THE FLUCTUATION THEORY OF THE STELLAR MASS LOSS

C.D. Andriesse
Kapteyn Sterrenwacht, University of Groningen
present address:
Department of Reactor Physics KEMA, Arnhem
The Netherlands

That we cannot give a justification - or sufficient reasons - for our guesses does not mean that we may not have guessed the truth.
K.R. Popper[1]

§1

If studied in detail, the mass flows from the stars appear to be variable. "Mere inspection of solar wind data reveals large variations on a time scale of several days" (Hundhausen, 1972). Also, "it is obvious that mass loss from *hot stars* is not a stationary phenomenon but that variations on short timescale occur" (de Jager et al., 1979). And a *cool supergiant* has "a photosphere fluctuating in both brightness and radial velocity, (and) an expanding chromosphere ... uncoupled from motions in the photosphere" (Goldberg, 1979).

The idea that fluctuations in the mass flow are as significant as the very existence of the flow has led to the development of a fluctuation theory of the stellar mass loss. A general theory for fluctuations in non-equilibrium systems - and such are stellar atmospheres (Pecker et al., 1973) - has been developed long ago (cf. Becker, 1961, or Landau and Lifshitz, 1969). In developing the general theory to a specific stellar theory, however, the arguments have not come up in their logical order (Andriesse, 1979, 1980a, 1980b). The present sketch of this theory improves on that order and is offered as a framework for further study.

[1] Objective Knowledge, Clarendon Press (Oxford, 1972) p. 30. Truth is meant to be correspondence with the facts, the only idea of truth which makes rational criticism possible. In the field of science opposite ideas are still alive. For example, professor M. Kuperus, who has criticized some assumptions in the present theory as unfounded, denies that the correct description of the facts, to which they lead, says anything in favour of their correctness.

C. Chiosi and R. Stalio (eds.), Effects of Mass Loss on Stellar Evolution, 213–227.

§2

Our starting point is the concept of *thermodynamic fluctuations*. Landau and Lifshitz (1969) introduce it as follows:

"Let us assume that there is a quantity x such that, if it has a definite value (considerably different from its mean fluctuation), a definite state of partial equilibrium can be described by it. In other words, the relaxation time for the éstablishment of partial equilibrium for a given value of x is assumed to be much less than the relaxation time required to reach the equilibrium value of x itself. This condition is satisfied by a wide class of quantities of physical interest. We shall call the fluctuations of such quantities thermodynamic fluctuations[2]."

In a stellar atmosphere we discern between two relaxation times, which in general are much different. There is a local short timescale τ_d, describing the dynamical processes at the stellar surface, and an overall long timescale τ_t, describing the relaxation of the stellar body to thermal equilibrium. They are obtained from

$$\tau_d = (3\bar{\gamma} - 4)^{-\frac{1}{2}}(R^3/GM)^{\frac{1}{2}}, \tag{1}$$

where γ is the adiabatic exponent (5/3 for an ideal particle gas and 4/3 for a pure photon gas; the bar denotes a special mean, cf. Baker, 1972), G is the gravitational constant and R and M are the radius and mass of the star; and

$$\tau_t = |\Omega|/L, \tag{2}$$

where L is the luminosity of the star and Ω its potential energy given by

$$\Omega = - G \int_0^R dr\ 4\pi r\, \rho(r) \int_0^r dx\ 4\pi x^2\, \rho(x) \tag{3}$$

if $\rho(r)$ is the mass density. It is often possible to put $\bar{\gamma} \simeq 5/3$, so that $\tau_d \simeq (R^3/GM)^{\frac{1}{2}}$, and, for stars where ρ is only mildly peaking towards the centre, to put $\Omega \simeq - GM^2/R$, so that $\tau_t \simeq GM^2/(RL)$.

