THE EVOLUTION OF THE MAGNETIC INCLINATION AND BEAM RADIUS OF PULSARS

XINJI WU
CCAST (World Laboratory) and Department of Geophysics, Peking University

WEN XU
Department of Physics, Peking University

Abstract

A new parameter K is defined which becomes very important in our study of pulsar evolution. The distribution of pulsars in the K *vs.* $\log t_c$ diagram reveals some constraints which can best be described by theoretical curves.

The results show that the evolution of a pulsar is characterized by two different stages corresponding to the decrease of the emission-cone width and the alignment of the magnetic axis, respectively. The decay time of the alignment of the magnetic inclination is 1.5×10^7 yr. The time scale is 6×10^4 yr when the value of ρ_t drops to e times its final value ($t \to \infty$). Investigation of the birth values $\alpha(0)$ lends support to a random orientation of the pulsar magnetic axis with respect to the spin axis. The evolution limit of ρ is shown to have a Gaussian distribution around a central value of 5.6° with $\sigma = 2.0°$. As a result of this K analysis, new methods are suggested to determine the birth values $\alpha(0)$, the final values of ρ_0, and the pulsar's overall geometry.

Introduction

Polarization has played a key role in our understanding of the evolution of pulsars. According to the polar-cap model, we get the relation:

$$K = \frac{\sin \Delta\psi}{\sin \Delta\phi} = \frac{\sin \alpha}{\sin \rho} \tag{1}$$

where $\Delta\psi$ is the position-angle swing of linear polarization, $\Delta\phi$ the apparent beamwidth, α the magnetic inclination, and ρ the width of the emission cone. The equation has a very clear physical meaning. On the left side, only observable quantities appear, where the right side exactly describes a pulsar's "relative" geometry. The ratio is defined as the K parameter, which can be deduced easily from the observables $\Delta\psi$ and $\Delta\phi$.

K varies with time when α and ρ vary with time individually. The evolution of K therefore results from the evolution of both α and ρ. According to Candy and Blair's model (1986), the angle α decreases as

$$\sin \alpha(t) = \sin \alpha(0) e^{-t/\tau}. \tag{2}$$

The angular width of the emission cone has a time dependence of the form:

$$\rho(t) = \rho_0 (1 - e^{-2t/\tau})^{-\gamma/(n-1)} \tag{3}$$

$$\rho \propto p^{-\gamma} \tag{4}$$

where $\alpha(0)$ and ρ_0 are constants, γ is taken as 1/3 and n as 2.5 in this paper. τ is the alignment time scale, and t is the pulsar age which is related to characteristic age by:

$$t = (\tau/2) \ln(1 + 4t_c/(n-1)\tau) \tag{5}$$

Candy and Blair (1986) drew a diagram in which $(d\psi/d\phi)_{\max}$, the maximum rate of position-angle swing of linear polarization, varies with characteristic age and discussed the evolution of pulsars. There are two fatal weaknesses. First, the assumption that $\beta = \rho/2$ is not necessarily true; the accurate formula is

$$\left(\frac{d\psi}{d\phi}\right)_{\max} = \frac{\sin \alpha}{\sin \beta} = \frac{\sin \alpha}{\sin \rho/2} \frac{\sin \rho/2}{\sin \beta} \simeq \frac{\sin \alpha}{\sin \rho/2} \frac{2}{Q} \tag{6}$$

where the Q is defined by Wu *et al.* (1986) and $Q \simeq \beta_n$, the parameter used by Lyne and Manchester (1988). Values of both Q and β vary from 0 to 1. Second, according to Lyne and Manchester (1988), there are 7 pulsars with the largest value of $(d\psi/d\phi)_{\max}$ (=50); and these pulsars cover a wide age range of $10^{5.9}$ to $10^{7.4}$ yr. Therefore, no age value can be said to have a "maximum" value of $(d\psi/d\phi)_{\max}$ within this range.

In this paper, we use the new parameter K instead of $(d\psi/d\phi)_{\max}$ to construct a diagram which agrees quantitatively with the predictions of the evolution model. The data used in this paper come from tables 1 and 2 of Lyne and Manchester (1988).

