CONSTRAINTS ON STELLAR INTERIOR PHYSICS FROM HELIOSEISMOLOGY

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1. Introduction

Traditional observations of the properties of stars generally provide tests of only the gross aspects of stellar structure and evolution. The limitation lies in the amount and precision of the available data of relevance to the structure of the stellar interior, *i.e.*, the determination of stellar effective temperatures, surface composition, luminosities and, in a few cases, masses. Additional constraints on the observed stars, such as the common age and composition normally assumed for stars in clusters or multiple systems, clearly increase the information. However, detailed information on the physics and processes of stellar interiors requires more extensive data, with a dependence on stellar structure sufficiently simple to allow unambiguous interpretation. Such data are offered by observations of stellar pulsation frequencies: they can be observed with great accuracy and their dependence of stellar structure is generally well understood. In particular, the richness and precision of the observed frequencies of solar oscillation are now offering a detailed view on the interior properties of a star.

Reviews on solar oscillations were provided by, *e.g.*, Gough & Toomre (1991). The modes are characterized by a degree l measuring, approximately, the number of wavelengths in the stellar circumference. For each l, there is a set of possible modes of oscillation, characterized by the radial order n, and with angular frequencies ω_{nl}. In a spherically symmetric model the frequencies are independent of the azimuthal order m. This degeneracy is lifted by rotation or other departures from spherical symmetry.

In the Sun, modes are observed at each l between $l = 0$ and at least 2000, with cyclic frequencies $\nu = \omega/2\pi$ between about 1000 and 5000 μHz. The relative standard deviations are less than 5×10^{-6} in many cases, making

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these frequencies by far the most accurately known properties of the Sun. The observed modes are essentially standing acoustic waves, propagating between a point just below the photosphere and an inner turning point at a distance r_t from the centre such that $c(r_t)/r_t = \omega/\sqrt{l(l+1)}$, c being the adiabatic sound speed. Over the observed range of modes r_t moves from very near the centre of the Sun to just below the solar surface. This variation of the region to which the mode frequencies are sensitive permits inverse analyses to determine localized properties of the solar interior.

The analysis of the observed frequencies is generally aimed at determining differences between the Sun and reference solar models and hence inferring the errors in the assumed physics or other properties of the models. The quality of the helioseismic data has inspired considerable efforts to improve the solar model computations, by including as far as possible known processes and by using the most precise description of the physics available. In particular, diffusion and gravitational settling, which have often been neglected in the past, have a substantial effect on the models at this level of precision and hence must be included.

Here I shall use as reference the so-called Model S of Christensen-Dalsgaard *et al.* (1996), which includes diffusion and settling of helium and heavy elements; OPAL equation of state (Rogers, Swenson & Iglesias 1996) and opacity (Iglesias, Rogers & Wilson 1992) were used, as well as nuclear reaction rates largely from Bahcall & Pinsonneault (1995). This, as well as other models discussed here, were calibrated to present solar luminosity and radius, as well as to the observed surface ratio $Z_s/X_s = 0.0245$ between the heavy-element and hydrogen abundances, by adjusting the initial composition and the mixing-length parameter.

2. Results of helioseismic inversion

The differences between the observed and model frequencies are small, of order 0.3 % or less. This motivates analysis in terms of linearized relations between the frequency differences and differences in suitable sets of model variables. Here I use (c^2, ρ), ρ being density. It is possible to form linear combinations of the frequency differences, in such a way as to obtain a measure of the sound-speed difference $\delta c/c$, localized to a small region of the Sun, while suppressing the influence of the difference in ρ, of uncertainties in the modelling of the near-surface region of the Sun and of observational errors (for details, see for example Basu *et al.* 1996). Differences between the Sun and Model S obtained in this manner are shown in Fig. 1, based on frequencies resulting from a combination of observations from the BiSON network (Chaplin *et al.* 1996) and the LOWL instrument (Tomczyk *et al.* 1995). As indicated by the horizontal bars, the points provide averages of

$\delta c^2/c^2$ over limited regions in the Sun, from the centre to near the surface. Furthermore, the estimated random errors in the result, based on the quoted errors in the observed frequencies, are minute, less than 2×10^{-4} in most of the solar interior. Thus it is in fact possible to measure a property of the solar interior as a function of position, with great precision.