The fact that $\tau_d << \tau_t$ implies that the stellar atmosphere can reach a partial equilibrium, different from thermodynamic equilibrium. The quantity of interest, describing this state of partial equilibrium, is the mass loss rate $\dot{M}$. We shall discuss it in the dimensionless form (first proposed by Williams, 1967),

$$\lambda = GM\dot{M}/(RL), \tag{4}$$

which is the ratio of the mechanical power in the mass loss and the thermal power of the star.

[2] They should not be confused with the fluctuations about thermodynamic equilibrium, which are termed non-thermodynamic fluctuations, and to which e.g. the fluctuation-dissipation theorem applies.

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§3

Why is λ the relevant parameter and not another quantity describing the dynamical state of a stellar atmosphere, like e.g. the degree of turbulence? That is because the outward mass motion is the cause of most, if not all, nonthermal phenomena observed in stellar atmospheres (Thomas, 1979) and thus is fundamental[3]. It originates from the basic instability to an outward mass flow in the outer mantle of any star.

Consider the force balance. In the presence of some velocity u we must add an inertial force $m\dot{u}$ and a frictional force $m\nu u$ to the well known static force given by gravitation and pressure gradient; in this instance m is the mass-element considered, which we shall take to be $4\pi r^2 \rho\, dr$, and ν is a friction constant, which shall be discussed below. Thus

$$m\dot{u} + m\nu u = 4\pi r^2 dp + \frac{GMm}{r^2}, \tag{5}$$

where dp is the pressure difference over the shell between r and r + dr, G the gravitational constant and M the stellar mass within r. To simplify the problem we may first study the steady state for which $m\dot{u} = 0$. The friction constant ν is, apart from a dimensionless constant ξ, the atomic collision frequency, or

$$\nu = \xi q/\ell = \xi q n \Sigma = \xi q \rho \Sigma/\mu, \tag{6}$$

where q is the atomic velocity $(kT/\mu)^{\frac{1}{2}}$, kT the thermal energy, μ the atomic mass, ℓ the mean free path, n the atomic density and Σ the atomic collision cross-section; Maxwell's relation for ℓ in gases has been adopted. With this relation for ν the steady-state value of ρu is described by

$$\rho u = \frac{\mu}{\xi q \Sigma}\left[\frac{1}{\rho}\frac{dp}{dr} + \frac{GM}{r^2}\right], \tag{7}$$

which, as we see, is not conserved (constant).

Consider next the flow $\dot{m} = 4\pi r^2 \rho u$ - which has no direct relation with the mass loss rate $\dot{M}$ - and determine the sign of $d\dot{m}/dr$. If this sign is negative, the star is unstable to an inward flow, if positive, to an outward flow. In calculating $d\dot{m}/dr$ the following substitutions can be made:

$$\begin{aligned} p &= \rho q^2, \quad \text{or} \quad \frac{dq}{dr} = \frac{1}{2q\rho}\left[\frac{dp}{dr} - q^2\frac{d\rho}{dr}\right], \\ p &\propto \rho^\gamma, \quad \text{or} \quad \frac{dp}{dr} = \gamma\frac{p}{\rho}\frac{d\rho}{dr}. \end{aligned} \tag{8}$$

[3] This point of view is not shared by the authors of other theories of the stellar mass loss, where stellar winds appear as products rather than as producers of those non-thermal phenomena, like acoustic waves (Fusi-Pecci and Renzini, 1975) or coronas (Hearn, 1975, 1979), or as products of the radiation pressure on atmospheric particles, like ions (Castor et al., 1975) or grains (Gilman, 1972). But as the winds have to be fed by material from the stellar body, there has to be a motor to bring this material up to the atmospheric level where the supposed mechanisms start to work. So these theories treat the tail of the problem rather than the head.

