Part 4. The Interaction between Convection and Pulsation

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Stochastic Excitation in Solar-Type Stars

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Abstract. The most convincing evidence to date of solar-type oscillations in other stars comes from recent observations of β Hydri (Bedding et al., 2001) and α Cen A (Bouchy & Carrier, 2001). It is the current belief that the convection dynamics in the outer layers of sun-like stars is the source for driving the intrinsically stable modes to the observed amplitudes. Comparing such observations with theoretical models will help us improve our understanding of the interaction between convection and pulsation.

In this contribution I review the mechanisms responsible for mode damping in stars with convective envelopes, and the basic mechanism of stochastic driving by turbulent convection. The application of a stochastic excitation formalism to the Sun is discussed and compared with recent measurements and numerical simulations. Amplitude predictions for models of Procyon, α Cen A and β Hydri are compared with observations.

1. Introduction

Sun-like stars possess surface convection zones, and it is in these zones, where the energy is transported principally by the turbulence, that most of the driving of the intrinsically stable modes takes place. Mode stability is governed not only by the perturbations in the radiative fluxes (via the κ-mechanism) but also by the perturbations in the turbulent fluxes (heat and momentum). The study of mode stability therefore demands a theory for convection that includes the interaction of the turbulent velocity field with the pulsation.

Predictions of amplitudes of solar-like oscillations in other stars were carried out first by Christensen-Dalsgaard & Frandsen (1983) and later by Houdek et al. (1999) and by Samadi et al. (2001). Christensen-Dalsgaard & Frandsen obtained amplitudes of modes by postulating equipartition between the energy of an oscillation mode and the kinetic energy in one convective eddy having the same turnover time as the period of the oscillation. This simple formula for excitation was proposed by Goldreich & Keeley (1977b), who used it to estimate amplitudes for the solar case, assuming damping rates determined solely by a scalar turbulent viscosity (Goldreich & Keeley, 1977a). Houdek et al. (1999) assumed the excitation mechanism proposed by Balmforth (1992b). In Balmforth's calculations the linear damping rates were obtained from solving the equations of linear nonadiabatic oscillations in which convection was treated with the time-dependent, nonlocal convection formalism by Gough (1976, 1977). As discussed by Balmforth (1992a) for the solar case and by Houdek et al. (1999) for other stars, it is the perturbed momentum flux which crucially contributes to mode damping and to making all modes stable in these stars. Recently

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Samadi & Goupil (2001) proposed a turbulent-excitation model based on Goldreich et al.'s (1994) work. They included a more sophisticated model for the excitation process due to the entropy perturbations. Furthermore, with their formulation the effect of assuming different turbulent spectra in the excitation model on the mode amplitudes can be investigated consistently.

In this contribution we adopt Balmforth's excitation model to predict amplitudes of radial p modes for the Sun, α Cen A, β Hydri and Procyon. Stellar models and damping rates are computed in the manner of Houdek et al. (1999).

2. The stochastic excitation model

2.1. Mode parameters

Were solar p modes to be genuinely linear and stable, their power spectrum could be described in terms of an ensemble of intrinsically damped, stochastically driven, simple-harmonic oscillators, provided that the background equilibrium state of the star were independent of time (Fig. 1); if we assume further that mode phase fluctuations contribute negligibly to the width of the spectral lines, the intrinsic damping rates of the modes, $\Gamma/2$, could then be determined observationally from measurements of the pulsation linewidths Γ.

The power (spectral density), P, of the displacement ξ_{nl} of a damped, stochastically driven, simple-harmonic oscillator, satisfying

$$I_{nl}\left[\frac{\mathrm{d}^2\xi_{nl}}{\mathrm{d}t^2}+\Gamma_{nl}\frac{\mathrm{d}\xi_{nl}}{\mathrm{d}t}+\omega_{nl}^2\xi_{nl}\right]=f(t)\,, \tag{1}$$

which represents the pulsation mode of order n and degree l, with linewidth Γ_{nl} and frequency ω_{nl} and inertia I_{nl}, satisfies