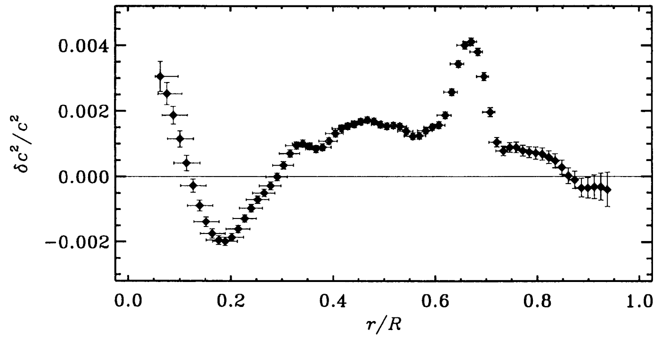

Figure 1. Inferred difference in squared sound speed, in the sense (Sun) – (model). The horizontal error bars mark the first and third quartile points of the averaging kernels, whereas the vertical error bars show 1-σ errors, as progated from the errors in the observed frequencies. From Basu *et al.* (1997).

It is evident, also, that at a superficial level the agreement between the Sun and the model is excellent: we have been able to predict the solar sound speed with a precision of better than 0.2 %. This has required improvements in the modelling inspired by the high accuracy of the observations; however, the calculation involves no adjustment of parameters to fit the model to the data. On the other hand, the difference between the Sun and the model is highly significant, given the very small error in the inferred difference. Thus, in that sense, the model is hardly satisfactory.

The oscillations depend essentially only on the dynamical properties of the Sun, *e.g.*, pressure, density and sound speed. Since, approximately, $c^2 \propto T/\mu$, where T is temperature and μ the mean molecular weight, helioseismology constrains T/μ but not T and μ separately. This has ramifications for the possibility of helioseismic constraints on the solar neutrino production (*e.g.* Antia & Chitre 1995; Christensen-Dalsgaard 1997).

The dependence of the oscillation frequencies on azimuthal order m carries information about the solar internal angular velocity. Inversion of this dependence shows that in the convection zone rotation varies with latitude approximately as on the surface, with modest dependence on r. Near the base of the convection zone there is a rapid transition, over less than about $0.1R$, to rotation depending little on r and latitude in much of the radiative interior (*e.g.* Thompson *et al.* 1996; Kosovichev 1996).

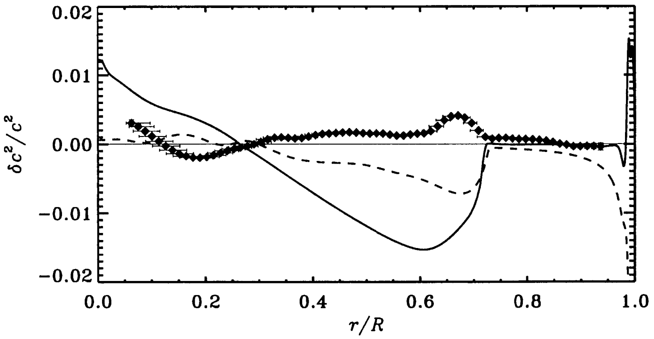

Figure 2. Relative differences in squared sound speed between models with modified physics and the standard case, in the sense (modified model) – (standard model). The solid line shows the effect of neglecting settling, while the dashed line shows the effect of using the Los Alamos Opacity Library rather than the OPAL tables. Symbols show the inferred difference between the Sun and the standard model, as in Fig. 1.