It is thus assumed that the radiation pressure can be neglected. It can further be assumed that, to a first approximation, $d^2p/dr^2 = 0$. Simple algebra then leads to

$$\frac{d\dot{m}}{dr} = \frac{2\pi\gamma q\mu}{\xi\Sigma}\frac{d\ln\rho}{d\ln r}\left[4 - \gamma\frac{d\ln\rho}{d\ln r} - \frac{\gamma - 1}{\gamma}\frac{r}{h}\right] \tag{9}$$

with the scale height $h = r^2kT/(GM\mu)$. As $d\ln\rho/d\ln r$ is negative, the sign of $d\dot{m}/dr$ is positive when

$$\frac{\gamma - 1}{\gamma}\frac{r}{h} > 4 - \gamma\frac{d\ln\rho}{d\ln r}, \tag{10}$$

which then is the condition for instability to an outward flow. For stellar mantles we have $d\ln\rho/d\ln r$ in between -1 and -2, say -3/2, whereas $\gamma \simeq 5/3$, so that instability occurs when $h/r < 4/65$. The value of r where this limit is reached, r_c, lies far below the photosphere (in the photosphere itself $h/r \sim 10^{-3}$).

It can be concluded that, whether or not the criterion is satisfied for a convective rather than a radiative transport of energy from the core to the surface, there should be a transport of matter to the surface in the outer mantle of any star.

§4

A complete analysis of the outward flow has to include the inertial force and appropriate timescales, as well as the interaction with the radiation field and, as we have to do with a plasma, with eventual magnetic fields. In view of the tremendous complexity of the problem, the solution may not be expected soon.

In the absence of a complete analysis, observations and their qualitative interpretation may be a guide. Of the various dynamical phenomena in the solar photosphere and chromosphere, the *spicules* could be the most relevant tracers of outflow (Kiepenheuer, 1968). These small surges become visible in Hα when, in the lower chromosphere, they reach the local sound speed q of about 30 km s^{-1}. In some way or other they extract energy from the radiation field, streaming up along the (locally) almost vertical magnetic field lines. Without discarding the possibility that other, larger structures carry more mass away from the (sub)photosphere, we may take them as hints for the development of the above instabilities to an outward mass flow. They come and go, randomly spread (around centres of magnetic activity) over the solar surface, in a dynamical time τ_d of about 1 hour. So the outward mass motions may well be of a *stochastic* nature. This fits in the notion that they originate from an instability to fluctuations (of u above zero) and also with the fact that the mass loss, to which they sometimes lead, is a fluctuating phenomenon. These surges, considered as events, are initially subsonic, but are amplified outward by virtue of some coupling to the radiation field, reach the sound velocity q in the lower chromosphere[4] and are transsonically heated from there on (Cannon and Thomas, 1977), whereas *some of them* are sufficiently amplified further that they surpass the escape velocity v.

§5

Without having a complete analysis of the surges, we may find the average value to which u is amplified from the principle that the free energy of any system tends to a minimum. The free energy contains the potential energy Ω, given by (3); the thermal energy H, given by

$$H = \int_0^R dr\ 4\pi r^2\, \rho(r)\, kT(r)/\mu, \tag{11}$$

k being Boltzmann's constant, T(r) the absolute temperature and μ the mean atomic mass; the kinetic energy K, given by

$$K = \int_0^R dr\ 4\pi r^2\, \rho(r)\, \langle u^2(r)\rangle/2; \tag{12}$$

and the magnetic energy, which we shall neglect here.

Looking for a minimum in the atmosphere at the stellar radius R - this minimum will be reached somewhere in the chromosphere and thus at a distance which is slightly farther out than the photospheric R - we require that

$$\frac{d\Omega}{dR} + \frac{dH}{dR} + \frac{dK}{dR} = 0. \tag{13}$$

This leads to

$$\frac{1}{2}\langle u^2(R)\rangle = \frac{GM(R)}{R} - \frac{kT(R)}{\mu} = \frac{1}{2}v^2 - \frac{kT(R)}{\mu}, \tag{14}$$

so that the average value of u, $\langle u^2(R)\rangle^{\frac{1}{2}}$, lies below the escape velocity v - but only just. We find that the difference v - $\langle u^2(R)\rangle^{\frac{1}{2}} \simeq h(R)/\tau_d$, with h(R) the atmospheric scale height, which only is a modest velocity. Therefore, relatively small fluctuations of u above its average value, pertaining to the state of partial equilibrium of the atmosphere, would be sufficient to lead to mass loss. Because of the closeness of this value to v we can say that u fluctuates *in half of the cases* to values above v.

