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# Towards an accurate model of redshift-space distortions: a bivariate Gaussian description for the galaxy pairwise velocity distributions

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**Abstract.** As a step towards a more accurate modelling of redshift-space distortions (RSD) in galaxy surveys, we develop a general description of the probability distribution function of galaxy pairwise velocities within the framework of the so-called streaming model. For a given galaxy separation $\vec{r}$, such function can be described as a superposition of virtually infinite local distributions. We characterize these in terms of their moments and then consider the specific case in which they are Gaussian functions, each with its own mean $\mu$ and variance $\sigma^2$. Based on physical considerations, we make the further crucial assumption that these two parameters are in turn distributed according to a bivariate Gaussian, with its own mean and covariance matrix. Tests using numerical simulations explicitly show that with this compact description one can correctly model redshift-space distorsions on all scales, fully capturing the overall linear and nonlinear dynamics of the galaxy flow at different separations. In particular, we naturally obtain Gaussian/exponential, skewed/unskewed distribution functions, depending on separation as observed in simulations and data. Also, the recently proposed single-Gaussian description of redshift-space distortions is included in this model as a limiting case, when the bivariate Gaussian is collapsed to a two-dimensional Dirac delta function. More work is needed, but these results indicate a very promising path to make definitive progress in our program to improve RSD estimators.

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## 1. Key concepts

The exact relation between real- and redshift-space correlation function is provided by the streaming model (Fisher 1995; Scoccimarro 2004):
$1 + \xi_S(s_\perp, s_\parallel) = \int dr_\parallel \; [1 + \xi_R(r)] \; \mathcal{P}(r_\parallel - s_\parallel|\vec{r})$ , where $r^2 = r_\parallel^2 + r_\perp^2$ and $r_\perp = s_\perp$.
$\mathcal{P}(v_\parallel|\vec{r})$ is the line-of-sight pairwise velocity distribution at separation $\vec{r}$.

### 1.1. *Modelling the velocity distribution*

At each separation $\vec{r}$ we describe the velocity distribution $\mathcal{P}$ as
$\mathcal{P}(v_\parallel) = \int d\mu d\sigma \; \mathcal{P}_L(v_\parallel|\mu,\sigma) \; \mathcal{F}(\mu,\sigma)$ , where $\mathcal{P}_L$ is a generic local velocity distribution parameterizable by its mean $\mu$ and standard deviation $\sigma$. $\mathcal{F}$ is the joint distribution of $\mu$ and $\sigma$.

### 1.2. *Local Gaussianity (LG)*

We explore the specific case in which the local distribution $\mathcal{P}_L$ is a Gaussian function:
$\mathcal{P}(v_\parallel) = \int d\mu d\sigma \; \mathcal{G}(v_\parallel|\mu,\sigma) \; \mathcal{F}(\mu,\sigma)$, where $\mathcal{G}(v_\parallel|\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(v_\parallel-\mu)^2}{2\sigma^2}\right]$.

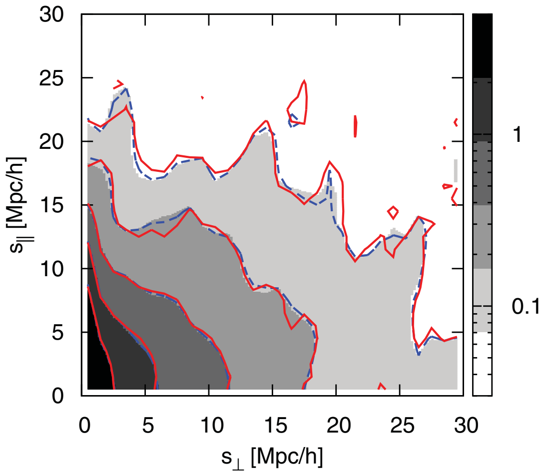


**Figure 1.** The redshift-space correlation function $\xi_S$ measured from the simulated sample. The grayscale contours correspond to the direct measurement; the blue dashed contours correspond to fitting each local distribution of pairwise velocities $\mathcal{P}_L$ with a Gaussian function and measuring its two moments $\mu$ and $\sigma 2$ to empirically build their distribution function $\mathcal{F}$; the red solid curves are instead based on the further assumption that $\mathcal{F}$ is described by bivariate Gaussian. In practice, the contours demonstrate the impact of reducing the degrees of freedom in the form of the distribution function of pairwise velocities. The level of fidelity of the red solid contours when compared to the gray-scale ones shows the goodness of the bivariate Gaussian assumption. Note that the "unsmoothed" appearance of $\xi_S$ is not at all an issue, reflecting the limited number of "local samples" involved in the specific evaluation. The goal of this exercise is to show that the same $\xi_S$ can be obtained when using the directly measured velocity distribution, or its modelization under the increasing assumptions of the LG and GG models.

### 1.3. *Gaussian (local) Gaussianity (GG)*

We then make the further assumption that $\mathcal{F}(\mu, \sigma)$ is a bivariate Gaussian: $\mathcal{P}(v_\parallel) = \int d\mu d\sigma\ \mathcal{G}(v_\parallel|\mu, \sigma)\ \mathcal{B}(\mu, \sigma)$, where $\mathcal{B}(\mu, \sigma) = \frac{1}{2\pi\sqrt{\det(C)}} \exp\left[-\frac{1}{2}\Delta^T C^{-1} \Delta\right]$, with $\Delta_1 = \mu - \bar{\mu}$, $\Delta_2 = \sigma - \bar{\sigma}$ and $C$ representing the $\mu$-$\sigma$ covariance matrix.

## 2. Validation of the model

To test the goodness of the LG and GG descriptions we directly measure the local distributions of pairwise velocities from the MultiDark Bolshoi simulation (Riebe *et al.* 2013). The main result of our analysis is reported in Fig. 1 and discussed in the corresponding caption. More details can be found in Bianchi *et al.* (2014) togheter with additional results and a discussion on their potential implications in modelling RSD.

## References


Bianchi, D., Chiesa, M., & Guzzo, L. 2014, *arXiv*:1407.4753
Fisher, K., B. 1995, *ApJ*, 448, 494
Riebe, K. *et al.* 2013, *Astronomische Nachrichten*, 334, 691
Scoccimarro, R. 2004, *Phys. Rev D*, 70, 083007