7 - MECANIQUE CELESTE (CELESTIAL MECHANICS)

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A. Introduction

During 1985-1987 Celestial Mechanics has been intensively developed in all its branches embracing physical bases, mathematical aspects, computational techniques and astronomical objectives. Commission 7 has organized three IAU conferences : Symposium No. 114 "Relativity in Celestial Mechanics and Astrometry" (Leningrad, May 1985), Colloquium No. 96 "The Few Body Problem" (Turku, June 1987) and Topical Session "Resonances in the Solar System" of the X-th European Regional Astronomy Meeting (Prague, August 1987). Members of the commission have broadly participated in the NATO Advanced Study Institute "Long-Term Dynamical Behaviour of Natural and Artificial N-Body Systems" (Cortina d'Ampezzo, August 1987) and some other international and regional conferences. Prospects of the actual celestial mechanics investigations have been discussed at a session of Commission 7 at the XIX-th IAU General Assembly (New Delhi, November 1985). Three papers dealing with the unsolved problems of celestial mechanics were primarily addressed to the rising generation of celestial mechanicians (V.A. Brumberg and J. Kovalevsky, CM. 39, 133, 1986; P.K. Seidelmann, CM. 39, 141, 1986). Several books and specific topic proceedings in celestial mechanics were published during these years :

E.P. Aksenov, Special Functions of Celestial Mechanics, Nauka, Moscow, 1986 (in Russian).

V.I. Arnold, V.V. Kozlov and A.I. Nejshtadt, Mathematical Aspects of Classical and Celestial Mechanics, VINITI, Moscow, 1985 (in Russian).

V.G. Golubev and E.A. Grebenikov, The Problem of Three Bodies in Celestial Mechanics, Moscow State University, Moscow, 1985 (in Russian).

K.V. Kholshevnikov, Asymptotic Methods of Celestial Mechanics, Leningrad State University, Leningrad, 1985 (in Russian).

BHA : Space Dynamics and Celestial Mechanics (ed. K.B. Bhatnagar), Reidel, 1986.

BUM : Natural Satellites (eds J.A. Burns and M.S. Matthews), Univ. Arizona Press, 1986.

CAH : Gravitation in Astrophysics (eds. B. Carter and J.B. Hartle), Plenum Press, 1987.

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DDM : The Stability of Planetary Systems (eds. R.L. Duncombe, R. Dvorak and P.J. Message), CM. 34, Nos. 1-4, 1984.

DLS : Fundamental Astronomy and Solar System Dynamics (eds. R.L. Duncombe, J.H. Lieske and P.K. Seidelmann), CM. 37, No. 3, 1985.

EIL : Astrometric Techniques, IAU Symp. No. 109 (eds. H.K. Eichhorn and R.J. Leacock), Reidel, 1986.

FMS : Resonances in the Motion of Planets, Satellites and Asteroids (eds. S. Ferraz-Mello and W. Sessin), Univ. of Sao Paulo, 1985.

HAI : 300 Years of Gravitation (eds. S.W. Hawking and W. Israel), Cambridge Univ. Press, 1987.

KKM : Reference Systems (eds. B. Kolaczek, J. Kovalevsky and I.I. Mueller), Reidel, 1987.

KNY : Proc. of the Twentieth Symposium on Celestial Mechanics (eds. H. Kinoshita, H. Nakai and M. Yoshikawa), Tokyo, 1987.

KOB : Relativity in Celestial Mechanics and Astrometry, IAU Symp. No. 114 (eds. J. Kovalevsky and V.A. Brumberg), Reidel, 1986.

SZB : Stability of the Solar System and its Minor Natural and Artificial Bodies (ed. V. Szebehely), Reidel, 1985.

WIB : The Earth's Rotation and Reference Frames for Geodesy and Geodynamics (eds. G. Wilkins and A. Babcock), IAU Symp. No. 128, Reidel, 1986.

The amount of periodical publications in celestial mechanics is abundant. It is sufficient to look through "Celestial Mechanics" Journal (3 volumes or 12 issues per annum). So, following the example of the 1985 report by J. Kovalevsky, this report will not claim to cover all fields of interest of celestial mechanics and to give a detailed bibliography. In particular, taking into account the incoming symposium on "Applications of Computer Technology to Dynamical Astronomy" (Gaithersburg, July 1988) no mention is made here on the computational techniques. The main attention is given to domains not included in the previous report, such as relativistic celestial mechanics, resonance theory, asymptotic methods, artificial satellite motion, etc.

In preparing this report the reviews by N. Borderies, J. Chapront, T. Damour, A. Deprit, N.V. Emel'yanov, S. Ferraz-Mello, J.D. Hadjidemetriou, J. Henrard, K.V. Kholshevnikov, P.K. Seidelmann, M. Soffel, Z.H. Yi and individual reports of many other commission members are highly appreciated.

In the course of the report, Astronomy and Astrophysics Abstracts numbers are used except for the papers published in the most common journals or the proceedings indicated above.