[4] This means that the relatively small fraction of the atmospheric gas, involved in the surges, will not lead to notable broadening of the Fraunhofer lines in excess of the thermal broadening, as these absorption lines are formed in a (transition) layer below the chromosphere, where u < q.

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§6

We now can find a relation for the loss rate $\dot{M}$ or λ. Equation (5) can be rewritten as

$$\dot{u} = - \nu u + g', \tag{15}$$

where $g' = (1/\rho \; dp/dr + GM/R^2)$ is the net static acceleration, which is close to zero. Since $\dot{M} = 4\pi R^2 \rho(R) u_{>v}$ and $\ddot{M} = 4\pi R^2 \dot{\rho}(R) u_{>v} + 4\pi R^2 \rho(R) \dot{u}$, where $u_{>v}$ denotes the velocities in excess of the escape velocity v, this relation is equivalent with

$$\ddot{M} = - \nu \dot{M} + 4\pi R^2 \left[\dot{\rho}(R) u_{>v} + \rho(R) g' \right]. \tag{16}$$

Multiplying with (GM/RL), assuming $\nu = 1/\tau_t$, and approximating $\dot{\rho}(R)$ by $\rho(R)/\tau_d$, we find that

$$\dot{\lambda} = - \lambda/\tau_t + A(t), \tag{17}$$

$$A(t) = 4\pi R \frac{GM\rho(R)}{L} \left[u_{>v}/\tau_d + g' \right]$$

$$\simeq 4\pi R \frac{GM\rho(R)}{L} u_{>v}/\tau_d . \tag{18}$$

The time-behaviour of A(t) is determined by the fluctuating acceleration $u_{>v}/\tau_d$, which develops on a dynamical timescale. Thus is A(t) a stochastic 'force', with an autocorrelation of the form (Andriesse, 1979)

$$\langle A(t)A(t + y)\rangle \simeq \langle A^2 \rangle e^{-y/\tau_d} . \tag{19}$$

The 'force' is associated with a large number of events, both spread in time and over the surface of the star.

A formal integration of equation (17) leads to

$$\lambda = \lambda_o e^{-t/\tau_t} + \int_0^t dy \; A(t - y) e^{-y/\tau_t} . \tag{20}$$

This shows that in the absence of the stochastic 'force' the mass loss λ will decay from some initial value λ_o to zero in the thermal time τ_t. This is the reason why we have approximated ν by $1/\tau_t$: any large deviation of the outward flow from its average value will disappear only by the dissipation of excess energy in this deviation and this occurs on the thermal time-scale. In the presence of the stochastic 'force' A(t), this decay to zero is stopped at the average value

$$\langle \lambda \rangle = \langle \int_0^t dy \; A(t - y) e^{-y/\tau_t} \rangle, \tag{21}$$

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where the brackets denote an ensemble (surface) average over the events[5]. It is important to realize that the average of the stochastic 'force' is finite and positive.

§7

Before treating relation (17) further, we like to draw the attention to its general nature. It is a Langevin equation, which is characteristic for the description of thermodynamic fluctuations, apparent in the Brownian motion or in the electronic noise, and can be found in any textbook on these subjects (cf. Becker, 1961; Middleton, 1960). Being equivalent to the Fokker-Planck equation, it is rooted in the statistical theory for the approach to equilibrium of systems with many degrees of freedom. This led the author to simply take it as an *Ansatz* in his first paper on the fluctuation theory, without any other justification than that it is generally valid for thermodynamic fluctuations[6].

There is, however, an important difference in the application of the Langevin equation to the problem of the stellar mass loss on the one hand, and to the problem of the Brownian motion or the electronic noise on the other hand. In the first case the stochastic 'force' works in one direction only, so that its average is finite and positive, whereas in the second case this 'force' works in all directions, so that its average is zero. By a trick we can translate the first problem in the second though.