$$P\propto P_{\mathrm{L}}\,P_{\mathrm{f}}=\frac{\Gamma_{nl}/2\pi}{(\omega-\omega_{nl})^2+\Gamma_{nl}^2/4}\,P_{\mathrm{f}}\,, \tag{2}$$

assuming $\Gamma_{nl}\ll\omega_{nl}$, where $f(t)$ describes the stochastic forcing function. The total mean energy in the mode is

$$I_{nl}\,V_{nl}^2\;:=\;\frac{1}{2}\omega_{nl}^2 I_{nl}\langle|A_{nl}|^2\rangle\;\propto\;\frac{P_{\mathrm{f}}(\omega_{nl})}{I_{nl}\Gamma_{nl}}\;\propto\;\omega_{nl}^2 I_{nl}\int_0^\infty P(\omega)\,\mathrm{d}\omega\,, \tag{3}$$

where A_{nl} is the displacement amplitude (angular brackets, $\langle\rangle$, denote an expectation value); i.e., it is directly proportional to the injected power of the stochastic forcing, P_{f}, at the frequency ω_{nl} and indirectly proportional to Γ_{nl}. Thus, in order to compute the root-mean-square velocity amplitude, V_{nl}, we need to model the damping rate and the rate of energy of the random forcing.

2.2. Damping rates

Basically, the damping of stellar oscillations arises from two sources: processes influencing the momentum balance, and processes influencing the thermal energy equation. Each of these contributions can be divided further according to their physical origin, as illustrated in Fig. 2. A detailed discussion of the processes

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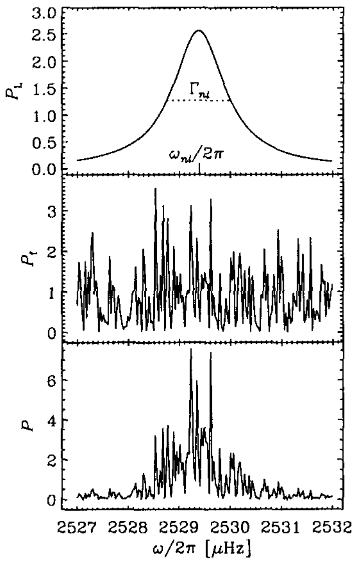

Figure 1. Power spectra of a randomly excited, damped, harmonic oscillator. P_f represents the spectral density of the random force and P is the product of the Lorentzian P_L and P_f (adopted from Kosovichev, 1995).

was given by Houdek et al. (1999 and references therein). Here we limit the discussion to the convection dynamics only.

Vibrational stability is influenced crucially by the exchange of energy between the pulsation and the turbulent velocity field. The exchange arises either via the pulsationally perturbed convective heat flux, or directly through dynamical effects of the fluctuating Reynolds stress. In fact, it is the modulation of the turbulent fluxes by the pulsations that seems to be the mechanism predominantly responsible for the driving and damping of solar-type acoustic modes. It was first reported by Gough (1980) that the dynamical effects arising from the turbulent momentum flux (also called turbulent pressure p_t) perturbations contribute significantly to the damping Γ_t. Detailed analyses (Balmforth, 1992a) reveal how damping is controlled largely by the phase difference between the momentum perturbation and the density perturbation. Therefore, turbulent pressure fluctuations must not be neglected in stability analyses of solar-type p modes. Results of modelled solar damping rates were discussed recently by Houdek et al. (2001).

2.3. Acoustical noise generation rate

Intrinsically stable modes can be stochastically excited by the turbulent convection. The process can be regarded as multipole acoustical radiation (e.g. Unno, 1964), e.g., the volume expansion of the convective eddies results in a monopole