B. Relativistic Celestial Mechanics

(Main contributor : V.A. Brumberg)

In recent years the general theory of relativity (GRT) in its most simple applications is no longer seen as a theory under verification, but must be considered as a necessary framework in the discussion of high-precision observations and for the construction of accurate dynamical ephemerides. Disregarding the importance of general relativity as the physical foundation of celestial mechanics and from a purely operational point of view the distinction between relativistic and Newtonian description of the motion of celestial bodies is displayed 1) mathematically by the structure of the field equations and the equations of motion and 2) physically by the way we compare the theoretical and observational data. The first question is the subject of relativistic celestial mechanics, the second one belongs to relativistic astrometry. Only the consistent simultaneous solution of the two questions leads to physically meaningful results. The essence of the second question is the fact that in contrast to the inertial coordinates of Newtonian mechanics the coordinates of general relativity have no physical meaning and cannot be considered as the measurable quantities. Therefore, expressed in terms of the coordinates the conclusions of the relativistic dynamical theories are not unique, depend on the type of the coordinates employed and cannot be directly confirmed or refuted by observations. Only in terms of the measurable quantities these conclusions become unique and may be compared with observations. The IAU Symposium 114 mentioned above gave an insight into these both questions and was stimulatory to further investigations in this domain. The crucial problems on this way are as follows :

I. Experimental Verifications

The vast program of the parametrized post-Newtonian formalism performed during the past two decades by C.M. Will with the aim to test general relativity and other theories of gravitation had led to the experimental verification of general relativity as the theory best fitted to observations (KOB, 355). As reported by P.K. Seidelmann, E.J. Santoro and K.F. Pulkinnen (KOB, 99) there are some discrepancies between the planetary ephemerides and the observational data but they may be due either to systematic discrepancies in the observational data or inadequacies in our knowledge of a model of the Solar system.

II. Gravitational Radiation

For the first time after Einstein, Infeld and Fock a significant progress was achieved in formulating the equations of motion of celestial bodies taking into account the gravitational radiation. The culminating contributions were made by T. Damour (33.066.077; CAH, in press) and S.M. Kopejkin (40.066.087). This made it possible to investigate the orbital motion of the binary pulsar PSR 1913+16 by the rigorous celestial mechanics methods (solution of the relativistic two body problem with account of the gravitational radiation terms) and to confirm the previous results obtained on the basis of the approximate linearized theory of gravitational radiation. Besides, these investigations give an insight into the general structure of the equations of motion in general relativity.

III. Matching

The modern approaches to derive the equations of motion do not attempt to present solution in the global form valid for the whole space. Instead, the space-time is splitted up into several sub-domains with specific physical characteristics, for example : the internal region with dominant influence of the gravitational field of the body under consideration, the buffer region where the internal and external fields have comparable effects and the external region where the gravitational field of other bodies dominates. The solutions valid for each region separately are combined by the matching technique. This driving idea has been exposed in details by K.S. Thorne and J.B. Hartle (40.067.014). T. Damour has presented a general review of the problem of motion in Newtonian and Einsteinian gravity (HAI, 128). This review may be consulted as an excellent introduction into the modern state of the problem of motion in general relativity. The matching procedure allowed to derive the equations of motion not only for the Solar system bodies but for the condensed objects, like the black holes, as well. In the modified form the matching procedure is applied by S.M. Kopejkin (Sternberg Astron. Inst. Trans., 59, in press) to obtain the equations of motion of celestial bodies taking into account their shape, axial rotation and internal structure. The physical characteristics of the bodies (sphericity, rigid body distribution of velocities inside the body, etc.) are described in the body's proper reference frame and this local solution is matched to the global one. This procedure removes the terms of non-physical origin and makes the whole technique more rigorous mathematically and more meaningful physically.

IV. Reference Frames

Relativistic treatment of the reference frames is at present of particular interest. The concept of reference frame is often differently used in physics and astronomy leading sometimes to misunderstanding. For astronomical purposes it is suitable to follow the operational definition by J. Kovalevsky (Bull. Astr., 10, 87, 1985) considering that a reference frame results from materialization of a coordinate system by means of some reference astronomical objects. The most widespread approach in the relativistic theory of astronomical reference frames is to construct the proper reference frame of a fictitious or actual observer with the aid of the Fermi normal coordinates (the time axis is the proper time of the observer, the three space axes are the space-like geodesics orthogonal to the world-line of the observer). In application to the geocentric frame one has at first to construct the proper reference frame for the massless Earth and then to substitute the corresponding coordinate transformation into the full metric incorporating the gravitational influence of the Earth. This approach has been developed in papers presented at the IAU Symposium No 114 by B. Bertotti, C. Boucher, T. Fukushima et al., M. Fujimoto and E. Grafarend (KOB, 233, 241, 145, 269) and reviewed by N. Ashby and B. Bertotti (Phys. Rev., D34, 2246, 1986). These reference frames were examined and applied to specific astronomical problems by M. Soffel et al. (Veröff. Bayerisch. Komm. intern. Erdmessung, 48, 237, 1986). Another, but somewhat similar way based on a finite linear transformation from barycentric to geocentric coordinates is exposed by N.V. Pavlov (38.066.211.212). In discussing the astronomical observations the infinitesimal transformations are sometimes used based on the possibility of local splitting of the space-time at the point of observation into the time axis and the three dimensional space. In this manner R.W. Hellings (41.066.196) and V.A. Brumberg (EIL, 19) introduce the local inertial coordinate system in the infinitesimal vicinity of the point of observation and derive the formulas for the relativistic reduction of observations.