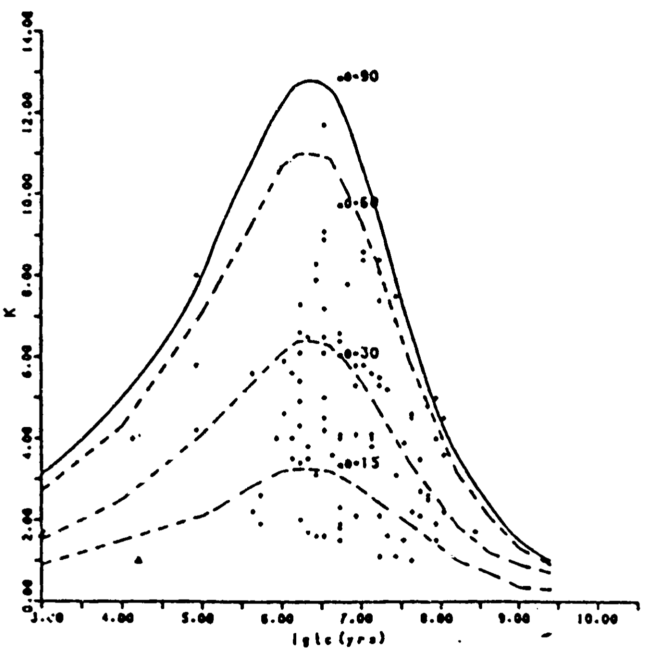

Figure 1 The K *vs.* $\log t_c$ distribution and theoretical fitted curve (solid line) from eq.(6) using $n = 2.5$, $\gamma = 1/3$, $\tau = 1.5 \times 10^7$ yr, and $\rho_0 = 2.9°$. The dashed lines have different values of $\alpha(0)$.

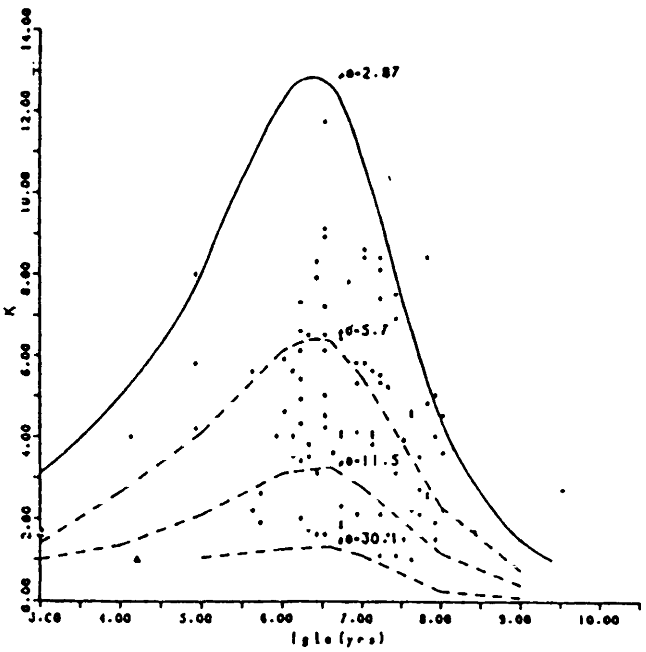

Figure 2 See figure 1 for the key, but use $\alpha(0) = 90°$. The dashed lines have various values of ρ_0.

The K parameter age distribution

As stated above, K values can only be determined if we know the observed polarization angle swing and the pulse width. Plots giving K *vs.* $\log t_c$ show a distribution which can be best explained by our calculated K curves: (from eqs. 1, 2, 3 and 4)

$$K = \frac{\sin \alpha(0) e^{-t/\tau}}{\sin \left(\rho_0 (1 - e^{-2t/\tau})^{\gamma/(n-1)}\right)} \qquad (7)$$

The observed distribution shows a maximum corresponding to a characteristic age of 5×10^6 yr. The evolution time scale is determined by a best fit in which all the observed data points are exactly under the theoretical curve.

