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Normal Fitting classes of finite groups

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The thesis is concerned primarily with the definitions and applications of the normal Fitting classes called transfer classes. Following the example of other authors, the various definitions and results are presented for finite groups and classes of finite groups rather than for only soluble groups.

Construction of the transfer classes is done in Chapter Two via normal Fitting pairs. First, each finite group G is assigned a set of subgroups of G, denoted $C(G)$, such that if H is a group and $\alpha : H \to G$ is an injective homomorphism taking H to a normal subgroup of G, then $C(H\alpha) = \{X\alpha \mid X \in C(G),\ X \leq H\}$. When every member of $C(G)$ is isomorphic to a fixed group U, for every finite group G, homomorphisms $d(C)_G : G \to \text{aut } U/B(U)$ are constructed, where

$$B(U) = \langle x \in \text{aut } U \mid [x, U] < U\rangle(\text{aut } U)' .$$

This is done using the determinant-like mapping δ, from $\text{aut } U \text{ wr } S_n$ to $\text{aut } U/B(U)$, where n is the cardinality of $C(G)$ and S_n is the symmetric group on n letters. That $\left(d(C), \text{aut } U/B(U)\right)$ is a normal Fitting pair is proved, and a transfer class is defined as the normal Fitting class corresponding to such a normal Fitting pair.

Although the concept to which the name Fitting map is given here, is different to that for which Zappa [5] originally introduced the term, in

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1975, the two concepts are very similar, so the same name is used. Before proceeding to discuss the transfer classes it is shown that every Fitting map can be constructed from particular 'primitive' Fitting maps.

When C is a primitive Fitting map the Fitting pairs defined previously are refined, and the new normal Fitting classes resulting are shown to be contained in the corresponding transfer classes. In special cases, further information is obtained, relating these various normal Fitting classes to one another.

The third chapter investigates the relation between transfer classes and the normal Fitting classes found by Blessenohl and Gaschütz [2] using the chief series of soluble groups. For the final result of the chapter, some of the transfer classes are used to construct a set of normal Fitting classes whose intersection is contained in the appropriate Gaschütz-Blessenohl class. To obtain that result, propositions are proved relating to the determinant of an automorphism on a chief factor, M/N, of a finite q-group G, for some prime q, to the determinant of the automorphism induced on the $GF(q)$-module $\prod_{C \in H} C/Q(C)$, where the direct product is taken over H, the set of cyclic subgroups of G lying in M but not in N.

Since transfer classes were defined they have been used in the proof of two notable results. In [4], Cossey conjectured that $\underline{\underline{S}}_q \underline{\underline{S}}_p \subseteq \underline{\underline{S}}_*$, for p, q prime and p not dividing $q - 1$. By 1977 this had been disproved by Bryant and Kovács [3], and this result appears in the fourth chapter. In [1] Berger determined $\underline{\underline{X}}_*$ for various Fitting classes $\underline{\underline{X}}$, including the Fitting class of soluble groups $\underline{\underline{S}}$. His result, for the case of $\underline{\underline{S}}$, is also included in the fourth chapter.

References

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