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Semi-Analytic Formulas of Velocity Stirring Rates in Particle Disks

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Abstract. We have obtained viscous stirring and dynamical friction rates of planetesimals based on three-body orbital integrations, taking into account the Rayleigh distribution of eccentricities and inclinations. We have found that the evolution of root mean square eccentricities and inclinations calculated by using these rates agrees with N-body simulations quite well.

1. Introduction

In order to study the early stages of planetary accretion from a very large number of planetesimals, numerical simulations based on the coagulation equation are generally used (e.g., Wetherill & Stewart 1993). The coagulation equation is written as

$$\frac{dn_m(t)}{dt} = \frac{1}{2}\int_0^m A_{m',m-m'} n_{m'}(t) n_{m-m'}(t) dm' - n_m(t)\int_0^\infty A_{mm'} n_{m'}(t) dm'. \tag{1}$$

In the above, $n_m(t)dm$ is the mass distribution function of planetesimals, and $A_{mm'}$ is the accretion rate. In those simulations, planetesimals with broad size distributions are divided into a number of discrete size bins, and the evolution of size distributions is calculated according to the collision rate between each pair of these size bins. These techniques are also used to the studies of the evolution of particle disks around planets. Since the accretion rate $A_{mm'}$ depends on the mean eccentricity and inclination of particles, it is essential to derive accurate velocity evolution equations which can be used in these simulations.

2. Evolution Equation and the Stirring Rates of Planetesimals

Recently, evolution equations for mean square eccentricity and inclination of particles with the Rayleigh distribution of eccentricities and inclinations have been derived (Ohtsuki 1999, Stewart & Ida 2000), by improving earlier works (e.g., Stewart & Wetherill 1988, Ida 1990). For example, in a system of bimodal population of particles with masses m_1 and m_2, the evolution equation for the mean square eccentricities of component 1 particles is written as

$$\begin{aligned}\frac{d\langle e_{m_1}^2\rangle}{dt} &= a^2\Omega \int n_s(m_2) m_2' h_{m_1,m_2}^4 \Big\{ m_2' \langle P_{VS}\rangle \\ &+ \frac{m_2'\langle e_{m_2}^2\rangle - m_1'\langle e_{m_1}^2\rangle}{\langle e_{m_1}^2\rangle + \langle e_{m_2}^2\rangle} \langle P_{DF}\rangle \Big\} dm_2\end{aligned} \tag{2}$$

In the above, $m_2' = m_2/(m_1+m_2)$, and $h_{m_1,m_2} = [(m_1+m_2)/3M_\odot]^{1/3}$. $\langle P_{VS}\rangle$ and $\langle P_{DF}\rangle$ are the viscous stirring and the dynamical friction rates of eccentricities, respectively. We obtain a similar equation for inclinations.

Ohtsuki (1999) obtained $\langle P_{VS}\rangle$ etc. for particles in planetary rings with low optical depth by 3-body orbital integrations, and excellent agreement was found between the velocity evolution calculated using these stirring rates and N-body simulations (Ohtsuki & Emori 2000). Semi-analytic formulas of these rates for ring particles have been also derived (Ohtsuki 2000).

On the other hand, Stewart & Ida (2000, hereafter SI00) derived analytic formulas of $\langle P_{VS}\rangle$ etc. for planetesimals. SI00 compared the velocity evolution calculated by their formulas with N-body simulations, and found fairly good agreement. However, the following discrepancies remained unexplained: (i) Evolution in the low velocity cases disagrees with N-body simulations. (ii) The dynamical friction terms in the analytic formulas have to be reduced by 30% to obtain better agreement with N-body simulations. The formulas derived by SI00 are fairly good approximation in relatively high velocity cases; in fact, numerical simulations of planetary accretion using the coagulation equation and SI00's formulas show good agreement with N-body simulations (Inaba et al. 2000). However, more accurate formulas for low velocity cases are also needed to simulate planetary accretion with the effect of collisional fragmentation (Wetherill & Stewart 1993). Ida (1990) obtained stirring rates in low velocity cases, but distribution of e and i was not taken into account.

In the present work, we have obtained stirring and dynamical friction rates of planetesimals with a Rayleigh distribution of eccentricities and inclinations by 3-body orbital integrations, using the method described in Ohtsuki (1999, 2000).

3. Comparison with N-body Simulations

We have compared the velocity evolution calculated by the evolution equation (2) together with the three-body stirring rates $\langle P_{VS}\rangle$ etc. with N-body simulations for one- and two-size component systems, and found excellent agreement in both cases (Ohtsuki, Stewart, & Ida 2000).

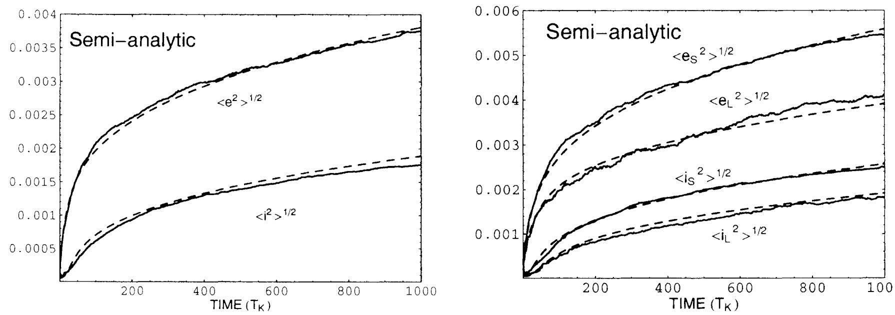

Figure 1. Comparison of the evolution of root mean square eccentricities and inclinations calculated by the semi-analytic formulas (dashed lines) with N-body simulations (solid lines). Left: Single component case with $N = 1000$ and $m = 10^{24}$g. Right: Two-component case with $N_S = 800$, $N_L = 200$, $m_S = 10^{24}$g, and $m_L = 4 \times 10^{24}$g.

We also tried to derive semi-analytic formulas for $\langle P_{VS} \rangle$ etc. which reproduce our 3-body results. Preliminary results show that these semi-analytic formulas can reproduce the velocity evolution in one- and two-size component systems quite well (Fig.1).

4. Summary

In the present work, we have obtained the viscous stirring and dynamical friction rates of planetesimals due to gravitational encounters using three-body orbital integrations (Ohtsuki et al. 2000). We confirmed that the evolution of root mean square eccentricities and inclinations calculated by using these stirring rates agrees with the result of N-body simulations quite well, even in the low velocity cases. These stirring rates based on three-body orbital integrations can be used to produce more accurate numerical simulations of planetesimal accumulation.

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