The fitted results are $\rho_0 = 2.87°$ and $\tau = 1.5 \times 10^7$ yr when we assume $\gamma = 1/3$ and $n = 2.5$. When $\gamma = 1/3$ is assumed, a better fit is obtained. The fit gives a strict restriction on the possible range of τ because τ controls only the horizontal shift of the curve and has no influence on its shape. It is obvious from the diagram that we cannot cover all the data points unless τ is as large as 1.5×10^7 yr. Our result is consistent with Lyne and Manchester's (1988) value of 10^7 yr. The only two pulsars located outside the curve can be covered if a larger $\tau = 1.5 \times 10^8$ yr or $\rho_0 < 2.87°$ is assumed.

Discussion of $\alpha(0)$ and ρ_0

To explain the scatter of K values in the K *vs.* $\log t_c$ plane, there are three possibilities. First, dispersion of the birth values $\alpha(0)$ can completely account for it. Second, dispersion of ρ_0 can explain the scatter. Third, dispersion of both $\alpha(0)$ and ρ_0 combine to account for the scatter.

1. $\alpha(0)$ scatter

The dispersion of dots under the best fitting curve is naturally explained if $\alpha(0)$ is not equal to 90° in eq.(7). Pulsars are shown to have a birth distribution which ranges over all values of $\alpha(0)$, rather than having all been born as orthogonal rotators $\alpha(0) = 0°$. This accounts for the random locations of the dots at different heights under the fitted curve. Thus we can divide pulsars into groups with different $\alpha(0)$, each typical value of $\alpha(0)$ is shown in the K *vs.* $\log t_c$ diagram. The evolution curve is presented in figure 1. Each group has a common $\alpha(0)$ and therefore a similar evolution history. If $\alpha(0)$ is the only reason for the K dispersion, any pulsar can only evolve from the left to the right along these lines. However, some pulsars with large magnetic inclination (near 90°, *e.g.* some pulsars with an interpulse) are not located near the $\alpha(0) = 90°$ curve. This fact shows that the values of ρ_0 are not constant, and there are some pulsars with larger values of ρ_0 than 2.87°.

2. ρ_0 scatter

We expect that all the pulsars do not have exactly the same physical conditions. The cone radius limit ρ_0 may have some deviation from its average value. The dispersion of the pulsar distribution in the K *vs.* $\log t_c$ diagram can be explained also by the scatter of the values of ρ_0. We compute the evolution curves in figure 2 assuming that $\alpha(0) = 90°$ and the values of ρ_0 range from 2.87° to 30°. We can then divide pulsars into groups with different values of ρ_0. The distribution of ρ_0 can thus be obtained under this assumption. The distribution of ρ_0 is shown in figure 3 [assuming $\alpha(0) = 90°$ for every pulsar] which shows a Gaussian-like distribution with a long tail. There is no reasonable explanation for the long tail. Obviously, it results from the scattering of the values of $\alpha(0)$. The conclusion is that both $\alpha(0)$ and ρ_0 have appreciable scatter and thus particular distributions.

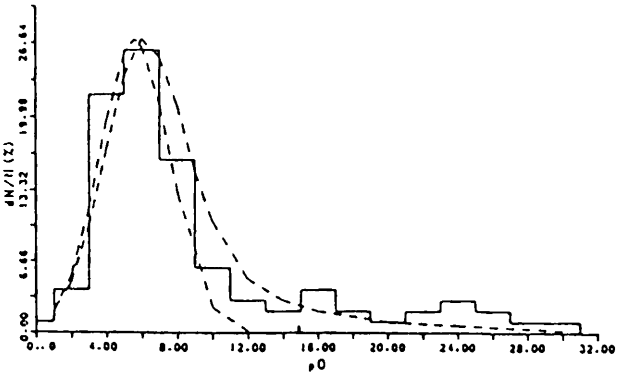

Figure 3 Normalized distribution histogram of ρ_0, computed under the assumption that $\alpha(0)$ is constant. The dashed curve with long tail is the fitted curve of eq.(8).

