# LIMITATIONS ON THE ACCURACY POSSIBLE IN ASTROMETRIC OBSERVATIONS OF THE SATELLITES OF THE MAJOR PLANE' 

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Astrometric accuracy has two components; the accuracy with which an image can be centred on a CCD or photographic plate and the accuracy with which two image centres can be mapped on to standard co-ordinates. Photographic plates cover a sufficient area of sky that several standard stars can be measured with the satellite images and the mapping to standard coordinates can be done in one operation. CCDs have much smaller areas and the number of stars with sufficiently accurate positions may not be sufficient. The surface densities of the best available catalogues are shown in Table I.

TABLE 1. Available Catalogues

| Catalogue | Mean Position Error <br> $(\operatorname{arcsec})$ | Number of Stars | Stars/Square Degree |
| :--- | ---: | ---: | ---: |
|  | $0.5(>1.0)$ | $1.9 \times 10^{7}$ | 461 |
| GSC | 0.27 | $1.8 \times 10^{5}$ | 8.7 |
| PPM (North) | 0.11 | $2.0 \times 10^{5}$ | 9.7 |
| PPM (South) |  |  |  |

The accuracy with which the scale can be measured is shown in Table II using the 1-metre Jacobus Kapteyn Telescope (JKT) on La Palma as an example.

Centering accuracy depends on whether the image is under- or oversampled, the signal-to-noise and the centering algorithm used. Experience with an ongoing trigonometric parallax programme shows that repeated CCD exposures of the same field may be mapped on to each other with an

TABLE 2. Characteristics of 1-metre JKT

| Focal Ratio | $\mathrm{f} / 15$ | $\mathrm{f} / 8$ |
| :--- | :---: | :---: |
| Detector <br> Field Diameter <br> (arcsec) | CCD | Photography |
| Field Area |  |  |
| (square Deg) | 400 | 5400 |
| Expected Number of Stars | 0.010 | 1.77 |
| GSC | 4.6 | 816 |
| PPMN | 0.1 | 15 |
| PPMS | 0.1 | 17 |
| Error of Scale $\left(\times 10^{4}\right)$ |  |  |
| GSC | 12 | 0.9 |
| PPMN | 7 | 0.5 |
| PPMS | 3 | 0.2 |

error of 0.05 pixels. As our CCDs are typically $1000^{*} 1000$ pixels we have to establish the scale with a fractional error $\left(\times 10^{4}\right)$ of 0.5 and the rotation within 0.003 degrees. Table II shows that only the PPM catalogue used with the photographic camera can yield sufficient accuracy and only the GSC has sufficient stars to calibrate a typical CCD exposure. The best policy is to set up fiducial fields by photography using the PPM as primary standards to find the positions of a sample of stars within one CCD field.

Three photographic plates of the Pleiades taken with the JKT. There are approximately sixty PPM (North) stars on each plate and the mean error from the PDS measures is $\pm 0.30$ arcsec in one co- ordinate, showing that the PPMN accuracy has been degraded by errors of plate measurement. Within the CCD fiducial field the inter-agreement of the measures was $\pm 0.18 \mathrm{arcsec}$ in one co-ordinate. Six CCD images of the fiducial field which contains nine stars of roughly the fourteenth magnitude, were measured and the agreement with the photographic positions was $\pm 0.05 \mathrm{arcsec}$ in one co-ordinate on one image. The mean of the six CCD images was used to correct the photographic star positions; after correction the error improved to $\pm 0.008$ arcsec which is 0.03 pixels. Thus we have more than achieved the desired accuracy in centering the images but the accuracy of the scale and orientation of the chip are still limited by errors in the star positions.

Thirty stars were measured in the field of M92 and treated in the same way as the Pleiades. The five images were taken on different nights. Thirteen stars were measured in NGC 6823 and reduced relative to the GSC; four stars were measured in the field of Saturn on 1993 August 28/29 and also


Figure 1. Rotation of fields at different declinations with Hour Angle, Squares M92, declination 43.1, Circles Pleiades 24.5, Stars NGC 6823 23.2, Triangles Field of Saturn -14.2; the errors for M92 and the Pleiades are $\sim 0.001$ degrees and the other fields $\sim 0.01$
measured relative to the GSC. Fig. 1 shows the angle in degrees between the columns of the chip and north plotted as a function of hour angle. The scatter after correction for hour angle in each field is commensurate with the errors which implies that the telescope is moving elastically without significant hysteresis or plasticity.

TABLE 3. Coefficients of Pointing Errors

| Parameter | Hour Angle | Declination | Position Angle |
| :---: | :---: | :---: | :---: |
| Altitude of Pole | $\tan \delta \sin \tau$ | $\cos \tau$ | $\sec \delta \sin \tau$ |
| Azimuth of Pole | $\tan \delta \cos \tau$ | $\sin \tau$ | $\sec \delta \cos \tau$ |
| Non-perpendicularity |  |  |  |
| of axes | $\tan \delta$ | . | $\sec \delta$ |
| Collimation | $\sec \delta$ | . | $\tan \delta$ |
| Flexure of | $\sec \delta(\sin \phi \sin \delta$ |  | $\sin \phi \sec \delta$ |
| Declination Axis | $+\cos \phi \cos \delta \cos \tau)$ | $\sin \phi \cos \delta$ |  |
| Flexure | $\sec \delta \cos \phi \sin \tau$ | $\sin \phi \cos \phi \sin \delta \cos \tau$ | . |
| of Tube | $\cdot$ |  |  |

Some of the rotation in Fig. 1 is caused by imperfections of the equatorial mounting. The correction formulae for hour angle, declination and field rotation were developed by Bessel (1841), Arend (1951). The corrections


Figure 2. Rotation after correction for errors of the equatorial mounting, symbols as in Fig. 1
are regularly determined so that the telescope can be accurately pointed. The effect of applying these corrections to Fig. 1 is shown in Fig.2. The discrepancies are reduced but still significant.

Alternatively the measured position angles may be used to find the pointing coefficients of the equatorial mounting. If this is done the residuals show no correlation with position on the sky and have a standard deviation of 0.003 degrees. However the pointing coefficients are then significantly different from those used to point the telescope. These discrepancies may arise from errors in the star positions or from Bessel's model of the equatorial telescope being incorrect in this case e.g. the assumption that the telescope tube and declination axis flex proportionately to the sine of the zenith distance may be false.

It is possible to calibrate CCD astrometry adequately and absolutely from calibration fields. However these fields should have star positions more accurate than those currently available and should lie close in the sky to the target field.

## References

Bessel, F.W. (1841) Theorie eines, mit einem Heliometer versehen AequatorealInstruments, Astronomische Untersuchungen, Erster Band, pp. 1-54
Arend, S. (1951) Théorie de l'équatorial visuel et de l'équatorial photographique., Monographies Observatoire Royal de Belgique, 2., pp. 51-57