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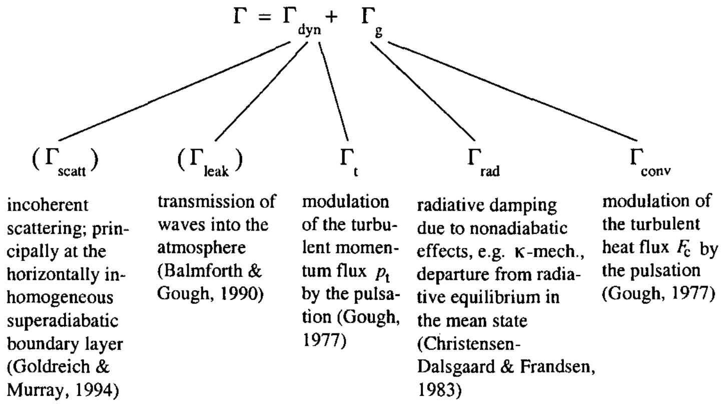

Figure 2. Physical processes contributing to the linear damping rate $\Gamma/2$. They can be associated with the effects arising from the momentum balance ($\Gamma_{\rm dyn}$) and from the thermal energy balance ($\Gamma_{\rm g}$). The contributions $\Gamma_{\rm scatt}$ and $\Gamma_{\rm leak}$ are in parentheses because they have not been taken into account in the computations reported in this paper. The influence of Reynolds stress on solar modes, contributing to $\Gamma_{\rm t}$, has been treated by Goldreich & Keeley (1977a) in the manner of a time-independent scalar turbulent viscosity. The linewidth of the oscillations is influenced also by nonlinearities, both those coupling a mode to others (Kumar & Goldreich, 1989) and those intrinsic to the mode itself.

radiation. The turbulent-excitation model predicts not only the right order of magnitude for the p-mode amplitudes (Gough, 1980), but it also explains the observation that millions of modes are excited simultaneously. Acoustical radiation by turbulent multipole sources in the context of stellar aerodynamics has been considered by several authors (see Houdek et al. 1999 and references therein). Because of the lack of a complete convection theory, the mixing-length approach still represents the main method for computing the turbulent fluxes in stars with convectively unstable regions. One of the main assumptions in the mixing-length formulation is the Boussinesq approximation. It neglects acoustic wave generation by assuming the fluid to be incompressible. This is demonstrated with the linearized expression of the perturbed continuity equation:

$$\underbrace{\partial_t\rho' + \nabla\cdot(\rho\boldsymbol{u}) = 0}_{\text{exact}} \quad \rightarrow \quad \underbrace{\nabla\cdot(\rho\boldsymbol{u}) = 0}_{\text{anelastic appr.}} \quad \rightarrow \quad \underbrace{\nabla\cdot\boldsymbol{u} = 0}_{\text{Boussinesq}}\,, \tag{4}$$

where ρ' is the Eulerian density perturbation and $\boldsymbol{u}$ is the convective velocity field. Consequently a separate model is needed to describe approximately the acoustical noise generated by the turbulent motion of the convective eddies. Such a model was proposed by Lighthill (1952); in this model the density fluctuations are the same between a real fluid with highly nonlinear motion and a fictitious acoustic medium with linear motion upon which an external stress system T_{ij} is acting. Expressed mathematically,

$$\mathcal{L}\rho' = \partial_i\partial_j T_{ij}\,, \tag{5}$$

where $\mathcal{L}$ is a linear wave operator and $T_{ij} = \rho u_i u_j$ is a nonlinear stress tensor. In other words, it is the nonlinearity of the fluid motion that generates the acoustical noise.

Balmforth (1992b) reviewed the theory of acoustical excitation in a pulsating atmosphere, and, following Goldreich & Keeley (1977b), he derived the following expression for the rate of energy injected into a mode with frequency ω by quadrupole emission through the fluctuating Reynolds stress:

$$P_{\mathrm{R}} = \frac{\pi^{1/2}}{8I} \int_0^M \left(\frac{\partial \xi}{\partial r}\right)^2 \rho \ell^3 u^4 \tau \mathcal{S}(m,\omega)\,\mathrm{d}m\,, \tag{6}$$

where ℓ,u,τ are the length, velocity and correlation time scales, respectively, of the most energetic eddies, determined by the mixing-length model. The function $\mathcal{S}(m,\omega_{\mathrm{r}})$ accounts for the turbulent spectrum, which approximately describes contributions from eddies with different sizes to the noise generation rate P_{R}, and which we implemented as did Balmforth (1992b). The displacement eigenfunction of a p mode is described by ξ (r and M are radius and total mass).