3. Effects of modifying the physics

To evaluate the significance of the comparatively close agreement between the "standard" Model S and the Sun it is necessary to consider other models with differing assumptions or physics. An important example is the inclusion of settling and diffusion. Figure 2 compares the difference between a model without these effects, but using otherwise the same physics, and Model S, with the difference between the Sun and Model S. It is evident that neglect of settling would very considerably worsen the agreement between the Sun and the models (see also Cox, Guzik & Kidman 1989; Christensen-Dalsgaard, Proffitt & Thompson 1993; Bahcall *et al.* 1997). The figure also shows the effect of replacing the OPAL opacities with the older Los Alamos Opacity Library (Huebner *et al.* 1977). Clearly the revision of the opacity has improved the agreement between the model and the Sun considerably, although less so than the inclusion of settling. This also suggests that the apparent improvement brought about by including settling is not compromised by opacity errors, as suggested by Elliott (1995).

Very considerable effort has gone into work on the equation of state (*e.g.* Däppen 1992), with corresponding improvements in the agreement between the resulting models and the observed frequencies (*e.g.* Christensen-Dalsgaard, Däppen & Lebreton 1988). Detailed analyses have demonstrated the ability of the helioseismic data to probe subtle properties of the thermodynamic state of the solar plasma (*e.g.* Christensen-Dalsgaard & Däppen 1992; Vorontsov, Baturin & Pamyatnykh 1992; Elliott 1996); this allows the use of the solar convection zone, where the structure depends largely on the equation of state, as a laboratory for plasma physics.

4. What is wrong with the solar model?

Despite the improvements in solar modelling and in the agreement between the model and the Sun, the remaining highly significant discrepancy between the model and the Sun indicates a lack in our understanding of stellar evolution; although modest in the solar case, the error could quite possibly have more substantial effects in other stars where conditions are more extreme than in the Sun.

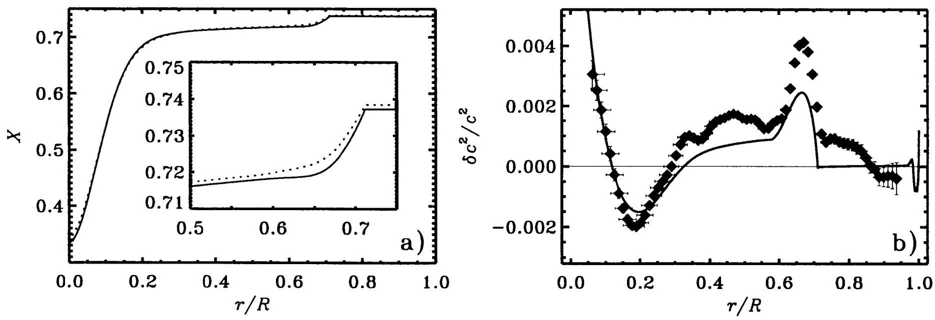

Figure 3. (a) Profiles of the abundance X by mass of hydrogen. The solid line shows the profile in Model S of Christensen-Dalsgaard *et al.* (1996), whereas the dotted line shows a modified profile aimed at trying to match the sound-speed difference shown in Fig. 1 between the Sun and the model. (b) Difference in squared sound speed between the model with modified X-profile and Model S. The symbols show the inferred difference between the Sun and Model S, as in Fig. 1. Adapted from Bruntt (1996).

The dominant features in the sound-speed difference occur in regions of the model with strong composition gradients (*cf.* Fig. 3), resulting from nuclear burning in the core or helium settling from the convection zone. These gradients would be affected by "non-standard" processes causing mixing in convectively stable regions. Mixing in the core could increase the hydrogen abundance at the centre of the model while reducing it at the edge of the core; the central sound speed would therefore be increased, and the sound speed at the edge of the core reduced, as required by Fig. 1. Similarly, weak mixing beneath the convection zone would increase the hydrogen abundance and sound speed in this region, again potentially according for the observed bump. As a toy model of such processes, Fig. 3 also shows an artificially modified hydrogen profile and the corresponding change in the sound speed, confirming that redistribution of hydrogen can in fact largely account for the observed behaviour.

Such suggestions evidently require physical mechanisms for the mixing. Just beneath the convection zone the steep gradient in the helioseismically inferred rotation rate is likely to be associated with circulation which could cause mixing (Spiegel & Zahn 1992; Gough *et al.* 1996; Elliott 1997). Mixing might also be caused by instabilities associated with the spin-down of the Sun from the usually assumed initial state of rapid rotation (*e.g.* Chaboyer *et al.* 1995), or by penetration of convection beyond the unstable region. Independent evidence for mixing beneath the convection zone is provided by the destruction of lithium and beryllium (*e.g.* Chaboyer *et al.* 1995).