§8

Assume a normal distribution of the velocities u around their average value, which in practice is equal to the escape velocity v. This means that the probability for a certain value of the stochastic 'force' p(A) is given by

$$p(A) = \frac{1}{\sigma\sqrt{2}}\, e^{-A^2/\sigma^2}, \tag{22}$$

σ being the standard deviation. However, equation (18) stipulates that only those values of u, which are in excess of v, lead to a (positive) 'force' A and, consequently, that negative values of A should be discarded. With the notation A_+ for positive, A_- for negative and $A_\pm$ for both positive and negative values of A we have

[5] This means that $\langle\lambda\rangle = \tau_t\langle A(t)\rangle$ would be wrong, as it implies that A(t) has instantaneously the same value over the whole surface of the star.

[6] It led him in a few days to the main result of the theory, found in November 1977. However, it also led to a long delay in its publication, only in March 1979, as the referees were sceptical to the point of denying that it has a physical basis.

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$$\langle A_\pm \rangle = \int_{-\infty}^{\infty} dA\ Ap(A) = 0, \tag{23}$$

$$\langle A_+ \rangle = \int_{0}^{\infty} dA\ Ap(A) = \sigma/\sqrt{2\pi}. \tag{24}$$

But

$$\langle A_\pm{}^2 \rangle = \int_{-\infty}^{\infty} dA\ A^2 p(A) = \sigma^2/2, \tag{25}$$

so that

$$\langle A_+ \rangle = \langle A_\pm{}^2 \rangle^{\frac{1}{2}}/\sqrt{\pi}. \tag{26}$$

This suggests that the average of the stochastic 'force' A_+, relevant for the stellar mass loss, can be found from the root-mean-square average of the stochastic 'force' $A_\pm$, relevant for the Brownian motion or the electronic noise (except for the factor $\sqrt{\pi}$).

Similarly, assume a normal distribution for $\lambda_\pm$. This implies that we consider for a moment negative mass losses also, like negative displacements considered in the theory for the Brownian motion or the electronic noise. Then,

$$\langle \lambda_+ \rangle = \langle \lambda_\pm{}^2 \rangle^{\frac{1}{2}}/\sqrt{\pi}. \tag{27}$$

The latter assumption of a normal distribution may be less justified than the former, as λ cannot be completely stochastic. It results from the stochastic 'force' plus an independent systematic 'force' (equation 17). This means that (27) is only true by order of magnitude.

We can now treat A as a symmetric 'force' with an average of zero and apply the approximations (26) and (27) to find the physically relevant average.

§9

The integration of (17) can be performed with the physically illuminating time-step method of Einstein and Hopf (cf. Becker, 1961). Take a time τ such that

$$\tau_d \ll \tau \ll \tau_t, \tag{28}$$

within which λ has changed its value many times but $\langle\lambda\rangle$ has hardly changed. With the abbreviation

$$\int_0^\tau dt\ A(t) = B_o \tag{29}$$

we have for the change of λ after such a time τ

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$$\lambda_1 - \lambda_o = - \tau/\tau_t \lambda_o + B_o. \tag{30}$$

More in general, with

$$\int_{j\tau}^{(j+1)\tau} dt\ A(t) = B_j, \tag{31}$$

where j is an integer number, we can write

$$\lambda_{j+1} - \lambda_j = - \tau/\tau_t \lambda_j + B_j; \tag{32}$$

or, using

$$\alpha = 1 - \tau/\tau_t, \tag{33}$$

$$\lambda_{j+1} = \alpha\,\lambda_j + B_j. \tag{34}$$

The last equation is a recursion formula relating the final value λ_n to the initial value λ_o through the intermediate values λ_1, $\lambda_2 \ldots \lambda_{n-1}$. We can eliminate these intermediate values by multiplying the equation for λ_j with α^{n-j} and then adding the newly found equations as

$$\lambda_n = \alpha^n \lambda_o + (\alpha^{n-1} B_o + \alpha^{n-2} B_1 \ldots + B_{n-1}). \tag{35}$$

