3 George Street,
Edinburgh, 26*th August* 1960.

The Editor,
T.F.A.

Dear Sir,

Calculation of Yields

In their interesting letter Messrs. Cornwall and Ager raise two points:

(*a*) whether the assumptions underlying the method I suggested for calculating the yield to redemption make it unsuitable in a growing fund;

(*b*) on a more philosophic plane exactly what is meant by the "yield" on a security.

As to the first I cannot agree that there is any assumption, implicit or other, in my method that the annual amounts of i_2A are retained for the investor's use and are not re-invested. The real difference between the suggested methods is that I assume that if the amounts i_2A are re-invested they will earn the same rate of interest as other investments made contemporaneously. Messrs. Cornwall and Ager assume that they will earn a different rate. This is clearly seen if the two equations for the yields are written in the forms

$$A+i_2As'_{\overline{n}|}=C+gCs'_{\overline{n}|} \quad . \quad . \quad . \quad . \quad (1)$$

and

$$A+i_3As''_{\overline{n}|}=C+gCs'_{\overline{n}|} \quad . \quad . \quad . \quad . \quad (2)$$

showing that

$$i_3s''_{\overline{n}|}=i_2s'_{\overline{n}|}$$

and that both methods imply the re-investment of the whole of the income from the investment. The important difference in principle is whether the desired yield is to be accumulated at the same rate as the interest payments received from the investment, or at some other rate.

This decision is independent of the expected future progress of the fund, and from this point of view neither method can claim any theoretical advantage over the other. The method now suggested however seems to me to involve an anomaly similar to that which my previous suggestion sought to avoid, namely, the assumption that future investments will be made not at the ruling market rate but at some hypothetical rate determined by the original conditions of the investment. Since $i_3s''_{\overline{n}|}=i_2s'_{\overline{n}|}$ if two investments are compared, one giving a higher value of i_2 than the other, then so long as the terms of the investments are the same that investment must give a higher value for i_3 though this is not necessarily true for investments of differing terms. I should prefer to regard i_3 not as the yield on the original investment, but as the average return over a period of years obtained on a series of investments, assuming that future interest rates follow a defined course. The resulting average rate is of greater importance to the actuary engaged in fixing an interest basis for long term contracts than is the annual income he may hope to receive by investing in one security rather than another, but this is a separate problem.

If, normally, both methods tend to the same conclusions in making a comparison of different securities and if one gives the actuary a more informative answer than the other, it is reasonable to ask why it should not be preferred. My reasons are that it involves a conception of a yield which cannot be fitted into the general structure of yields as they are commonly interpreted. I regard the yield on an investment as the annual return which will be received from a unit investment, the capital remaining intact. This definition expresses the obvious truth that the yield on a redeemable security purchased at par is identical with the coupon rate, and that the yield on a perpetuity is the running yield. But by the method suggested in the letter above we find that there are circumstances in which the purchase at par of a 3% redeemable stock yields 3·454% and the purchase at 60 of a 3% perpetual stock yields $4\frac{1}{2}$%. I can attach a meaning to these results, but not if I am asked to regard them as expressing the "yield" on these investments. My suggested method would give the more usual answers of 3% and 5% respectively.

If the equation

$$A(1+i_3)^n = C + gCs'_{\overline{n}|}$$

is written in the form

$$\begin{aligned} A(1+i_3)^n &= (1+i_1)^n[Cv_1{}^n + gCa'_{\overline{n}|}] \\ &= (1+i_1)^n A' \end{aligned}$$

where A′ is the price of the security to yield the re-investment rate i_1 then

$$(1+i_3) = (1+i_1)(A'/A)^{1/n}$$

On this definition (and dealing only with the present case that $i > i_1$), the amount after one year of a unit invested at the "yield" is the amount at the re-investment rate of interest increased by the capital profit due to the change in interest rates, expressed as an annual rate over the term of the investment. "Yields" on securities of different terms will therefore vary with term in accordance with variations in the function $(A'/A)^{1/n}$. This function does not lend itself easily to analysis and I have been unable to prove generally that it decreases as n increases, though experiments suggest this is so. It may however be shown that when $i=gC$ and hence $C=A$ the ratio $(A'/C)^{1/n}$ decreases as n increases. Hence the suggested method involves the assumption that the " yield " on a redeemable security purchased at par decreases as the term lengthens. It is also clear that in the limit $(A'/A)^{1/n}$ tends to unity and hence this method gives the yield on a perpetuity as the assumed re-investment rate of interest. Both these results, apart altogether from their bizarre appearance in the normal yield structure, seem to me to involve an inherent bias against longer dated investments which is not desirable in a comparative index. The converse is of course true if $i_1 > i$ when the bias is in the opposite direction.

To illustrate these features I have compiled the table below for the securities A and C in my previous note. The results for security A relate to a low coupon stock standing at a discount when interest rates are expected to fall, while those for security C relate to a high coupon stock standing at a premium when interest rates are expected to rise.

Term (yrs.)	*Security A* (conventional yield 5¼%) Yield		*Security C* (conventional yield 3%) Yield	
	By Formula in *T.F.A.* vol. 26	By suggested Formula	By Formula in *T.F.A.* vol. 26	By suggested Formula
10	5·31	5·15	3·10	3·28
20	5·34	5·04	3·14	3·49
30	5·35	4·95	3·15	3·64
40	5·34	4·88	3·14	3·75
50	5·33	4·82	3·13	3·84
60	5·31	4·77	3·11	3·91
70	5·30	4·73	3·09	3·97
80	5·29	4·70	3·07	4·02
90	5·28	4·68	3·06	4·06
100	5·27	4·66	3·05	4·10
∞	5·25	4·50	3·00	4·50

Neither series of figures tells the whole story, but in the conditions of an expanding fund where the difficulty may well be to find securities sufficiently long-dated to match the liabilities then it seems to me easier in current conditions to gauge where advantage lies from my series of yields than from those brought out by the alternative formula, important though the figures it produces are in relation to the interest basis of premium calculations. When to this factor there is added the difficulty of interpreting the results for perpetual stocks and stocks purchased at par it does not seem to me that the suggested method is advantageous as a means of comparing the merits of different securities.

Yours faithfully,

D. W. A. DONALD.

EDITOR'S NOTE

Mr. C. G. Myers has also written proposing a definition of the yield identical with that in Messrs. Cornwall and Ager's letter. The latter writers, who had an opportunity of seeing Mr. Donald's letter before it appeared in print, wrote subsequently agreeing that the result of their method is " really the average return on the series of investments arising from the capital originally invested ". This may not be a " yield " in Mr. Donald's sense of the word and it is not intended directly as a means of comparing investments of differing terms but they suggest that the concept is a useful one.