Stellar Rotation
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Oscillations of Fast Rotating Stars: p-Modes in Centrifugally Flattened Polytropes

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Abstract. Oscillation modes of rapidly rotating stars have not yet been calculated with precision, rotational effects being generally approximated by perturbation methods. We developed a mathematical formalism and a numerical method which fully account for the deformation of the star by the centrifugal force. The method has been first tested in the case of Maclaurin spheroids and then applied to uniformly rotating polytropic stars.

The ratio of centrifugal and gravity forces being generally small in stars, the effect of rotation on gravito-acoustic stellar oscillations has been mostly studied with perturbation methods. Although fully justified in the context of helioseismology, this approach might not be accurate enough for rapidly rotating stars. We are developing a mathematical formalism and a numerical method in order to treat the full problem. As a first step, we considered how the centrifugal flattening of the star affects the oscillation spectrum, neglecting the action of the Coriolis and centrifugal forces on the wave motion itself. Note that this is justified in the high frequency limit. Perturbations are also assumed to be adiabatic and the Cowling approximation is used.

Except in the particular cases of spheroids and spheres, the eigenvalue problem associated with acoustic resonances in an arbitrary axially symmetric cavity is not separable. A 2-D eigenvalue problem must then be solved numerically. As eigenfrequencies are very sensitive to the cavity shape, we expect a better numerical accuracy if grid points are exactly on the stellar surface. If (r, θ, ϕ) denote the usual spherical coordinates and, $r = S(\theta)$, the surface, we choose coordinate systems $(r = f(\zeta, \theta), \theta, \phi)$ such that $\zeta = \zeta_0$ describes the surface. The variables are then expanded on the spherical harmonics and the governing equation are projected on each spherical harmonic. Following a method described in Rieutord & Valdettaro (1997), the coupled differential equations of the variable ζ are discretized on the Gauss-Lobatto grid associated with Chebyshev polynomials and the resulting algebraic eigenvalue problem is solved.

This method has been used to calculate the acoustic spectra of Maclaurin spheroids and rotating self-gravitating polytropes. For uniform spheroids, the eigenfunctions are separable using spheroidal coordinates and the related Hamiltonian system is integrable. According to the semi-classical quantization performed by Arvieu & Ayant (1987), modes can be classified by the number of zero of the eigenfunctions along each spheroidal coordinates. Moreover, modes divide in two classes depending on their internal caustics which is either a spheroid or a hyperboloid. We also noticed that, for 10 percent flatness, the difference between frequencies calculated by a first order perturbative method and

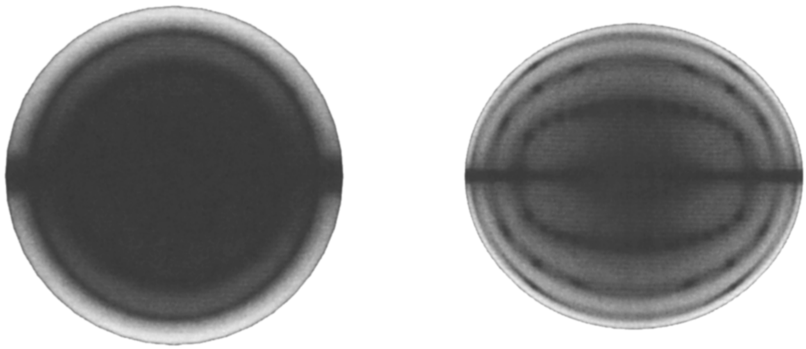

Figure 1. Evolution of the $(\ell,n) = (1,3)$ mode as the rotation of a $n = 3$ polytrope increases; the iso-contours of the vertical kinetic energy have been chosen to see the low levels near the centre.

the actual frequencies can be much higher than the accuracy of spatial asteroseismology missions (Lignières et al. 2001). The case of rotating self-gravitating polytropes is not as simple since their surface is not an exact spheroid so that the eigenfunctions are no longer separable and the related Hamiltonian system is non-integrable. This poses a problem for mode classification and so for the study of the spectrum in general. By progressively increasing the rotation from the non-rotating case, we used the quantum numbers defined in the non-rotating case to classify the modes. But, as the flatness affects differentially the eigenstates, the spectrum structure changes and modes with originally distinct frequencies tend to reach identical frequencies. When this occurs between modes having the same absolute value of the azimuthal number and the same parity with respect to the equator, an avoided crossing takes place, the classification becoming ambiguous during the crossing. As in the example shown in Figure 1, the low degree and low order modes that we have studied are of the whispering gallery type.

We are now investigating the acoustic spectrum structure looking for some regular behaviours. Such regularities might be used to identify modes in observed spectra of fast rotating stars. Also, the organization of the phase space of the related Hamiltonian system should inform us about the spectrum structure and may lead to discover manifestation of quantum chaos in stellar pulsations.

References

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Jean-Paul Zahn, André Maeder and Don VandenBerg waiting in front of empty glasses. Philippe Eenens Jr. looking on the side.

Henny Lamers and Marcelo Borges Fernandes, the master and disciple.