If we square λ_n and consider the average, we have $<\lambda_n^2> = <\lambda_o^2> \equiv <\lambda^2>$, whereas $<B_i B_j> \equiv <B^2>$ if $i = j$ and 0 if $i \neq j$, since the B_j are assumed to be statistically independent. This results in

$$<\lambda^2>(1 - \alpha^{2n}) = <B^2> \frac{1 - \alpha^{2n}}{1 - \alpha^2}, \tag{36}$$

which after substitution of $\alpha^2 = 1 - 2\tau/\tau_t$ leads to

$$<\lambda^2> = \tau_t <B^2>/2\tau.$$

It should be noted, that B as given by equation (31) depends on τ. We can express $<B^2>/2\tau$ in the autocorrelation of A.

§10

By definition,

$$<B^2> = \int_0^\tau dt \int_0^\tau dt' <A(t)A(t')>, \tag{38}$$

whereas $<A(t)A(t')>$ will only depend on the time difference $y = t' - t$,

$$<A(t)A(t')> = <A(t)A(t + y)>, \tag{39}$$

given in equation (19). In evaluating the double integral (38) we introduce, besides the time difference y, the time sum $z = t' + t$, with which

$$<B^2> = \frac{1}{2}\int_0^{2\tau} dz \int_{-\tau+|\tau-z|}^{\tau-|\tau-z|} dy\ <A(t)A(t + y)>$$

$$\simeq \tau \int_{-\infty}^{\infty} dy\ <A^2>\ e^{-|y|/\tau_d}$$

$$= 2\tau\tau_d<A^2>. \tag{40}$$

The replacement of the integration limits by $\pm\ \infty$ is justified by the fact that $<A(t)A(t + y)>$ is virtually zero at $y = \tau >> \tau_d$. Combining the results (37) and (40) and taking notice of the fact that we have considered symmetrical functions λ and A, we find

$$<\lambda_{\pm}{}^2> = \tau_d\tau_t<A_{\pm}{}^2>. \tag{41}$$

Using the approximations (26) and (27) this results in the averages, relevant to the stellar mass loss

$$<\lambda_+> \simeq \sqrt{\tau_d\tau_t}\ <A_+>. \tag{42}$$

The problem is reduced now to that of the average of the stochastic 'force'.

§11

The energy, associated with the stochastic 'force' acting on the atmosphere with mass m_a, is

$$K(R) = m_a<u^2(R)>/2 \simeq m_a v^2/2 = m_a/M(R)\cdot|\Omega(R)|. \tag{43}$$

By a *thought experiment* we now prove that

$$m_a/M(R) = \tau_d/\tau_t, \tag{44}$$

with which the above relation reduces to

$$K(R) \simeq \tau_d/\tau_t\cdot|\Omega(R)|^7. \tag{45}$$

To this end we need to realize that m_a is that minute fraction of the

[7] This implies an equipartition of kinetic (mechanical) power $K(R)/\tau_d$ and thermal power $|\Omega(R)|/\tau_t = L$ in the atmosphere.

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stellar mass for distances in excess of R that does not strongly interact with the radiation field (optical depths below unity). Give a small displacement δ to the stellar atmosphere, which due to gravitational interaction is communicated to the stellar body. The reaction to δ of the atmosphere on the one hand, and of the stellar body on the other, must be governed by the conservation of momentum. As the appropriate relaxation times are τ_d[8] and τ_t, respectively, we have

$$m_a\delta/\tau_d = M(R)\delta/\tau_t \tag{46}$$

which proves relation (44).

