greater subdivision of the difference between the limits of the variable ultimately points. Now, it is evident, that if the difference, as in the case proposed, between the limits of the variable be unity, that this common factor may be represented by $\frac{1}{n+1}$. The effect, therefore, is really equivalent to taking a certain proportion only of each term; and this effect is precisely that which is indicated when an average has to be taken, provided the proportion correspond to the number of terms, as it obviously does in a definite integral whose limits are zero and unity: for as n increases, the limiting ratio of $\frac{1}{n}$ to $\frac{1}{n+1}$, or $1 : \frac{1}{1+\frac{1}{n}}$, becomes more and more equal to unity. Now, whatever law of facility of error, or of deviation among a set of observations be supposed, it has been well shown by Professor De Morgan, (*Ency. Metrop.*, art. "Probab.,") that the average term and the most probable value approach nearer and nearer to an equality as the number of data or values increases; and this is precisely the same condition as that under which the value or summation of the definite integral more and more accurately represents the limiting value of the average term. It may also be seen, by reference to an article in the July number of the *Edinburgh Review* (No. 185, p. 19,) on Probabilities, said to be by Sir John Herschel, that the same conditions, above declared to be inherent in definite integration, and therefore in averaging upon the system of limits, have to be also stipulated for in the postulates, whenever the law of the results has to be determined in its utmost generality.

E. J. F.

[NOTE.—We have received from Mr. William Wylie, of the Colonial Life Assurance Company's Office, in Edinburgh, ingenious solutions of the first and third of these Problems.—ED. *A. M.*]

ON THE DETERMINATION OF SURPLUS.

To the Editors of the Assurance Magazine.

GENTLEMEN,—I have been very much gratified with the article in the *Assurance Magazine* on the Determination of the Surplus of a Life Assurance Company. It may perhaps interest some one to see the process which I have used for the same purpose.

It should be premised that it is the practice in the American Companies to assure at the age of the nearest birthday, so that no material error can arise from assuming the day of the date of the policy as the birthday of the party assured.

In the first place, I arrange the policies according to the year of birth, as in the article referred to, but grouping them according to the age at which they were assured, and the consequent premium paid: thus—

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Born in the year ——.
Assured at the age x; premium for £1, p_x.

No. of Policy.	Date of Policy.	Decimal of a Year to Jan. 1.	Sum assured.	Product of the two Columns.
——	——	t_1	A_1	A_1t_1
——	——	t_2	A_2	A_2t_2
		&c.	&c.	&c.
		Σt_1	ΣA_1	ΣA_1t_1
Assured at the age $x+1$; premium for £1, p_{x+1}.				
——	——	t'_1	A'_1	$A'_1t'_1$
——	——	t'_2	A'_2	$A'_2t'_2$
		&c.	&c.	&c.
		$\Sigma t'_1$	$\Sigma A'_1$	$\Sigma A'_1t'_1$
		$\int$	$\int \Sigma A_1$	$\int \Sigma A_1t_1$

I here suppose the balance-sheet to be required for January 1st: if it is for any other date, the decimal of the year is taken from the date of the policy to that date; the column being easily filled up from a table previously prepared. The products (A_1t_1) I calculate to the nearest dollar; perhaps for the pound sterling they had better be calculated to tenths.*

Now, let V_{x+n} be the present value of £1 payable at death, at the age $x+n$; a_{x+n} the present value of an annuity of £1 at the same age. Then the present value of the first policy at the last birthday is A_1V_{a+n}; at the next birthday it is A_1V_{x+n+1}; and on the 1st January it is

$$A_1V_{x+n} + A_1t_1\,(V_{a+n+1} - V_{x+n}).$$

The value of the second policy, January 1st, is

$$A_2V_{x+n} + A_2t_2\,(V_{x+n+1} - V_{x+n}), \text{ \&c.}$$

The value of all the sums assured at the age x is

$$V = V_{x+n}\Sigma A_1 + (V_{x+n+1} - V_{a+n})\,\Sigma A_1t_1\,;$$

and the value of all the sums assured of those born in the same year is, on 1st January,

$$\int V = V_{x+n}\int\Sigma A_1 + (V_{x+n+1} - V_{x+n})\int\Sigma A_1t_1\,.$$

It will be observed that the only difference in the labour of obtaining this true value, and an approximate one, is that employed in filling the columns for t_1 and A_1t_1,—a labour which requires no repetition. If the third column had indicated the exact day of birth, the result would have been mathematically exact.

Again, the present value of the future payments, after the last payment was made on the first policy, was $A_1p_x a_{x+n}$; immediately before the next

* It should be remembered that the columns t_1 and A_1t_1 are constant while the policy is in force, the footings only having to be corrected from year to year as the policies lapse.

payment is made, it will be $A_1 p_x(1+a_{x+n+1})$; and on the 1st January it is

$$A_1 p_x a_{x+n} + A_1 t_1 p_x(1+a_{x+n+1} - a_{x+n}).$$

The value of the future payments on the second policy, January 1st, is

$$A_2 p_x a_{x+n} + A_2 t_2 p_x(1+a_{x+n+1} - a_{x+n}), \text{ \&c.}$$

The present value, January 1st, of the future premiums on all the policies issued at the age x, on lives born in the year ——, is

$$v = a_{x+n} p_x \Sigma A_1 + (1 + a_{x+n+1} - a_{x+n}) p_x \Sigma A_1 t_1.$$

The corresponding value for those issued at the age $x+1$ is

$$v' = a_{x+n} p_{x+1} \Sigma A'_1 + (1 + a_{x+n+1} - a_{x+n}) p_{x+1} \Sigma A'_1 t'_1, \text{ \&c.}$$

The sum of all these values is

$$\int v = a_{x+n} \int p_x \Sigma A_1 + (1 + a_{x+n+1} - a_{x+n}) \int p_x \Sigma A_1 t_1.$$

I use the true values of V_x and p_x, and apply an appropriate "loading" to $\int V$ and $\int v$, the amount of which must depend on the nature of the risks, and must be estimated for each particular Company.

I have pointed out how the exact value of all the sums assured may be obtained, when the system of assurance is such that the date of the policy cannot be taken for the date of birth. For the present value of the future payments in such cases, the exact formula is more complicated, inasmuch as both dates must be elements of the calculation. I have no doubt, however, that a formula may be obtained by which this value may be approximated to within strictly defined limits.

I ought to mention that I have found the columns t and At very useful in estimating the probable mortality in a Company in a financial year.

Your obedient Servant,

C. GILL.

Mutual Life Assurance Company,
New York, June 1851.

[NOTE.—In our article on this subject, we strongly insisted on the very small difference arising between the results of a class valuation, and one in which each policy has been separately valued. The following remarkable confirmation of this has been handed to us by a friend. It occurred in a Company in which the sums assured amounted to £2,689,719, and the annual premiums to £81,225.

	£
Value of the sums assured, each policy being separately valued .	1,576,411
Do. of future premiums, do. do. .	902,553
	£673,858

	£
Value of the sums assured, the policies being valued in classes .	1,576,521
Do. of future premiums, do. do. .	902,839
	£673,682
Difference . .	£176