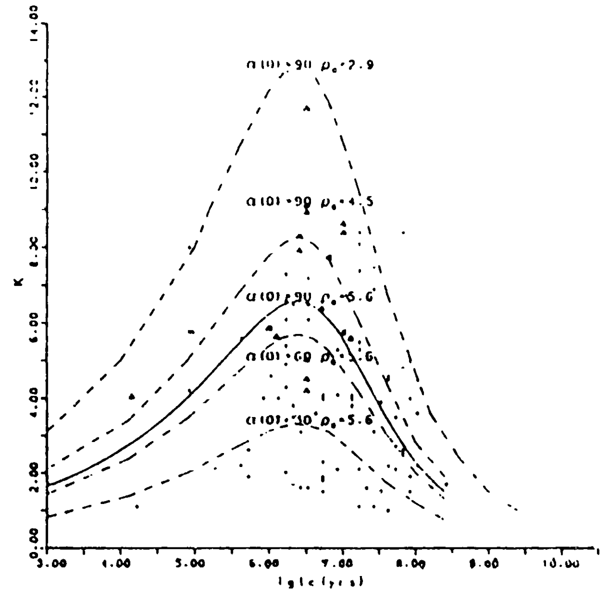

Figure 4 A comprehensive explanation of the pulsar distribution in the K *vs.* $\log t_c$ diagram. The solid line has $\rho_0 = \bar{\rho}_0 = 5.6°$ and $\alpha(0) = 90°$. The squared dots have $\alpha \sim 90°$ in the results of Lyne and Manchester (1988).

3. $\alpha(0)$ and ρ_0 scatter

The theoretical fitted curve in figure 3 assumes that a) the distribution of $\alpha(0)$ is a random distribution with a density function of $1/2 \sin\alpha(0)$ and b) that ρ_0 has a Gaussian distribution. After some calculation and use of the approximation $\sin\rho = \rho$, we can reproduce the normalized distribution function

$$P(\rho_0) \simeq \int_0^{90} d\alpha(0) \sin^2\alpha(0)\; e^{-(\rho_0 \sin\alpha(0) - \overline{\rho_0})^2/2\sigma^2)} \tag{8}$$

which is plotted in figure 3 as a dashed line having a long tail. The value of $\overline{\rho_0}$ is 5.6° and $\sigma = 2.0°$. The theoretical fitted curve is in agreement with the histogram of ρ_0.

If this situation occurs, each $\alpha(0)$ line shown in figure 1 is broadened into a zone, and $\rho_0 = \overline{\rho_0} \pm \sigma = 5.6° \pm 2°$. The effect of ρ_0 on the distribution can be best understood if one considers those pulsars having a magnetic inclination near 90° in tables 1 and 2 of Lyne and Manchester (1988) [shown in figure 4 as squared dots]; these pulsars fall in a zone which is located exactly in the upper section near the highest $K_{\max}$. Evidently the assumption that ρ_0 should have a distribution is inevitable.

A comprehensive understanding of the K *vs.* $\log t_c$ diagram can be made by assuming different values of $\alpha(0)$ and ρ_0. For simplicity, we divide the diagram into two regions by the line with $\rho_0 = \overline{\rho_0} = 5.6°$ and $\alpha(0) = 90°$. Above the line, the effect of ρ_0 is most clear. Below the line, the scattering of $\alpha(0)$ is mainly affected. In this way, $\alpha(0)$ and ρ_0 can both be determined. A new method of determining pulsar geometry is suggested by the K *vs.* $\log t_c$ analysis. The values of α, ρ and β can determined by eqs. (2), (3) and (6). Given a random distribution of $\alpha(0)$, some young pulsars with small magnetic inclination are expected which were also mentioned by Lyne and Manchester (1988).

Conclusions

In this paper, our standpoint is the observed K distribution. Comparison between the data and the theoretical evolution model makes it possible to understand pulsar evolution not only qualitatively but also quantitatively. We conclude that 1. The magnetic inclination α evolves on a scale of 1.5×10^7 yr and the radius of emission cone ρ evolves much quicker with a time scale of 6×10^4 yr (ρ drops to e times its final value).

2. At birth the magnetic inclination angle has a a random orientation with respect to the spin axis. Proszynski (1979) and Wu *et al.* (1982) showed that the current $\alpha(t)$ distribution shows a preference for

small angles. The present α distribution thus reflects the effects of both formation and evolution.

3. The cone radius evolution limit ρ_0 is shown to have a Gaussian distribution with $\overline{\rho_0} = 5.6°$ and $\sigma = 2.0°$. Thus pulsars may have different ρ values for the same age.

4. Important parameters, such as the magnetic inclination α, the beam radius ρ, the magnetic "impact" angle β, and Q or β_n can be obtained using the evolution model.

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