A similar expression is obtained for the emission of acoustical radiation by low-order multipole sources through the fluctuating entropy. The ratio of the noise generation rate between the fluctuating entropy, P_{s}, and Reynolds stress is (Goldreich et al., 1994):

$$\left(\frac{P_{\mathrm{s}}}{P_{\mathrm{R}}}\right)^{1/2} \approx \frac{4}{\alpha\Phi}\gamma_1\,, \tag{7}$$

where α is the mixing-length parameter (which is the ratio between the mixing length ℓ and the local pressure scale height), γ_1 is the first adiabatic exponent and Φ is the eddy-shape parameter, which is of order unity. Assuming typical values for α, γ_1 and Φ for the solar case, the value of this ratio is ~ 3. Consequently the noise generation rate due to the fluctuating entropy is about one order of magnitude larger than the contribution from the fluctuating Reynolds stress (see left panel of Fig. 3). In the right panel of Fig. 3 the results are depicted from hydrodynamical simulations by Stein & Nordlund (2001). From this comparison it is obvious that there is still controversy as to whether the fluctuating entropy or Reynolds stress is the dominating source of excitation

Another way to study the properties of the turbulent-excitation model is to compare the depth and frequency dependence of the integrand of P_{R} with solar measurements and with hydrodynamical simulations of the Sun. The results are depicted in Figure 4: the contours in the top panel show the integrand of Eq. (6) multiplied by $\mathrm{d}m/\mathrm{d}\ln p$, the maximum values of which are plotted by the dot-dashed line. These maximum values are compared with the positions of the horizontal, solid lines, which represent the error bars of Chaplin & Appourchaux's (1999) measurements of the locations of the driving regions. For modes with frequencies $\gtrsim 2.3\,\mathrm{mHz}$ the computed locations of the excitation regions are in good agreement with the observations. These modes have frequencies larger than the acoustical cut-off frequency, illustrated by the solid curve (the dashed curve is the isothermal approximation to the acoustical potential), and are therefore propagating. For these modes the frequency dependence of

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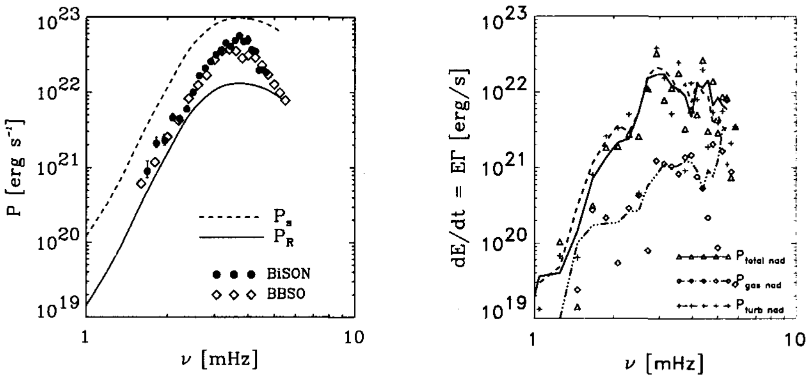

Figure 3. Noise generation rate as function of frequency for a solar model. Left: Results, obtained with Eqs. (6) and (7), are compared with observations by BiSON (Chaplin et al., 1998) and from the BBSO (Libbrecht, 1988). The contribution from the fluctuating entropy $P_{\rm s}$ is about one order of magnitude larger than the contribution from the fluctuating Reynolds stress $P_{\rm R}$. Right: Results are obtained from hydrodynamical simulations (adopted from Stein & Nordlund 2001). The contribution from the fluctuating Reynolds stress ($P_{\rm turb}$) is on average about four times larger than the contribution from the fluctuating entropy ($P_{\rm gas}$).

$P_{\rm R}$ is predominantly determined by the turbulent spectrum – described by the function $\mathcal{S}$ in Eq. (6). Modes with frequencies smaller than the acoustic cut-off frequency are evanescent. The frequency dependence of $P_{\rm R}$ for these modes is predominantly determined by the shape of the eigenfunctions, i.e. by the term $(\partial\xi/\partial r)^2/I$, and consequently by the structure of the equilibrium model.