Evidently, one may use any coordinates in constructing the reference frames. But if a coordinate system is not dynamically adequate to the problem under consideration both the solution of the dynamical problem and the transformation to the observational data will contain a number of extra terms caused only by the inadequate choice of the reference frame. These terms cancel out in the expressions of measurable quantities and the resulting relativistic effects turn out to be much smaller than the relativistic perturbations in the coordinate description of the dynamical problem. On the contrary, if the coordinate system is dynamically adequate the coordinate solution of the dynamical problem will not contain any large term of non-dynamical origin and will insignificantly change in converting to the measurable quantities. Reasoning from these considerations V.A. Brumberg and S.M. Kopejkin (KKM, in press) have developed a relativistic theory of the reference frames satisfying two conditions : 1) all basic non-rotating reference systems are built in the harmonic coordinates, 2) their metric tensors represent the dynamically adequate solutions of the Einstein field equations for the corresponding problems. In such a way, the barycentric reference systems (BRS), the geocentric reference system (GRS), the topocentric reference system (TRS) and the satellite reference system (SRS) have been constructed.

V. Time Scales

The theory of reference frames is in close relation with the investigations on the time scales and the system of astronomical units. In many respects this is a question of definitions and agreements. The operational definition of TAI is exposed by B. Guinot (CM, 38, 155, 1986). The relation TDB-TDT based on the BDL analytical planetary and lunar theories of motion has been obtained by L. Fairhead, P. Bretagnon and J.-F. Lestrade (WIB, in press) and by T.H. Hirayama et al. (KNY, 75). Using the JPL numerical ephemerides this relation was studied by D.C. Backer and R.W. Hellings (Ann. Rev. Astron. Astroph., 24, 537, 1986). The dependence of the astronomical units of measurement upon the reference frame has been investigated by T. Fukushima et al. (CM,38, 215, 1986) and R.W. Hellings (41.006.196). The consistent theory of transformations BRS→GRS→TRS (SRS) enables to treat these questions in straightforward manner.

VI. Modern Theories of Motion in the Solar System

Relativistic terms are taken into account in the planetary theories in the barycentric harmonic coordinates in numerical (JPL, ITA) or analytical (BDL) forms (see KOB). Relativistic terms in the lunar theory by J.F. Lestrade and M. Chapront-Touzé (32.094.028) and by V.A. Brumberg and T.V. Ivanova (39.094.017) as well as relativistic terms in the artificial satellite theory by C.F. Martin, M.H. Torrence and C.W. Misner (40.052.033) are found in the BRS and their amplitude may attain several meters. In converting to the GRS their amplitude reduces to the centimeter level. This was shown explicitly by M. Soffel, H. Ruder and M. Schneider for the Moon (41.094.009) and by S.Y. Zhu et al. for Earth satellites (IAU Colloq. No. 96). Considering that the equations of lunar and satellite motion in BRS are known it is easy to obtain the equations in GRS with the help of the transformation BRS → GRS. Formulas for the relativistic reduction of observations are given within sufficient practical accuracy in Japanese Ephemeris for 1985. More rigorous theory may be developed using the transformations BRS →

GRS →TRS (SRS) admitting the rigid body rotations of the systems. Particular attention is drawn now to the pulsar timing formulas. The most detailed results are obtained by T. Damour and N. Deruelle (Ann. Inst. H. Poincaré, 44, 263, 1986) and D.C. Backer and R.W. Hellings (Ann. Rev. Astron. Astroph. 24, 537, 1986).

C. Analytical Methods and Orbital Resonance

(Main contributors : S. Ferraz-Mello and J. Henrard)

Perturbation theory as a tool of investigation and resonance phenomena in the Solar System as a reservoir of interesting problems are continuing to attract many investigators.

Perturbation theories used in Celestial Mechanics are now almost exclusively based upon Lie Transforms methods as a technique and K.A.M. theory as a justification. In that frame two aspects have received continued attention : the formal or algebraic aspect (how to set up a perturbation theory) which we review under the title "Intrinsic Perturbation Theories" and the convergence aspect (what does a perturbation theory tell us) which we review under the title "General Estimates". The theoretical results relative to this second aspect have to be supplemented with the information given by numerical experiments (see "Planetary Theories") and the study of the chaotic behaviour of Hamiltonian systems (see "Chaotic Motions").

Applications of Perturbation Theories to the Solar System are broken down in this report in the three traditional chapters : Planetary Theories, Satellite Theories and Asteroid motions, each with its own physical but also mathematical peculiarities, although they have many similarities. A new chapter quite different from them : the Planetary Rings has been added recently and has already blossomed into a very rich field of research.

