Cor. 2. If the first corollary be proved independently, the theorem may be deduced from it by projection.

Cor. 3. The dual of this theorem is also true. It may be stated as follows:—If lines l, m, n, be drawn through any point to the three diagonal points of a complete quadrangle, and the harmonic conjugates l', m', n', of l, m, n, be taken with respect to the sides of the quadrangle that pass through these points, then l', m', n', will be concurrent.

Sixth Meeting, April 8th, 1887.

J. S. MACKAY, Esq., LL.D., in the Chair.

On the value of $\Delta^n 0^m/n^m$, when m and n are very large.

By Professor TAIT.

I had occasion, lately, to consider the following question connected with the Kinetic Theory of Gases:—

Given that there are 3.10^{20} particles in a cubic inch of air, and that each has on the average 10^{10} collisions per second; after what period of time is it even betting that any specified particle shall have collided, once at least, with each of the others?

The question obviously reduces to this:—Find m so that the terms in

$$X^m = (x_1 + x_2 + x_3 + \ldots\ldots + x_n)^m$$

which contain each of the n quantities, once at least, as a factor, shall be numerically equal to half the whole value of the expression when $x_1 = x_2 = \ldots\ldots = x_n = 1$. Thus we have

$$X^m - \Sigma(X - x_r)^m + \Sigma(X - x_r - x_s)^m - \ldots\ldots = \tfrac{1}{2}X^m$$

or

$$\Delta^n 0^m / n^m = \tfrac{1}{2}.$$

It is strange that neither Herschel, De Morgan, nor Boole, while treating differences of zero, has thought fit to state that Laplace had, long ago, given all that is necessary for the solution of such questions. The numbers $\Delta^n 0^m$ are of such importance that one would naturally expect to find in any treatise which refers to them at least a state-

https://doi.org/10.1017/S0013091500001449 Published online by Cambridge University Press

ment that in the *Théorie Analytique des Probabilités* (Livre II., chap. ii., § 4) a closely approximate formula is given for their easy calculation. No doubt the process by which this formula is obtained is somewhat difficult as well as troublesome, but the existence of the formula itself should be generally known.

When it is applied to the above problem, it gives the answer in the somewhat startling form of "about 40,000 years."

P.S.—April 4, 1887.—Finding that Laplace's formula ceases to give approximate results, for very large values of m and n when these numbers are of the same order of magnitude, I applied to Prof. Cayley on the subject. He has supplied the requisite modification of the formula, and his paper has been to-night communicated to the *Royal Society of Edinburgh.*

Sur les cordes communes à une conique et à un cercle de rayon nul; Application à la théorie géométrique des foyers dans les coniques.

PAR M. MAURICE D'OCAGNE.

1. Etant donnés une conique K, dont l'équation est $K=0$, et un point P (α, β), l'équation générale des coniques qui passent par les points d'intersection de la conique K et du cercle P de rayon nul, qui a le point (α, β) pour centre, est

(1) $$K+\lambda[(x-\alpha)^2+(y-\beta)^2]=0.$$

Comme les quatre points d'intersection du cercle P et de la conique K sont imaginaires, le système (1) comprend un seul couple de droites réelles Δ et Δ'. Ces droites seront dites, par analogie avec une expression proposée par Chasles,* les *conjointes du point P et de la conique K.*

Parmi les couples de cordes imaginaires communes au cercle P et à la conique K se trouvent les *droites isotropes* passant au point P

$$(x-\alpha)+i(y-\beta)=0$$
$$(x-\alpha)-i(y-\beta)=0,$$

* *Journal de Liouville,* T. III., p. 385.