Magnetorotational Mechanism: 2D Simulation

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Abstract. In the paper we present results of numerical simulations of magnetorotational model of explosion of magnetized cloud. For the simulation we used a specially developed 2D implicit numerical scheme on Lagrangian triangular grid with grid reconstruction. Our results show that due to the amplification of toroidal magnetic field component and transfer of angular momentum, a compression wave in the envelope of the cloud appears and moves through rapidly decreasing density background. This wave turns into a shock wave and pushes out part of the envelope of the star. Quantitative estimates of mass and energy carried away are given.

1 Introduction

One of the possible mechanisms of supernova explosions is magnetorotational mechanism, suggested by Bisnovatyi-Kogan (1970). It is based on the transformation of the gravitational energy to the energy of explosion by magnetic field. The poloidal magnetic field in differentially rotating star is twisted and toroidal component of the magnetic field appears which grows with time. When the force produced by the magnetic field becomes comparable to the gravitational force it pushes the matter of the star outwards.

Simulation of the collapse of magnetized rotating gas cloud in 2D has been done in number of papers, see for reference Ardeljan et al. (1996b).

Realistic magnetic fields in the stars are rather small (the ratio between magnetic and gravitational energies is about $10^{-6} - 10^{-8}$). Such weak magnetic fields are the main difficulty for the numerical simulations, because of the very small hydrodynamic time scale and large time scale of the magnetic field amplification.

Numerical simulation of such stiff problem requires application of the implicit numerical methods, which are free from Courant restriction on the time step.

https://doi.org/10.1017/S0252921100070883 Published online by Cambridge University Press

2 Formulation of the Problem, Numerical Method

The problem of the collapse of rotating magnetized gas cloud is described by the set of magnetohydrodynamical equations with selfgravitation for gas with infinite conductivity.

For the initial conditions, we assume that the cloud is a rigidly rotating uniform gas sphere with the following parameters:

$$r = 3.81 \cdot 10^{16}\text{cm},\ \rho = 1.492 \cdot 10^{-17}\text{g/cm}^3,$$

$$M = 1.73 M_{\odot} = 3.457 \cdot 10^{33}\text{g},\ \gamma = 5/3,\ u^r = u^z = 0,$$

$$E_{\text{rot0}}/|E_{\text{gr0}}| = 0.04, \quad E_{\text{in0}}/|E_{\text{gr0}}| = 0.01, \quad E_{\text{mag1}}/|E_{\text{in1}}| = 0.05.$$

For our simulations we used implicit conservative Lagrangian operator difference scheme on triangular grid with reconstruction (Ardeljan et al. (1996a)). Grid restructuring procedure allows us to overcome grid overlapping situation in gas flows with nonuniform contractions or expansions or flows with vortexes. Due to the implicitness of the scheme it was possible to use time steps, which are much larger then Courant time step.

3 Results

The initial number of grid knots was 5000. At the initial time moment ($t = 0$) magnetic field is "switched off", because it does not significantly influence the hydrodynamical collapse stage which is rather short in comparison with time of the amplification of the magnetic field.

After a few consequent contractions and expansions the cloud comes to the differentially rotating stationary state, at $t_1 = 5.153$ and consists of a rapidly rotating dense core and an extended light envelope. The angular velocity of the cloud at this time presents a function with maximum of gradient at the transition zone between core and envelope.

At the moment $t_1 = 5.153$ an initial magnetic field of the following configuration (Ardeljan et al. (1996b)) (Figure 1):

$$H_{r0} = F_r(0.5r, 0.5z - 2.5) - F_r(0.5r, 0.5z + 2.5)\,,\ H_{\varphi 0} = 0\,,$$

$$H_{z0} = F_z(0.5r, 0.5z - 2.5) - F_z(0.5r, 0.5z + 2.5)\,,$$

$$F_r(r,z) = k\left(\frac{2rz}{(z^2+1)^3} - \frac{2r^3 z}{(z^2+1)^5}\right)\,,\ F_z(r,z) = k\left(\frac{1}{(z^2+1)^2} - \frac{r^2}{(z^2+1)^4}\right)\,,$$

has been "switched on" with the coefficient $k = 0.43$. This magnetic field is symmetrical about the equatorial plane, it fulfills $\text{div}\mathbf{H}_0 = 0$, but is not force-free. The choice of initial magnetic field configuration is connected with an attempt to avoid a singularity, which could create numerical problems. After the moment of "switching on" the magnetic field, the matter starts to move to the periphery of the envelope due to the magnetic force produced by