We have briefly covered under special headings two particular aspects as they do not enter readily in the adopted classification. They are "Resonance Sweeping" by which are analysed the effects of small non-conservative forces in shaping the Solar System as we see it now and "Chaotic Motions", still in his infancy as a topic of research in Celestial Mechanics, but which is likely to grow in importance both for theoretical and practical investigations.

I. Intrinsic Perturbation Theories

Several contributions have emphasized in the last few years the intrinsic characteristics of general perturbation theories. Perturbation theories have usually been specialized to selected sets of phase variables or to selected types of perturbations and it is not always easy to decide if the difficulties encountered in some procedures are physically significant or the reflect of an unappropriate choice of coordinates or of unperturbed first approximation. Ferraz-Mello (CM. 35.209, CM. 35.221) developed the equations of the method of Hori for first order resonances and emphasizes the fact that the unperturbed first approximation should already reflect the topology of the full system (Ferraz-Mello, Sessin CM.34.453). Deprit in a series of papers (CM.

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24.111, CM. 26.9, CM. 29.229, 39.042.059) and Kummer (41.042.087) use a group theoretical approach to describe the intrinsic properties of general perturbation techniques. This led Coffey et al (CM.39.365) to propose spherical charts rather than the usual cylindrical ones related to Delaunay's elements to describe the averaged main problem of an artificial satellite. The structure of the phase space in the vicinity of the critical inclination can then be fully explained. Action-Angle variables have been used successfully by Sessin and Ferraz-Mello (CM.32.307), Wisdom (IC. 63.272) and Henrard-Lemaitre (IC. 69.266, CM. 39.213) to treat a double resonance problem (see below under the heading : Asteroid motion). Pauwels (CM. 30.229) has shown how the phase space of the secular orbit-orbit coupling should be represented on a sphere rather than a plane. Sessin (CM.39.173) has proposed a new theory of integration for the so-called singularity of Poincaré.

The Ideal Resonance Problem (Garfinkel A.J. 76.157) has been revisited in an effort to bypass the difficulties of the Bohlin procedure. Jupp and Abdulla (CM. 34.411, CM. 37.183) use Lie series in the libration domain and Sessin (CM. in press) solves it using the method of Delaunay. Deprit (Adv. Astro. Sci. 46.521) shows how the "disencumbering" of the problem makes its physical mechanisms more transparent. Howland (CM. subm.) and Henrard-Wauthier (CM. subm.) use non-canonical transformations to reduce the problem to either a classical problem in perturbation theory or to the pendulum itself.

In the case of the elliptic restricted problem, the technique of Sessin allowing the reduction of a truncated averaged Hamiltonian to the circular case, through a suitable change of variables, was reconsidered by Tsuchida (FMS.149), Wisdom (CM.38.175), Henrard et al (CM.38.335) and Ferraz-Mello (FMS.37 and AJ. 94.808).

II. General Estimates

The well-known K.A.M. theory states that the series expansions of general perturbation methods gives information for all time but only on a peculiar closed subset of the phase space (the "good" tori). Nekhoroshev (Russ. Math. Surveys 32.1) using the same techniques has obtained important results valid on open subsets of the phase space by renouncing to have estimates for all times. These results have recently been refined and described in the context of Celestial Mechanics by Benettin, Galgani and Giorgilli (CM. 37.1, CM. 37.95).

III. Planetary Theories

The interest in Planetary Theories has continued to be strong especially in general planetary theories with a validity covering a time interval of the order of the million years or more. Classical theories have been revisited by Sukhotin (39.042.086) and by Knezevic (CM. 38.123). Laskar has continued his work on the numerical integration of the analytically averaged "autonomous" system (AA. 144.133, AA. 157.59, 41.042.002). An interesting new method called "synthetic theory" by Milani and Nobili has been developed. It consists in extracting by numerical averaging from the output of long term numerical integrations, information about frequencies, possible locking in resonance, exchanges in energy or momentum between planets, etc. The numerical integrations of Kinoshita and Nakai (5 outer planets for 5 million years - CM. 34.203) have been used for this purpose (Milani, Nobili 38.091.004, CM.35.269). Further progresses along these lines are coming from the LONGSTOP project (5 outer planets for 100 million years -Milani et al. 41.091.003, AA. 172.265, Carpino et al. AA. 181.182). Applegate et al (AJ.92.176) have also performed a numerical integration of the 5 outer planets for 210 million years.

Some perturbative terms like the 1.1 million years oscillation in the semi-major axis of Uranus and Neptune have been computed analytically only after being detected in the numerical integration (Milani et al. AA.172.265). While at some level of accuracy these results show a regular structure, the resolution is now such that one can wonder about the intrinsic numerical convergence of any planetary theory. The solar system could be chaotic with a very slow time constant (Milani et al. subm.). Mixing numerical and analytical methods has also been used successfully by Bretagnon (CM. 38.191) in order to improve the short period ephemeris of the planets.

