ON MEASURE OF SUM SETS III

CORRECTION

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In my paper on the continuous $(\alpha+\beta)$ – theorem (these Proceedings **12** (1961), 209-211) the proof of Lemma 6 assumes that if A and B are closed sets in the relative topology as subsets of the space P of positive reals then $C = A+B$ is closed.

I am indebted to Professor H. B. Mann for drawing my attention to the following counter-example:

$$\text{Let } A = \left\{\frac{1}{n};\ n = 3, 4, \ldots\right\}$$

$$B = A \cup \{1\}.$$

Then $A+B$ is not closed though A and B are closed in the relative topology.

Fortunately, Lemma 6 is still true though more argument is needed to prove it. It is necessary to consider A and B in the first instance as closed subsets of the reals in the absolute topology, with $\inf A = \inf B = 0$. Then $A(x)+B(x)$ being the sum of two compact sets is compact, so

$$C(x) = (A(x)+B(x)) \cap [0, x]$$

is compact. Thus C is closed.

If we then denote by A_ξ, etc. the set of all reals (including perhaps some negative reals) whose distance from A is less than ξ, then certainly $A_\xi + B_\xi = C_{2\xi}$. On applying Lemma 5 to the open sets $(A_\xi \cap P)$, $(B_\xi \cap P)$ Lemma 6 will follow in the modified form.

Now if A, B are closed in the relative topology in P, then the closure $\bar{A}$ of A in the absolute topology will be $A \cup \{0\}$. Then

$$C_1 = \bar{A}+\bar{B} = \{0\} \cup A \cup B \cup (A+B).$$

Lemma 6, in its new form, shows that

$$\mu^*(C_1(x)) \geqq kx.$$

Thus the theorem will follow if it can be shown that $\mu^*(C_1(x) \backslash C(x)) = 0$. Now $C_1(x) \backslash C(x) = \{0\} \cup (A(x) \backslash C(x)) \cup (B(x) \backslash C(x))$. It is enough to show that, for each $\varepsilon > 0$.

$$\mu^*(A(x) \backslash C(x)) < \varepsilon.$$

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Now the function $\phi(t) = \mu(A(x)\backslash(A(x)+t))$ is known to be continuous and $\phi(0) = 0$. Since $\inf B = 0$, there is $t \in B$ such that $\phi(t) < \varepsilon$. Since then

$$A(x)\backslash C(x) \subset A(x)\backslash(A(x)+t),$$

the result follows. Lemma 6 is then proved rigorously and the rest of the proof follows as in the original paper.

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