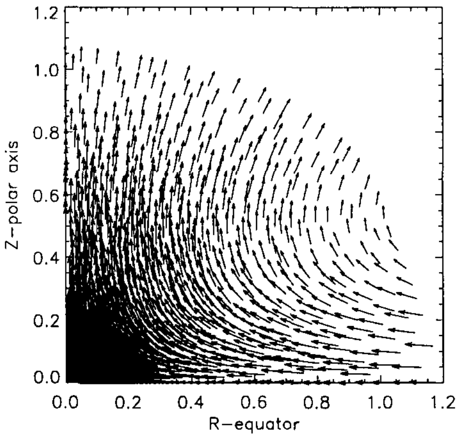

Fig. 1. Initial magnetic field at $t = 5.153$.

the initial configuration of the magnetic field, which leads to the appearance of a "finger" which grows at the outer boundary of the cloud. This artificial "finger" has no significant influence on the evolution of the inner parts of the cloud. Due to the quadrupole type of symmetry of the poloidal magnetic field the appearing toroidal component has 2 extrema. The first is situated at the equatorial plane in the zone between the core and the envelope and the second is situated above the equatorial plane close to the z-axis.

At $t = 10.234$ the toroidal part of magnetic energy reaches its maximum. While the total energy of the toroidal component of the magnetic field of the cloud is much smaller than its gravitational energy, the density of the toroidal magnetic energy in the regions near the extrema of H_φ becomes comparable to the density of the internal energy which pushes the matter out of the cloud. Starting from $t = 11.342$ the kinetic energy of the accelerated part of the envelope near equatorial plane becomes higher than its potential energy and can be ejected from the cloud. The ejection along the equatorial plane leads to a change of the shape of the cloud, which is stretched along r-axis (in r, z coordinates), (see Figure 2), related to the last computational moment $t = 32.634$.

At Figure 3 time evolution of the ejected mass of the envelope in percentage to the total mass of the cloud is presented. The ejected part of the matter contains about 0.09% of the gravitational energy of the stationary cloud after the collapse.

The model of a collapsing, rotating magnetized gas cloud, described above cannot be considered as a complete and final explanation of the mechanism of the supernova explosions. However the results of such simplified formulation give evidence in favour of the magnetorotational mechanism can as an explanation for the problem of the supernova explosions.

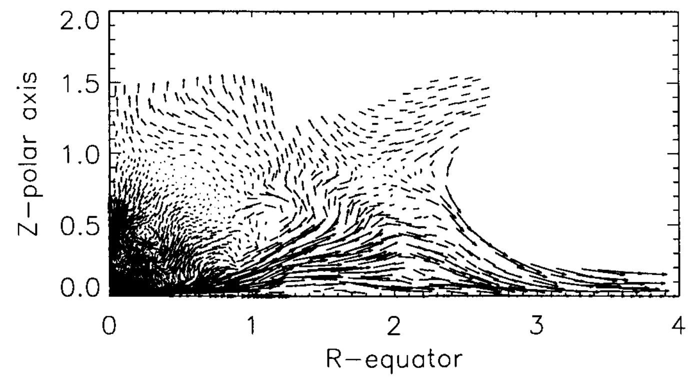

Fig. 2. Velocity field at $t = 32.624$.

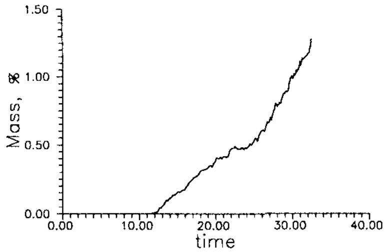

Fig. 3. Time evolution of the ejected mass in % to the total mass of the cloud.

Our recent simulations of the collapse of the star with more realistic equations of state and neutrino losses as in the paper by Ardeljan et al. (1987) show that the star contracts much stronger and rotates much more differentially, than for the simple equation of state used in this paper. We may expect a more efficient magnetorotational explosion.

Acknowledgements. S.G.M. and G.S.B.-K. are grateful to the partial support by the NSF grant AST-9320068, CRDF award # Rp1-173, and RFBR grants 96-02-16553, 97-02-26559. G.S.B.-K. thanks INTAS grant No.93-93. N.V.A. is thankful to the RFBR for the support by the grant 96-01-00838.

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https://doi.org/10.1017/S0252921100070883 Published online by Cambridge University Press