IV. Satellite Theory

Satellite theories (except the theory of the Moon which was reviewed three years ago) are not yet encountering these difficulties. The field has been reviewed by Ferraz-Mello (CM. 34.223 and FMS.37). Work has been continuing on the theory of the Galilean satellites of Jupiter (Thuillot CM. 34.245, Henrard CM. 34.255, Vu Thesis-Paris 1986). The Uranian satellites were studied with emphasis in the great inequalities of long period able to affect ephemerides and mass determinations. They were studied by Lazzaro et al. (38.101.018, 40.101.062 and AA. 182.150) with the classical Laplace Souillart theory and by Laskar (AA. 166.349) using his version of the method of general planetary theories (39.042.006). The theory of Laskar was compared by himself and Jacobson (AA. in press) to the existing earth based observations and allowed them a new determination of the masses within 15 % of the values found from Voyager II data.

Orbital resonances among Saturn's satellites were reviewed by Greenberg (38.100.143). Sato and Ferraz-Mello (FMS.105) and Salgado-Sessin (FMS.099) studied the resonance Enceladus-Dione. The resonance involving Hyperion is made more difficult by the great forced eccentricity (0.104) and present theories are simply not accurate enough to represent the motion of Hyperion as was shown by Taylor (38.100.078), Sinclair and Taylor (AA. 147.241), Taylor et al. (AA. in press) and Dourneau (Thesis-Bordeaux 1987). The evolution and the stability of the coorbital satellites of Saturn present new challenges. They have been studied by Lissauer et al. (IC. 64.425) and Sinclair (38.100.002). Hill's problem revisited by Henon and Petit (CM. 38.67, IC. 66.536) and Spirig and Waldvogel (40.042.029) could be a good model to analyze some of their properties. Greenberg (IC.70.334) studied the Laplacian libration and found a branch of solutions where the critical angle is different from 0 or π.

V. Asteroid Motion

The distribution of asteroids in the asteroid belt has continued to attract much attention. The field has been reviewed by Scholl (39.098.044, 40.042.103). The depletion of the outer asteroid belt seems to be well explained by the perturbation of Jupiter leading except in a few "protected" cases to close approaches (Milani, Nobili AA. 144.261, 39.098.029). The identification of families (Farinella et al. 41.098.038) and their possible explanation through collisions (Zappala et al. IC. 59.261) necessitate better theories (Knezevic 41.098.041). The role of secular resonances in shaping the belt has been studied by Kozai (CM. 36.47, 39.098.033), Nakai and Kinoshita (CM. 36.391) and Scholl and Froeschlé (41.098.042). Zellner et al. (39.098.094) have drawn the attention on the rather sharp upper limits in inclinations and eccentricities in the distribution of the Asteroids. Froeschlé and Scholl (AA.158.259) have shown that resonances can break up meteor stream into arcs of different sizes with distinctly different dynamical evolutions.

Numerical studies of Wisdom (AJ. 87.577, IC. 56.51), Murray and Fox (IC. 59.221) and Sidlichovsky and Melendo (41.098.009) have shown how the 3/1 and 5/2 Kirkwood gaps could be produced by close encounters with Mars due to large eccentricities induced by the Jupiter perturbations. In part of the phase space these large forced eccentricities are intermittent and a result of the chaotic character of the motion. Wisdom (IC. 63.272) has identified the mechanism producing this chaotic behavior as the separatrix crossings in the two critical arguments problem describing the averaged elliptic three body problem and not as previously thought as the effect of the short periodic terms. The effect of the eccentricity of the perturbing body has also been investigated numerically by Ferraz-Mello and Dvorak (AA.179.304) in another context (Resonance Enceladus-Dione). The 2/1 Kirkwood gap has been investigated numerically by Murray (IC. 65.70) and analytically by Henrard and Lemaitre (IC. 69.266, CM. 39.213). In the planar problem at least, a significant part of the volume of the phase space seems to be protected against increases in eccentricity large enough to induce close encounters with Mars. These resonances have also been studied by Gerasimov (A.Zh. 63.567) when second order harmonics are included (37.042.070) or not (A.Zh.63.763).

VI. Planetary Rings

(main contributor : N. Borderies)

1. Ring-satellite interactions : Encounters between two small bodies orbiting around a planet can be dynamically described by the Hill's problem. Petit and Henon (IC. 66.536) have shown that the number of parameters of the problem can be reduced to one, the reduced impact parameter, and have studied in details the family of solutions. Analytical developments relevant to this study are presented by these authors (CM. 38.67). In a subsequent paper (AA. 173.389), Petit and Henon generalized the interaction model by considering particles with finite sizes. This study leads to a numerical simulation of planetary rings (Petit, Ph.D. Thesis). Roques, in her thesis, has developed a different numerical model of the interactions between a satellite and a ring, in which she studied the response of the particles to the instantaneous perturbation exerted by a satellite. Theoretical work on disk-satellite interaction has been pursued by Sicardy (FMS. 167) and Meyer-Vernet and Sicardy (IC. 69.157). These papers deal with the torque exerted by a satellite on a disk at a Lindblad resonance. A powerful approach in the study of planetary rings is based on the concept of streamlines of the flow of particles. Borderies and Longaretti clarified certain problems of Celestial Mechanics posed by this approach (IC., in print).

