PHYSICAL CONDITIONS INSIDE WHITE DWARFS AND TYPE I SUPERNOVAE

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1. After the first suggestion by Schatzman (1963) that accreting white dwarfs, close to the limiting mass, might be the progenitors of type I Supernovae, the problem was not studied until Schatzman (1974) described the competition between collapse dominated by beta decay, which would lead to neutron star formation and collapse dominated by nuclear reactions leading to stellar disruption.

Obviously, the chemical composition determines the behaviour of the accreting white dwarf and, for example a pure carbon white dwarf, is likely to explode as an effect of accretion. This has been considered by Wheeler (1974), Taam (1980), Starrfield et al. (1981), Nomoto (1980, 1981), Sugimoto and Nomoto (1980), Chevalier (1981), Woosley et al. (1981). However the consequences of the physical conditions (solid state versus liquid state for example), considered by Canal and Schatzman (1978) have been discussed in a parallel series of papers by Canal and Isern (1980), Canal, Isern and Labay (1980a, b), Mochkovitch (1980), whereas others, like Nomoto and Miyaji (1980) studied the collapse of white dwarfs with a special chemical composition. The purpose of this paper is to discuss the role of the physical conditions inside a collapsing white dwarf, close to the instability limit, and to discuss briefly the present uncertainties concerning the exact physical conditions in a white dwarf.

It should be reminded that, at the beginning of the collapse, a large fraction of the interior of the white dwarf is likely to be in a solid state. The temperature of solidification at $\rho = 2 \cdot 10^{10}$ g cm^{-3} is $6.5 \cdot 10^{7}$ °K for pure Carbon and $1 \cdot 10^{8}$ for pure Oxygen. At these densities, for a random mixture of Carbon and Oxygen, the thermal runaway takes over the β capture, except for very low Carbon concentration. (Canal and Schatzman, 1976). It is quite clear that if we are not dealing with a random mixture the situation can be quite different.

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2. We can consider the following possible situations for a mixture of Carbon and Oxygen.

(A) Liquid mixture. Carbon and Oxygen are fully mixed. The number of pairs of Carbon nuclei is proportional to the square of the concentration. There is no thermonuclear runaway when the carbon concentration is sufficiently low (Canal and Schatzman, 1976).

(B) Solid mixture. The possibility of an eutectic has been considered by Stevenson (1980). Then we have the following possibilities:

(B)(a). No Eutectique.

(B)(a)(i). Oxygen solidifies first and sinks in the center of the star. This bas been considered by Mochkovitch (1980).

(B)(a)(ii). The solidification produces a mixture of randomly distributed Carbon and Oxygen nuclei. We can either have a mixture of small Oxygen and Carbon crystals, as has been considered by Kovetz and Shaviv (1970) as a complete random distribution. In the first case, the energy production goes like the concentration X_c; in the second case, as checked recently by Savedoff (1981), the energy production goes like the square X_c^2 of the concentration.

(B)(a)(iii). There is an ordered mixture of Carbon and Oxygen nuclei in the solid state. This case will be considered in the next section.

(B)(b). There is an Eutectic. This bas been considered by Mochkovitch (1980), and by Canal et al. (1980).

(B)(b)(i). If the concentration in number of Oxygen is larger than 33.2 %, solid oxygen falls to the center and a mantle of eutectic solidifies. This would favour collapse by β-capture.

(B)(b)(ii). If the concentration in number of Oxygen is less than 33.2 %, solid Carbon falls to the center and a mantle of eutectic solidifies. This would favour thermonuclear runaway. This case is not really different from the pure Carbon white dwarf case.

