

# A study on universality, non-extensivity and Lévy statistics of solar wind turbulence

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**Abstract.** A number of complex systems arising in diverse disciplines may have certain quantitative features that are surprisingly similar which are classified under the paradigm of “universality”. The non-extensive Tsallis statistical mechanics and Lévy flight patterns provide a novel basis for analyzing non-equilibrium complex systems that may exhibit long-range correlations. The present work studies the scope of employing non-extensive Gutenberg-Richter (G-R) type law for the magnitude distribution of energy of solar wind, in order to investigate the existence of a universal behavior as well as to compute the relations of degree of non-extensivity and Lévy statistics in solar wind turbulence with heliographic distance during different solar cycles.

**Keywords.** Solar wind, Turbulence

## 1. Introduction

For time series analysis of Alfvénic features of the magnetic field and plasma velocity fluctuations in the solar wind (SW), it is possible to consider the total specific energy  $E_T$  of the fluctuation field with the dynamical variable capable of representing the turbulence level in the SW as  $E_T = E_V + E_b$ . Here the kinetic specific energy  $E_V = \frac{1}{2}\langle V^2 \rangle_{1h}$  and magnetic specific energy  $E_b = \frac{1}{2}\langle b^2 \rangle_{1h}$  are derived from hourly variances of SW plasma speed and SW magnetic field expressed in Alfvén units such that  $V = V - \langle V \rangle_{1h}$  and  $b = B - \langle B \rangle_{1h}$  (Consolini and De Michelis 2011). A suitable generalization of the classical Boltzmann-Gibbs (BG) statistics called non-extensive Tsallis statistics where  $q > 1$  has been proposed by Tsallis (1988). This is related with anomalous diffusion processes generated as a result of self-organized states with distant correlations. To study the properties of Lévy statistics of SW turbulence, the non-extensivity parameter  $q$  can be associated with Lévy index  $\lambda$  using the relation  $q = (2 + \lambda) / (1 + \lambda)$  (Buiatti *et al.* 1999).

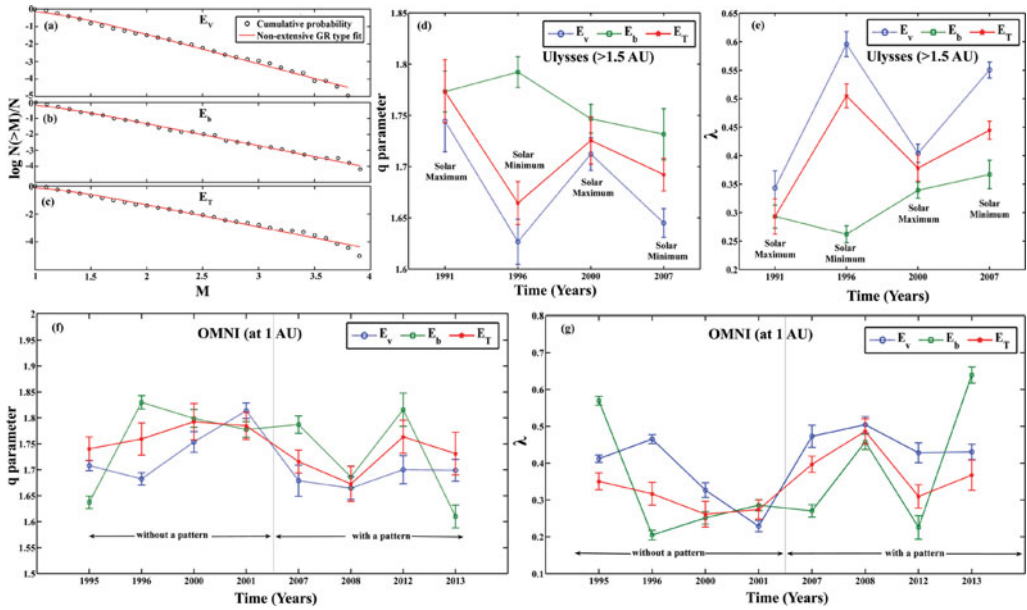
## 2. Data and Method

SW data for the present analysis has been taken from 1-min high resolution NASA OMNIWeb database. For a comparison in terms of spacecraft heliographic distance, the 4-min (averaged) Ulysses data has been taken from ESA database.

We employ a novel statistical model to analyze SW turbulence, called fragment-asperity interaction model, based on non-extensive Tsallis formalism, which has been introduced by Sotolongo-Costa and Posadas (2004) and revised by Silva *et al.* (2006). This approach guides to a non-extensive Gutenberg-Richter (G-R) type law for the earth quake magnitude distribution,

$$\log N(>M) = \log N + \left(\frac{2-q}{1-q}\right) \log \left[1 - \left(\frac{1-q}{2-q}\right) (10^{2M} \times a^{-2/3})\right]$$

where  $N$  is the total number of earthquakes,  $N(>M)$  the number of earthquakes with magnitude larger than  $M$  such that  $M \approx \log \epsilon$ .  $a$  is a constant of proportionality between the earthquake energy and the size of fragment,  $r$  ( $\epsilon \sim r^2$ ).



**Figure 1.** Cumulative probability graph with best regression fit using G-R type law for (a)  $E_V$  (b)  $E_b$  & (c)  $E_T$  for the year 2007. The variations in (d)  $q$  (e)  $\lambda$  for  $E_V$ ,  $E_b$  and  $E_T$  during solar minimum & maximum years from Ulysses data. The variations of (f)  $q$  and (g)  $\lambda$  for the energies during solar minimum & maximum years from OMNI data. The error bars specify MSE for the model regression fits for each year.

### 3. Results and Conclusion

Figures 1(d)-1(g) show the variations of  $q$  and  $\lambda$  for  $E_V$ ,  $E_b$  and  $E_T$  during solar minimum and maximum from OMNI (at 1 AU) and Ulysses ( $> 1.5$  AU) data. The solar maxima are characterized by a higher  $q$  (lower  $\lambda$ ) than the solar minima for  $E_T$  and  $E_V$  while  $E_b$  shows no such pattern. It is also seen that  $q$  derived from  $E_V$  closely follows  $q$  from  $E_T$  which in turn points to the fact that the non-extensivity of SW is modified mostly by SW speed fluctuations. The results from OMNI (Figs. 1(f) and 1(g)) show that a distinct pattern is seen in  $q$  and  $\lambda$  for all energies during 2007-2013 (solar cycle 24) when compared with 1995-2001 (solar cycle 23). The analysis from OMNI data show that there exists spatiotemporal anisotropy generated as a result of different non-equilibrium phase transitions seen in SW turbulence while comparing solar cycles 23 and 24. This can be attributed to variations in SW turbulence existed during the periods of deep minimum during the end of solar cycle 23 and beginning of solar cycle 24. By successfully using G-R type law for characterizing the SW turbulence, we also provide evidence for universal behavior that exists in different diverse disciplines of physics.

### References

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