2. Modelizations of the collisions effects : Theoretical work on the dynamics of a disk of colliding particles has been developed in several directions. Brophy and Esposito (IC., in print) have solved the Boltzmann equations for the equilibrium velocity distribution and have compared their solution with the approximate analytical solution of Goldreich and Tremaine. Shu and Stewart (IC. 62.360) used the Krook equation, which is simpler than the Boltzmann equation. This simplicity allowed them to solve analytically the problem for an unperturbed ring. Bor-

deries, Goldreich and Tremaine (IC. 63.406) have studied the case where the ring particles are closely packed together, such that their collective behaviour resembles that of a liquid. They found that density waves are unstable in such a ring and that eccentric modes can spontaneously develop in narrow rings. Araki and Tremaine (IC. 65.83) have studied the dynamics of dense particle disks by adapting the Enskog theory of dense hard sphere gases to inelastic spheres. They suggested that the complex ringlet structure in the B ring of Saturn represents adjacent solid and liquid phases. Collision and accretion process determine the distribution of particle sizes in a ring. A theoretical model has been recently developed in his thesis by Longaretti. He found that Saturn's rings could not consist of hard spheres because accretion would be too efficient and was lead to the conclusion that individual particles in these rings have a brief survival time.

3. Waves : Many density waves have been found in Saturn's rings. They are associated with the resonant forcing of the particle eccentricities. A smaller number of bending waves, associated with the resonant forcing of the inclinations, are also present. The main problem concerning density waves is that most of those which are observed in Saturn's rings are nonlinear : the perturbation in surface density is not small compared to the unperturbed surface density. Hence a lot of effort has been put in constructing nonlinear theories of density waves (Shu et al., APJ. 291.356, APJ. 299.542 and Borderies et al. IC., 68.522). The analysis of a density wave profile (Longaretti and Borderies, IC. 67.211) has allowed to check certain theoretical results such as the nonlinear dispersion relation which had been derived independently by the two groups mentioned above. Work on bending waves has been accomplished by Gresh et al. (IC. 68.481). Bending waves conform more closely than density waves to the linear model.

4. Ring confinement : Voyager images have revealed radial undulations at the edges of the Encke gap in Saturn's rings. These waves indicate that a satellite is present in the gap. Cuzzi and Scargle (AJ. 292.276) and Showalter et al. (IC. 66.297) have determined the location and size of the satellite responsible for the undulations and have discussed the implications of their analysis for the shepherding theory. The new data acquired by Voyager during its encounter with Uranus have permitted new tests of the theories. French et al. (SC. 231.480 and IC., in print) discovered that a normal mode $m = 2$ was spontaneously excited in the δ ring. Goldreich and Porco (AJ., in print) studied the sheperding of the Uranian rings. They found that the shepherding of the ϵ ring was consistent with the theory of confinement while, on the other hand, the α and β rings pose problems which lead the authors to question the self-gravity model for the rigid precession of these rings.

5. Neptune's arcs rings : With the discovery by Hubbard and Brahic (NA. 319.636) of incomplete rings around Neptune, the problem for the theoreticians has been to explain this observation. Lissauer (NA. 318.544) has proposed that the arcs could lie at the Trojan point of an undiscovered satellite of Neptune. Another satellite would be responsible for the radial confinement of the arc. An alternate theory has been proposed by Goldreich et al. (AA. 92.490). They invoke corotation resonances with a satellite moving on an inclined orbit. The same satellite would be responsible for the confinement of the arcs.

VII. Resonance Sweeping

The effect of small nonconservative forces in shaping the solar system as we see it now has first been brought to the attention of Celestial Mechanicians by Goldreich (MNRAS 130.159). This mechanism is particularly important when a resonance is encountered. It has continued to receive much attention in the last three years. Analytical tools have been developed by Lemaitre (CM. 32.109), Borderies and Goldreich (CM. 32.127) and Hadjidemetriou (ZAMP 37.776). Applications to the evolution of satellites due to tidal dissipation has been reviewed by Peale (BUM), Kovalevsky (39.091.038) and Henrard (40.091.031). Further contributions to the tidal evolution of the Moon are presented by Kovalevsky (40.042.098). Spin orbit resonances and their tidal evolution have been investigated by Sidlichovsky (39.042.044), Henrard (FMS 19) and Beletsky (40.042.091). The effects of this mechanism upon the accretion of planets has been investigated by Weidenshilling and Davis (IC. 62.12). Applications to the asteroid belt structure (D'yakov and Reznikov 37.098.011, 41.042.062, Gonczi et al. IC.51.633, Lemaitre CM. 34.329) shows that this mechanism provides an alternative explanation for the Kirkwood gaps.