We cannot simply return to equation (18) to relate K(R) with $<A_+>$, as it contains the atmospheric density. The following intuitive argument may be used. As the stochastic 'force' perturbs the gravitational energy of the star, one expects that the power $K(R)/\tau_d$ in these perturbations is proportional to both the 'force' $<A_+>$ and the gravitational energy $|\Omega(R)|$. With a (dimensionless) proportionality constant χ one has

$$K(R)/\tau_d = \chi <A_+>|\Omega(R)|. \tag{47}$$

Together with equation (45) this leads to

$$<A_+> \simeq \chi/\tau_t. \tag{48}$$

§12

Except for the uncertainty about the value of χ, we have obtained now a result for the average stellar mass loss. Combining relations (42) and (48) and omitting the subscript +, as it is clear that there are only positive values, we find the general relation

$$<\lambda> \simeq \chi \sqrt{\tau_d/\tau_t}. \tag{49}$$

The value of χ may be found by a comparison with measured data of mass loss rates. A more satisfactory procedure for finding χ is to consider a limiting case.

At the Eddington limit the outward radiative force approaches the inward gravitational force binding the star, so that at this limit the maximum loss of mass should be reached. By equating these forces we have

$$M = \frac{\kappa}{4\pi cG} L, \tag{50}$$

where c is the velocity of light and κ the mass absorption coefficient of the gas. The maximum of the mass loss rate is therefore given by

[8] If m_a would include an arbitrarily larger mass than just that of the atmosphere, part of it would strongly interact with the radiation field, and thus would the relaxation time be increased above τ_d.

$$\dot{M}_m = \frac{\kappa}{4\pi cG} L/\tau_t, \tag{51}$$

L/τ_t being the maximum rate with which L can change. As $1/\tau_t = L/|\Omega| \simeq RL/(GM^2)$ and as $\dot{M} = \lambda \cdot RL/(GM)$, we have for this maximum loss rate

$$\lambda_m = \frac{\kappa}{4\pi cG} \frac{L}{M} = 1, \tag{52}$$

by virtue of relation (50). This maximum value has been discussed before by Williams (1967) and Thomas (1973), but their argument was that, if a star does not at all thermalize the centrally produced power, it must use all of it in expelling mass - which is incorrect.

A star at the Eddington limit is virtually unbound, so that τ_d and τ_t must take on the same value: any perturbation of the stellar surface becomes resonant with the response of the stellar body. Thus,

$$(\sqrt{\tau_d/\tau_t})_m = 1, \tag{53}$$

which together with the above equations (49) and (52) suggests that

$$\chi \simeq 1. \tag{54}$$

But χ is not exactly equal to unity[9]. Andriesse (1980a) has discussed reasons for a somewhat smaller value, namely $(2\pi)^{-\frac{1}{2}} = 0.40$.

§13

The result of the fluctuation theory of the stellar mass loss rate is thus

$$\langle\lambda\rangle \simeq \sqrt{\tau_d/\tau_t}. \tag{55}$$

By inserting the definitions (1), (2) and (4), and using the approximations discussed in §2, this leads to

$$\langle\dot{M}\rangle \simeq L^{3/2}(R/M)^{9/4}/G^{7/4}. \tag{56}$$

Of course, the last equation should not be used in the cases where the mentioned approximations are incorrect. It is useful for main-sequence stars, but it becomes worse the further the stars have evolved from this sequence to the red. It gives about two orders of magnitude too much mass loss for evolved Hayashi-line stars, which have a strong concentration of matter at the centre. In these cases one should use the actual result (55) in conjunction with the definition equations of λ, τ_d and τ_t.

The only star, for which at this moment the physical parameters

[9] In taking the limit $\tau_d \uparrow \tau_t$ one violates the condition (28) used in deriving equation (49), which thus may not be valid close to that limit.

are known precisely enough to allow for a convincing refutation of the fluctuation theory, is the Sun. With its value for L, R and M one obtains $L^{3/2}(R/M)^{9/4}/G^{7/4} = 2.547 \times 10^9$ kg s^{-1} or 4.06×10^{-14} $M_\odot$ y^{-1}, whereas 40% of this value (see at the end of §12) would be a somewhat more realistic estimate of the mass loss rate. The value, derived from measurements of the solar wind (Hundhausen, 1972), is about 1.4×10^9 kg s^{-1}. Given the uncertainty in the determination of χ and the rough approximation of τ_t, one can say that the Sun does not refute the theory.