The lower panel of Fig. 4 shows the logarithm to the base 10 of the integrand of the total noise generation rate (entropy plus Reynolds stress) as functions of frequency and depth obtained from numerical simulations by Stein & Nordlund (2001). As for the model results (top panel), the driving region decreases with frequency; however, in the simulations it extends further out to the surface for modes with frequencies $\nu \lesssim 1.8\,$mHz.

3. Amplitude predictions

With the estimates of the damping rates and noise generation rate, the oscillation amplitudes can be obtained from Eq. (3). Fig. 5 plots the mean-square velocity amplitudes for a solar model and for three solar-type stars. For the Sun results are plotted for computations in which both observed (solid curve) and theoretical (dashed curve) damping rates were assumed. Both results are calibrated to the BiSON (Chaplin et al., 1998) observations (symbols) by scaling the maximum values of the computed amplitudes to the measurements. In the remaining panels of Fig. 5 the mean-square velocity amplitudes for models of α Cen A, β Hydri and

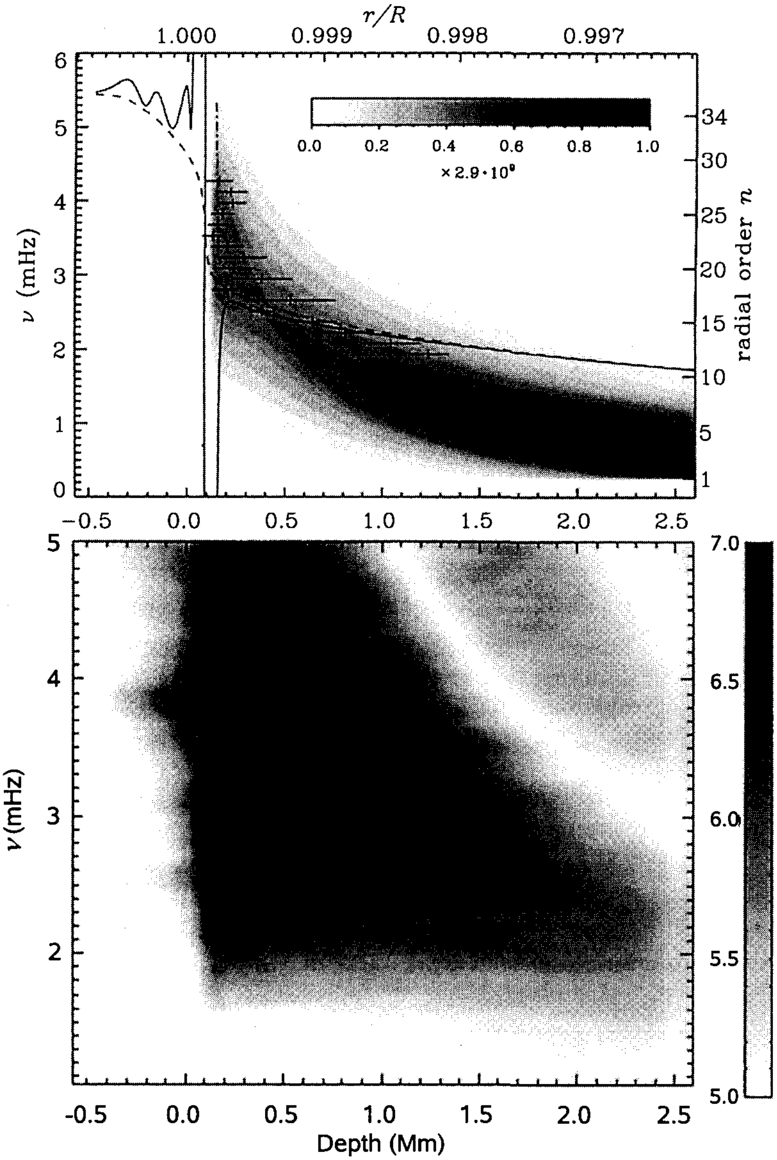

Figure 4. Top: Contours display the integrand of Eq. (6) multiplied by $\mathrm{d}m/\mathrm{d}\ln p$, the maximum values of which are plotted by the dot-dashed curve. The horizontal, straight lines represent error bars of the measured location of the excitation region by Chaplin & Appourchaux (1999). The solid and dashed curve display the exact and isothermal approximation of the acoustical potential. Bottom: Logarithm of integrand of noise generation rate from Stein & Nordlund's (2001) simulations.