VIII. Chaotic Motions

Chaotic behavior in Hamiltonian systems has been the subject of intensive research in Theoretical Physics for some times now. Henon (AJ. 69.73) and Froeschlé (AA. 9.15) were apparently the first to study it in the context of Celestial Mechanics leading to applications to galactic dynamics and to the asteroid belt (Froeschlé-Scholl AA. 48.389). It is only quite recently however that it was realized that this kind of behavior can actually be seen in the Solar System in the rotation of Hyperion (Wisdom et al. IC. 58.137, Peale 38.100.071) or could be an important mechanism in the explanation of the Kirkwood gaps (Wisdom IC. 56.51, IC. 63.272) or the depletion of the outer asteroid belt (Milani, Nobili AA. 144.261). The theory and computation of the Lyapunov characteristic exponents has been reviewed by Froeschlé (CM. 34.95) and the use of the Kolmogorov entropy as an estimate of the disorder of a system tested by Gonczi et al. (CM. 34.117). The threshold of chaotic and regular behavior has been analyzed by Innanen (AJ. 90.2377), Yokoyama (37.042.101), Galgani (40.042.130) and Simo (40.042.124).

D. Artificial Satellite Theory

(Main Contributor : André Deprit)

The scientific adventure speaks here in profusion : the refereed literature in periodic journals is but a fraction of what comes out in the form of technical memoranda, internal notes, prepublication of conference proceedings, and contractor's reports. One way to deal with such a wealth of information is to survey the major themes. Having gotten the lay of the land, the reader will find it easy to track the precise trails through the professional compilations of abstracts.

For the past thirty years, in fact since the seminal studies of Orlov, Robertson, Blitzer and Breakwell in the mid-50s, artificial satellite theory has lived under a dark cloud. Was the critical inclination an essential feature of the phase space or simply a spurious singularity caused by the coordinates or the perturbation algorithm ? The enigma has now been solved. In the main problem (J_2 only), along the family of mean circles of a given radius, there occurs a small segment where the orbits are unstable. The endpoints of that interval correspond to circles in orbital planes inclined over the equator at angles slightly above or below the critical value $\arctan 2 = 63 \deg .43$. These singular orbits owing to their resulting from a Hopf bifurcation testify to the intrinsic nature of the critical inclination (S.L. Coffey, R. Cushman, A. Deprit and B.R. Miller). The main problem of artificial satellite theory and, for the matter, any perturbed Keplerian system having a rotational symmetry, has now been shown to admit a two-dimensional bundle of spheres as its reduced phase space. Numerous systems have been analyzed in that framework : the Stark effect (A. Deprit), the Zeeman effect (S.L. Coffey, A. Deprit, B.R. Miller and C.A. Williams), the effect of radiation pressure on dust orbiting a planet (R. Cushman, A. Deprit and J.C. Van der Meer), the Gylden systems or Keplerian systems whose Gaussian parameter varies with time (C.A. Williams), the relativistic Two Body problem (D. Richardson).

In the global picture of the average phase space, circular solutions emerge as equilibrium positions; mission engineers know them as "frozen orbits". Since the time when these singular solutions were shown to present great advantages for controlling the altitude of low satellites (E. Cutting, G. Born and J. Frautnick, 1978), they have been set as nominal orbits for SEASAT-A, GEOSAT, NROSS, TOPEX/POSEIDON, ERS-1 and other spacecrafts carrying altimeters around the earth. Uphoff (1985) has proposed to keep orbiters on "frozen orbits" around Venus. At inclinations where other equilibria arise in the neighborhood of a frozen orbit, Brouwer's third reduction by eliminating the argument of perigee will fail (C.C. Tang, 1987). Furthermore, numerical contouring reveals that the position of a frozen orbit in the reduced phase space and its stability are affected significantly by high degree zonal harmonics in the gravity field of the planet (W.D. McClain, 1986) and by the luni-solar perturbations (M.E. Hough). Catastrophe Theory is likely to offer the means of exploring these intricate problems without getting lost in an avalanche of parameters and perturbations (K.R. Meyer).

The appeal to simplification in analytical theories never ages. Undoubtedly Brouwer's solution for the main problem in closed form as far eccentricities and inclinations are concerned marked a decisive break with the tradition. It prepared celestial mechanics to the impact of Geometric Dynamics. Intermediaries obtained by stamping out separable Hamiltonians are replaced by "natural" intermediaries resulting from intrinsic reductions in the sense of Meyer, Marsden and Weinstein. The question now is how much symmetry we need or want in the Hamiltonian of artificial satellite theory. The elimination of the parallax (A. Deprit) and the elimination of the perigee (K.T. Alfriend and S.L. Coffey) are now interpreted as two steps in a Lie transformation meant to confer spherical symmetry to the zonal problem of artificial satellite theory. Seeing Cid's inchoate attempt at producing a radial intermediary blossom into a second order problem integrable simply in terms of elliptic integrals (Cid, Ferrer, Sein-Echaluce) arouses expectations that major theories like those of the moon or the solar system will undergo the kind of disencumbering that Delaunay and Hill called for hundred years ago. In the mean time, theorists go on improving the techniques inherited from the XIXth century, witness a revision of Hansen's recurrences for obtaining the coefficients of high degree in the formulas of the Two-Body problem developed according to the powers of the eccentricity (R.Proulx).