Less convincing cases are provided by τ Sco, ζ Pup, AX Sgr, η Car, α^1 Sco, which roughly form a series of further evolved stars, whereas also cases exist of Wolf-Rayet stars and central stars of planetary nebulae. We need not discuss these cases here but note that at this moment no clear refutation has been found[10].

§14

As the theory is general, it should be useful in studying the stellar evolution, where so far mass loss has been ignored or taken into account in a rather crude way. Andriesse (1980c) has already shown that it sheds light on the relation between O and Of stars, whereas it contains the promise to understand enigmatic phenomena like η Carinae's 'outburst' and the formation of Wolf-Rayet stars. For this aspect the reader is further referred to a Note on evolution computations using the fluctuation theory, which recently has been submitted to Astronomy & Astrophysics.

Another aspect of the theory is, that it predicts an intrinsic scintillation of the stars, for the simple reason that the surges extract energy from the radiation field of the photosphere (§4). It is possible to explain the observed scintillation of early-type supergiants in terms of λ. Here the reader is referred to the qualitative discussion of the fluctuation theory (which may be read as a comment on the present paper) in the proceedings of the Erice-workshop of 1980 on red giants.

§15

As it stands, the above discussion is incomplete (§4), hazardous (§5) and heuristic (§12). It addresses phenomena of an enormous complexity, of which even the principles are poorly understood. Yet it arrives at a result of an extreme simplicity, i.e. with a high degree of falsifiability.

In the years to come it will be confronted with more facts than are available now. It cannot be excluded that some of these facts will refute its predictions, but until then one should take the simplicity,

[10] Recently, professor C. Chiosi, who claims to have been unaware of the present theory, seems to have discovered relation (55) by looking closely at the data on the loss rates of 27 O-stars.

to which the facts seem to obey, as a challenge.

Could it be that, according to the old rule *simplicitas veri sigillum*, the simplicity refers to a true law of nature? And that, according to Ockam's razor *entia non multiplicanda praeter necessitatem*, the complexity of phenomena in a stellar atmosphere is irrelevant for the establishment of this law?

Popper's view is that theories with a high degree of falsifiability, like the present one, come close to the truth, if many attempts to refute them have failed. Correct as this may be from a methodological point of view, the author nevertheless feels that the acceptance of this theory (as being true) does not depend on the attempted refutations but on the further clarification of its derivation.

The challenge of this theory lies in understanding the correctness of weakly founded steps in its argument.

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DISCUSSION

THOMAS: I regret that you will not continue work along these lines, which appears promising. What you did was a linear approximation of non-equilibrium. What about higher order approximations?

ANDRIESSE: They are certainly much more difficult and you also have to know more about the system (the photosphere) than just relaxation times. To give an example: the spread of the mass flow around the average value, hich is given by the linear approximation, can only be obtained by knowing the number of (mass-loss) events N. I believe that the spread will turn out to be about $1/\sqrt{N}$. But how can we predict N from elementary considerations?

CONTI: In the sun we do observe spicules, which are analogous to those you propose to be necessary for hot star winds. Do the solar spicules provide enough energy for the solar wind?

ANDRIESSE: If this could be proved, it would strengthen the ideas in the fluctuation theory. I believe that surges, like the spicules on the Sun, both provide the heat of the chromospheres-coronae and the kinetic energy of the winds. However, I cannot give any observational justification for this belief.

DUPREE: G. Brueckner of the U.S. Naval Research Laboratory has observed a few massive high velocity spicules at the solar limb by using a high resolution rocket-borne ultraviolet spectrograph. Assuming that they are distributed over the solar surface, he estimates that these spicules could provide enough energy to heat the corona.

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