3. THE ORDERED MIXTURE.

Following a suggestion of Jancovici (1982), it can be shown that the free energy of two Carbon nuclei, embedded in solid Oyxgen at T = 0 is lower than the free energy of a chain of two Carbon atoms imbedded in the same solid. Therefore it is quite clear that there will be a tendency for the Carbon nuclei to keep away from each others. This seems to be true, but to a lesser extent, for a mixture of an equal number of Carbon and Oxygen atoms (Dyson, 1971). We are then facing a situation already considered by Kirschnits (1960) where the rate of pycno-nuclear reactions can be strongly decreased by the crystalline structure of the mixture. As a consequence, we can expect that solid white dwarfs with $N_C \lesssim N_O$ could collapse by electron capture on oxygen, whereas for $N_C \gtrsim N_O$ the energy liberation per C - C reaction would favour explosive processes.

Let us now consider more closely the situation. Assume that we have a lattice of solid Oxygen and introduce 2 Carbon atoms. We compare the free energy of the system for the case where the two Carbon atoms are far apart, and for the case where the arc close together and form a chain of two nuclei.

The energy of the space charge background is the same in the two configurations. We then have to consider the difference in the electrostatic energy between the nuclei.

We first assume with Jancovici (1982) that the lattice is not disturbed by the replacement of one or two Oxygen nuclei by one or two Carbon nuclei.

. One Carbon of charge Z_2 is surrounded by 6 Oxygen of charge Z_1. Then

$$F_{CO} - F_{OO} = \frac{6\ Z_1\ Z_2\ e^2}{a} - \frac{6\ Z_1{}^2\ e^2}{a}$$

. Two Carbons are surrounded by 8 Oxygens. Then

$$F_{C^2} - F_{OO} = \frac{Z_2{}^2 + 10\ Z_1 Z_2 - 11\ Z_1{}^2}{a}\ e^2$$

and we have

$$\Delta F = -\ (Z_1 - Z_2)^2\ (e^2/a)$$

$$= -\ \frac{(Z_1 - Z_2)^2}{Z_1{}^2}\ \Gamma_1\ kT = -\ 0.0625\ \Gamma_1\ kT$$

with $\Gamma_1 = (Z_1{}^2\ e^2/a\ kT)$

where a is the edge of the sc lattice.

The result can be slightly improved by moving the Oxygen atoms, and by moving the two carbon atoms in the chain. This gives

$$\Delta F = - 0.0538\ \Gamma_1$$

The case of an equal mixture of two species has been considered by Dyson (1971). It should be noticed that the most bound C - O lattice ((bec lattice with Z_1 and Z_2 sites forming a (sc) sublattice) is more bound that the mixture of two (bc) lattices respectively of Carbon and Oxygen, the difference in the free energies being only, for each pair, only

$$\Delta F \cong - 3.95\ 10^{-4}\ \Gamma_1\ kT$$

We see that for a low concentration, the difference in free energy introduces for the reaction CC + OO → CO + CO a Boltzman factor which is $\exp(- f\Gamma_1)$ where f is of the order of 0.05. With $f = 0.054$ and $\Gamma_1 = 160$ (crystallization of Oxygen), the Boltzman factor is of the order of $2 \cdot 10^{-4}$. We shall conclude that the suggestion of Kirshnits (1960) is verified and that for a Carbon concentration $N_c \lesssim (1/2)(N_c+N_o)$ the rate of nuclear reactions is appreciably reduced by the potential barrier.

4. ASTROPHYSICS.

This result opens the possibility that, according to the chemical composition, and even for fast cooling and solidification of a Carbon-Oxygen mixture, a solid white dwarf could collapse, following electron capture on Oxygen, or experience some sort of explosive process due to the Carbon-Carbon nuclear reaction. The consequence of this sort of bifurcation, as far as the statistics of neutron stars and type I supernovae are concerned has been already discussed by Canal et al. (1980).

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DISCUSSION FOLLOWING E. SCHATZMAN'S TALK

SHAVIV: One has also to consider the possibility that in the liquid phase, before it solidifies it might very well be that for certain ratios of carbon to oxygen the one is not immiscible in the other and then things may start earlier than the phase you are talking about.

SCHATZMAN: This has been considered by J.P. Hansen and co-workers and for that kind of Z ratio which is 8:6 the non immiscibility in the liquid phase takes place at densities which are much lower than those we have to consider here. So it is only in the solid phase that it can possibly take place.