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The International Sun-Earth Explorers (ISEE), a collaboration between NASA and ESA, have reinforced the special emphasis that Molniya type satellites for communication over long distances have put on orbits with very large eccentricities. One would have believed, at first glance, the dominant perturbation is alternatively the drag on the lower part of the loops and the lunar and solar attractions on the upper part. A careful analysis, both numerical and analytical, has revealed, though, that the lunar attraction is mainly responsible for the decay of ISEE-1 and ISEE-2 (J.P. Amenabar et al. at the Computer Sciences Corporation, Silver Spring, MD, U.S.A.). This is but one of many instances where classical celestial mechanics is called upon nowadays to contribute to control theory in maneuvering satellites to extend their lifetime or to keep them close to their specified paths. On the other hand, tracking engineers are increasingly involved with the problem of extending extrapolations over very long arcs. Clearing dead satellites from the equatorial geosynchronous belt is one of the practical issues to which astrodynamicists contribute imaginative solutions. In those areas, competition for attention, merit, speed, versatility and portability is as vigorous as ever between general purpose numerical integrators, semi-analytical theories - numerical integration of numerically averaged equations advocated by P.J. Cefola and his group at the Charles Stark Draper Laboratory -, the stroboscopic method - devised by C. Lubowe in the late 50s at the Bell Telephone Laboratories, but re-invented by E.A. Roth at the European Space Operations Center (ESOC) in Darmstadt.

Yet, much of the computational story of these past three years has been the tremendous performance of UTOPIA and GEODYN in the midst of giant electronic developments. Satellite Laser Ranging (SLR) has emerged as a technology capable of providing geodesists and oceanographers with a vast array of new measurements and global coverage. The most fastidious orbit correctors operating in batch mode, UTOPIA and GEODYN, have now taken residence in the fastest vector preocessors available, respectively a CRAY at the University of Texas in Austin (UT) and a CYBER 205 at NASA Goddard Space Flight Center (GSFC). The transfer in both cases took two years; it was completed for the most part in the early months of 1986. In the new computer environment. David E. Smith and the Laboratory for Geophysics at GSFC set themselves to the task of analyzing in one batch all laser rangings to 17 satellites, deriving therefrom the first "universal" gravity model of the earth, GEM-T1. For his part, Byron Tapley with his team at the Center for Space Research at UT processes observations in single batches extending over 4000 days ($\approx$ 11 years) as a matter of routine. Copies of GEODYN and UTOPIA have been distributed to major institutions in the U.S.A. and outside. Nonetheless, independent work on the determination and analysis of precise orbits is being carried out at the University of Nottingam (V. Ashkenazi), at the Technische Hogeschool Delft (K.F. Wakker), at the Centre National d'Etudes Spaciales (CNES) of Toulouse (F. Nouel), to list but a few.

Vector processing is just one of the steps SLR is taking to improve its analyses. Refining the modelling of physical effects affecting the orbit of a close satellite is another one. In this regard, one should mention a flurry of studies caused by two unexpected phenomena : 1) a secular deceleration in LAGEOS's semimajor axis (D.P. Rubincam, F. Barlier) and 2) a secular acceleration of its node (C. F. Yoder et al.).

When the European Space Agency puts ERS-1 in orbit and, somewhat later, after NASA and CNES launch TOPEX/POSEIDON, a new era in orbit determination will be entered. The altimetry equipment on these platforms will produce information about the surface of the oceans with unprecedent accuracy at the level of 10 cm. Preparation of the missions is eliciting numerous studies ranging from a refitting of Kaula's solution of the variational equations in artificial satellite theory (Rosborough) to the possibility of separating permanent sea surface topography from time-varying features by comparing data at "crossover points" (G.H. Born, R.E. Cheney, J.G. Marsh, C.K. Shum, B.D. Schutz, B.D. Tapley, etc.). Unrelentingly, the orbit of SEASAT, the first satellite dedicated to studying the surface of the oceans, and that of GEOSAT-ERM are recomputed to evaluate the most recent models of the earth's gravity field in conjunction with Wahr's model of tides for the solid earth and that of Schwiderski for ocean tides.

The Global Positioning System (GPS) adds a purely geometric element to the problem of orbit determination from observations of close satellites. Studies of triangulation by clock carrying spacecrafts are pursued vigorously with a view of identifying the factors most likely to affect the accuracy at the level of 1m and below. Most critical among them would be the solar radiation pressure and the stochastic tropospheric delay at the zenith of the receiving station (S.M. Lichten at the Earth Orbiter and Navigation Systems Group, Jet Propulsion Laboratory). A GPS receiver on TOPEX/POSEIDON would bring the errors below the 10cm level (B.G. Williams, loc. cit.). Optimists talk with tongue in cheek of coming to 1mm by the mid-90s. Orbit determination by numerical integration of satellite orbits and differential corrections is now firmly established on the highest grounds of perfection in numerical engineering.

V.A. Brumberg,
President of the Commission.