initial Reynolds number R_{MRI} and Lundquist number S_{MRI}. (b) The critical Lundquist number for active MRI-driven turbulence as a function of the magnetic Prandtl number Pm.

with the kinematic viscosity, on the other hand, it increases with the decrease of R_{MRI}. The inverse correlation between $\langle\langle\alpha_{tot}\rangle\rangle$ and R_{MRI} for the cases with large viscosity could be originated from stable growth of a channel flow [see Masada & Sano 2008 (MS08)].

3. Discussion

We focus on the critical wavelength obtained from the linear theory to briefly explain the nonlinear behavior of the MRI. In the viscous fluid, the critical wavelength is given by $\lambda_{crit} \simeq v_A/\Omega$ despite the size of R_{MRI}. Even if R_{MRI} is much smaller than unity, the critical wavelength thus shifts to larger scale as the instability grows. Then the system always evolves toward a less dissipative state and is not saturated. On the other hand, in the resistive case, there is a critical point at which the critical wavelength switches from the decreasing function of the field strength to the increasing one. The MHD turbulence can thus decay in resistive fluid if $S_{MRI} \lesssim 1$ (see MS08 for more details).

Our results suggest that the nonlinear stage of the MRI can be anticipated by the shape of the critical wavelength. We can thus predict the nonlinear behavior of the MRI-driven turbulence in the system with both viscosity and resistivity from the linear dispersion relation (Pessah & Chan. 2008). In Fig. 1b, the critical Lundquist number $S_{MRI,c}$ is plotted as a function of Pm. This diagram implies that the MRI turbulence would be suppressed if S_{MRI} is less than a critical value. In the regime of $Pm \gg 1$, it is proportional to the square root of Pm and remains to be constant in the range $Pm \ll 1$. It is interesting that this can reproduce the result of the three-dimensional study for the doubly diffusive MRI performed by Fromang *et al.* (2007) qualitatively.

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