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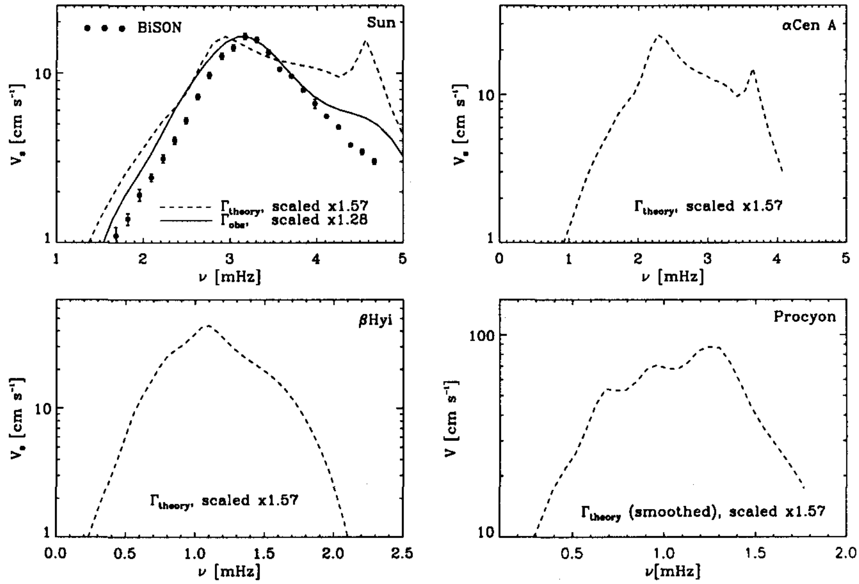

Figure 5. Mean-square velocity amplitudes for models of the Sun, α Cen A, β Hydri and Procyon, assuming Eq. (6) for the total noise generation rate.

Procyon are scaled by the factor 1.57, obtained from the scaled solar model using the theoretical damping rates.

Table 1. Comparison between theoretical $\hat{V}$ and observed $\hat{V}_{\rm obs}$ velocity amplitudes for three solar-type stars. Quoted amplitudes are peak values in $\rm cm\,s^{-1}$. A solar peak value of $\hat{V}_\odot = 23\,\rm cm\,s^{-1}$ is assumed. References to the observations are indicated.

Star	$M/M_\odot$	$L/L_\odot$	$T_{\rm eff}$ (K)	$\hat{V}/\hat{V}_\odot$	$\hat{V}$	$\hat{V}_{\rm obs}$	Reference
α Cen A	1.16	1.58	5799	1.6	37	35	Bouchy & Carrier (2001)
β Hydri	1.11	3.50	5800	2.8	65	50	Bedding et al. (2001)
Procyon	1.46	6.62	6395	5.7	132	60	Martic et al. (1999)

In Table 1 the peak values of the theoretical velocity amplitudes $\hat{V}$ are compared with recent observations $\hat{V}_{\rm obs}$. For the low-mass stars α Cen A and β Hydri the values of $\hat{V}$ and $\hat{V}_{\rm obs}$ are in fair agreement. However, for the more massive and hotter star, Procyon, the theoretical amplitude is larger by a factor of about 2.2.

For solar-type stars with masses $\gtrsim 1.35\,\rm M_\odot$ and which are hotter than the Sun, theoretical damping rates may be too small (possibly due to the lack of modelling another mechanism such as incoherent scattering, see Fig. 2) and the

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estimated convective velocities too large, leading to predicted amplitudes larger than the data suggest.

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