change cofinalities or cardinalities  $\leq \kappa$ . The possible cofinalities for  $\mu$  in the  $\mathbb{P}$ -generic extension depend on the Mitchell order of  $\mu$ . Inner model theory tells us that these assumptions are optimal in terms of consistency strength assuming  $\kappa \geq \aleph_2$ . (Note: this is joint work with Peter Koepke.)

We end with a short discussion on the connection between forcings that change cofinalities of several cardinals simultaneously and mutually stationary sequences of sets.

Abstract prepared by Dominik Thomas Adolf *URL*: http://www.uni-muenster.de/imperia/md/content/logik/dissadolf.pdf

CAROLIN ANTOS, *Foundations of Higher-Order Forcing*, University of Vienna, 2015. Supervised by Sy-David Friedman. MSC: 03Exx, 03E40, 03E70. Keywords: forcing, class forcing, class theory.

## **Abstract**

Forcing notions can be classified via their size in a general way. Until now two different types were developed: set forcing and definable class forcing, where the forcing notion is a set or definable class, respectively. Here, we want to introduce and study the next two steps in this classification by size, namely class forcing and definable hyperclass forcing (where the conditions of the forcing notion are themselves classes) in the context of (an extension of) Morse–Kelley class theory. For class forcing, we adapt the existing account of class forcing over a ZFC model to a model  $\langle M, \mathcal{C} \rangle$  of Morse–Kelley class theory. We give a rigorous definition of class forcing in such a model and show that the Definability Lemma (and the Truth Lemma) can be proven without restricting the notion of forcing. Furthermore we show under which conditions the axioms are preserved. We conclude by proving that Laver's Theorem does not hold for class forcings. For definable hyperclass forcing, we use a symmetry between MK\*\* models and models of ZFC<sup>-</sup> plus there exists a strongly inaccessible cardinal (called SetMK\*\*). This allows us to define hyperclass forcing in MK\*\* by going to the related SetMK\*\* model and use a definable class forcing there. We arrive at a definable class forcing extension from which we can go back to a model of MK\*\*. To use this construction we define a coding between MK\*\* and SetMK\*\* models and show how definable class forcing can be applied in the context of an ZFC- model. We conclude by giving an application of this forcing in showing that every  $\beta$ -model of MK\*\* can be extended to a minimal  $\beta$ -model of MK\*\* with the same ordinals.

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ANUSH TSERUNYAN, *Finite Generators for Countable Group Actions*; *Finite Index Pairs of Equivalence Relations*; *Complexity Measures for Recursive Programs*, University of California at Los Angeles, 2013. Supervised by Alexander S. Kechris and Itay Neeman. MSC: Primary 03E15, 37B10, 03D15, Secondary 37A35, 37A20. Keywords: Borel group actions, generating partitions, entropy, countable Borel equivalence relations, treeable-by-finite, finite index, recursive programs, complexity measures.

## **Abstract**

**Part 1:** For a continuous action  $\Gamma \curvearrowright X$  of a countable group  $\Gamma$  on a Polish space X, a finite Borel partition (coloring)  $\mathcal{P} = \{C_i\}_{i < n}$  of X induces the so-called *coding map* from X to the shift  $n^{\Gamma}$  by sending each  $x \in X$  to the sequence of colors that x encounters when moved by the group elements, i.e.,  $x \mapsto (i_{\gamma})_{\gamma \in \Gamma}$ , where  $\gamma \cdot x \in C_{i_{\gamma}}$ .  $\mathcal{P}$  is called a *generator* if its coding map is injective. A finite generator exists if and only if the action is embeddable into a finite shift action.

For  $\Gamma := \mathbb{Z}$  or any other amenable group, the existence of a finite generator is precluded by the existence of an invariant Borel probability measure of infinite entropy. It was asked by B. Weiss in the late 80s if the actions that do not possess any invariant Borel probability measure must admit a finite generator. For  $\sigma$ -compact (e.g., locally compact) actions, I give

a positive answer to this question for an arbitrary group  $\Gamma$ . I also prove that a 4-set generator always exists off of a (topologically) meager set, thus answering a question of A. Kechris from the mid-90s. Lastly, I show that any aperiodic action of  $\Gamma$  admits a  $\Gamma$ -equivariant Borel map to the aperiodic part of the 2-shift.

**Part 2:** We investigate pairs of countable Borel equivalence relations  $E \subseteq F$ , where E is of finite index in F. Our main focus is the well-known problem of whether the treeability of E implies that of F: we provide various reformulations of it and reduce it to one natural universal example. In the measure-theoretic context, assuming that F is ergodic, we characterize the case when E is normal. Finally, in the ergodic case, we characterize the equivalence relations that arise from almost free actions of virtually free groups.

**Part 3:** We consider natural complexity measures for recursive programs from given primitives and derive inequalities between them, answering a question asked by Yiannis Moschovakis.

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JOSEPH ZIELINSKI, *Compact Structures in Descriptive Classification Theory*, University of Illinois at Chicago, 2016. Supervised by Christian Rosendal. MSC: 03E15. Keywords: Borel reducibility, descriptive classification theory, compact metrizable structures, homeomorphism of compact metric spaces.

## Abstract

An equivalence relation, E, on a Polish topological space, X, is Borel reducible to another, F, on Y, when there is a Borel-measurable function from X to Y assigning F-classes as complete invariants for E. Descriptive classification theory is the programme whose aim is to assess the complexity of naturally arising isomorphism relations by identifying their positions in the Borel reducibility preorder.

This thesis considers the class of compact metrizable structures: compact Polish spaces equipped with closed relations. The natural isomorphism relation in this setting is homeomorphic isomorphism, whereby two such structures are isomorphic when there is a homeomorphism between them that preserves this relational structure. In work carried out jointly with C. Rosendal, we observe that the relation of homeomorphic isomorphism between compact metrizable structures in any countable relational language is Borel reducible to the orbit equivalence relation of an action of a Polish group. We then illustrate how this can be used to bound the complexity of other equivalence relations (e.g., topological group isomorphism between locally compact or Roelcke precompact Polish groups) by encoding the objects into compact structures.

Among the orbit equivalence relations of Polish group actions there are some of greatest complexity, in the sense that every other orbit equivalence relation is Borel reducible to them. Such relations were first exhibited by H. Becker and A. S. Kechris, arising from a universal Polish group acting by shift on the space of its closed subsets. Subsequently, it was shown by S. Gao and A. S. Kechris, and by J. D. Clemens, that the isometry relation between separable complete metric spaces has this same complexity. From this, M. Sabok showed that the same is true of the isomorphism relation between separable C\*-algebras.

Continuing this line of research, we establish the following:

Theorem. The homeomorphism relation between compact metric spaces is Borel bireducible with the complete orbit equivalence relation for Polish group actions.

This is done in two stages: in the first, the objects of Sabok's construction are encoded into compact metrizable structures; in the second, it is shown that the additional relational structure can be eliminated by encoding it into the topology of another compact space. We also present an improvement of this, from joint work with C. Rosendal. Using similar techniques to those described above, we replace the first stage with a short proof that the complete orbit equivalence relation of Becker and Kechris is Borel reducible to the homeomorphic