There appears to be no similarly simple explanation of potential core mixing. However, the Sun has been shown to be unstable to low-order, low-degree g modes, at least during earlier phases of its evolution (*e.g.* Christensen-Dalsgaard, Dilke & Gough 1974); it is conceivable that the nonlinear development of these modes can lead to mixing (Dilke & Gough 1972). The rotational spin-down might also lead to mixing of the core. If such processes were to be common to low-mass stars, they would have a substantial influence on our understanding of stellar evolution, including an increase in the estimated ages of globular clusters and hence in the discrepancy with the cosmologically inferred age of the Universe.

Unfortunately, substantial contributions to the difference between the model and the Sun might come from perhaps less interesting errors in the basic physics. Indeed Tripathy *et al.* (1997) showed that the sound-speed difference in Fig. 1 can be largely reproduced by modifications to the opacity of less than about 5 %. While this is certainly smaller than the generally assumed uncertainty in current opacity calculations, it remains to be seen whether the specific change required is physically plausible.

Finally, I note that there is evidence for errors in the equation of state in and below the helium ionization zone (*e.g.* Dziembowski, Pamyatnykh & Sienkiewicz 1992), even when using the OPAL equation of state (*e.g.* Basu & Christensen-Dalsgaard 1997). The effect is small in the Sun, but it might be substantial in lower-mass stars where non-ideal plasma effects could be stronger.

5. Relation to stellar astrophysics

The helioseismic results clearly give some confidence in modelling of stellar evolution. However, in part this relative success of the solar models undoubtedly stems from the fact that the Sun is a comparatively simple type of star: for example, at only slightly higher mass than solar the problems of a convective core would play a major role. The ability to cover a broad range of parameters makes investigations of other stars, "classical" as well as seismological, an essential complement to the solar studies, even though they can never be as detailed and precise as those obtained for the Sun. For example, discrepant period ratios in models of double-mode Cepheids and δ Scuti stars led to the prediction of a substantial increase in opacities (*e.g.* Simon 1982; Andreasen & Petersen 1988), at temperatures in the range $10^5 - 10^6$ K; this falls within the solar convection zone and hence would have no effect on solar structure. The opacity increase in the OPAL tables has in fact largely resolved this discrepancy (*e.g.* Moskalik, Buchler & Marom 1992). Similarly, properties of convective cores, including the important but highly uncertain question of mixing beyond the region of instability, might well be studied from observations of solar-like oscillations in other stars (Kjeldsen, these proceedings) or observation of sufficiently detailed spectra of oscillations in, for example, δ Scuti or β Cephei stars.

Independent stellar information may also help to compensate for the non-uniqueness in the physical interpretation of the helioseismic data, illustrated in the preceding section. For example, investigations of element abundances may provide further insight into the physics of mixing beneath stellar convection zones (*e.g.* Baglin & Lebreton 1990).

6. Concluding remarks

Major advances in helioseismology will result from the extensive new data from the GONG network (*e.g.* Harvey *et al.* 1996) and from the instruments on the SOHO satellite (*e.g.* Scherrer *et al.* 1996), as well as from the continued observations from other ground-based instruments. As a result, we shall be able to investigate in more detail solar structure, not least in the core and the convection zone, as well as rotation and other aspects of solar internal dynamics. In parallel with this, major advances in the study of stellar oscillations may lead to the definite detection of solar-like oscillations in other stars and a detailed analysis of oscillations in δ Scuti stars and other "classical" pulsators. Finally, the new very large telescopes and advances in stellar-atmosphere modelling (Gustafsson, these proceedings) are likely to lead to major improvements in our knowledge about stellar composition and the processes that control it, for a variety of stars.

In this way we shall obtain much firmer tests of stellar modelling, reaching beyond the fundamental properties of stars to the even more fundamental questions of their physical basis.

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Discussion of this paper appears at the end of these Proceedings.