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ARTICLE

Indexical utility: another rationalization of exponential discounting

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Abstract
This paper is about time preferences, the phenomenon that the very same things are usually considered the less valuable the farther in the future they are obtained. The utilities of those things are discounted at a certain rate. The paper presents a novel normative argument for exponential discount rates, whatever their empirical adequacy. It proposes to take indexical utility seriously, i.e. utilities referring to indexical propositions (that speak of 'I', 'now', etc.) as opposed to non-indexical propositions. Economic focus is only on the latter, while the former are neglected. The potential ignorance of when is now generates a difference between indexical and non-indexical utility that can be exploited for a novel argument in favour of exponential discount rates.

Keywords: Time preferences; indexical utility; exponential discounting; uncertainty about time

1. Introduction

This short note is about time preferences, the well-known phenomenon that the very same things are usually considered the less valuable the farther in the future they are obtained. The utilities of those things are discounted at a certain rate. There are normative arguments why the discount rate should be exponentially decreasing. Some kinds of hyperbolic discounting seem to better agree with the empirical data. This is not my issue. Here, I want to present a novel normative argument for exponential discount rates, whatever their empirical adequacy. My point will simply be to take indexical utilities seriously, i.e. utilities referring to indexical propositions as opposed to non-indexical, 'eternal' propositions. My utility of getting \$100 today is the same as my utility of getting \$100 at 1 November 2022, only if I am sure that today is 1 November 2022. Otherwise, the two utilities may fall apart. To my knowledge, the point is much discussed in philosophy regarding indexical belief, but not regarding preferences or utilities, while it is not discussed at all in economics. There, indexical references to time were just taken as a convenient short hand for references to absolute time. Taking indexical utilities seriously means distinguishing them from non-indexical utilities and describing their relation. We will see that a simple argument for exponential discount rates falls out from this.

Section 2 will provide a brief contextualization within the discussion on time preferences. Section 3 will semi-formally introduce and explain indexical utilities and their relation to non-indexical utilities. Section 4 will argue for an additional assumption and show how this assumption and the relation between indexical and non-indexical utilities entails exponential discount rates.

What is the point of this? Well, the normative justification of exponential discount rates is perhaps not in need of further strengthening, though it is interesting to see a novel approach to them. However, indexical utilities are definitely worth being seriously considered. My point then is that they are not gratuitous, but do have substantial theoretical consequences.

2. Time Preferences

The phenomenon of time preferences[1] is widespread, if not universal. It is well-known to have important economic consequences, in particular in the theory of interest, which occupied already medieval scholastics. Various explanations of that phenomenon have been offered. The most obvious one, perhaps, is the uncertainty of human life. I can make immediate use of \$100 now. But who knows whether I am still alive in a year? If not, \$100 in a year are useless.

The topic has been essentially advanced by the discounted utility (DU) model of Samuelson (1937). Frederick *et al.* (2002: 355f.) write: ‘Despite Samuelson’s manifest reservations, the simplicity and elegance of this formulation was irresistible, and the DU model was rapidly adopted as the framework of choice for analyzing intertemporal decisions.’ What is the formulation? It is about a consumption profile for the time periods 0, …, T (where T may also be infinity ∞). At any time t, a consumption bundle c_t is considered (‘c’ may also stand for ‘consequence’ or ‘choice’). And it is assumed that the agent has one and the same instantaneous cardinal utility function u for these bundles; that is, if u_t is the utility function at time t for possible bundles c_t, then $u_t = u$ for all t. The issue now is how the agent initially, at time 0, evaluates an entire consumption profile $(c_0, \ldots, c_T)$ by her overall utility function U_0. Samuelson proposed:

(1) $U_0(c_0, \ldots, c_T) = \sum_{t=0}^{T} d(t)u(c_t)$, where the discount function d is given by $d(t) = (1 - \delta)^t$ for some δ with $0 < \delta < 1$.

All kinds of idealizing assumptions go along with this representation (see Frederick *et al.* 2002: sect. 3.2–6): (i) that the instantaneous utilities $u(c_t)$ are independent from the consumption at other or earlier times; (ii) that they are independent from

[1]Frederick *et al.* (2002: 352) are terminologically more careful. They ‘use the term *time discounting* broadly to encompass *any* reason for caring less about a future consequence, including factors that diminish the expected utility generated by a future consequence, such as uncertainty or changing tastes’. And they ‘use the term *time preference* to refer, more specifically, to the preference for immediate utility over delayed utility’ (italics are theirs). The literature is usually sloppier and speaks just of time preferences, and I will do so here as well.

each other also in the sense of combining additively in the overall utility U_0; (iii) that the instantaneous utilities u_t are stationary, i.e. constant ($= u$) across time; and (iv) that the discount function, too, is invariant across the consumption profiles. Obviously, all assumptions may fail. We need not discuss this in detail.

There is a peculiar ambiguity in economic discussions of time preferences. On the one hand, the DU model presupposes a certain *stability* of the agent's preferences. Samuelson (1937: 155) states clearly that under the assumptions he goes on to specify it should be 'possible to arrive theoretically at a precise measure of marginal utility of money income to an individual *whose tastes maintain a certain invariance throughout the time under consideration* and during which time the prices of all goods remain constant' (my italics). The above assumption that the instantaneous utilities are constant over time may also be seen as reflecting the invariance of tastes.

The point is emphasized by Koopman's (1960) justification of (1) for the case of $T = \infty$. He assumes *stationarity* in his sense (his Postulate 4), i.e. that

(2) $$U_0(c_0, c_1, c_2, \ldots) \geq U_0(c_0, c'_1, c'_2, \ldots) \text{ iff } U_0(c_1, c_2, \ldots) \geq U_0(c'_1, c'_2, \ldots)$$

for all c_0, $(c_1, c_2, \ldots)$, and $(c'_1, c'_2, \ldots)$.[2] 'This expresses the idea that the passage of time does not have an effect on preferences' (1960: 294).[3] And he shows that stationarity in this sense together with some other postulates[4] implies (immediate) impatience for large classes of utility functions U_0. (His argument does not work for all utility functions.) Here, *impatience* means that

(3) $$U_0(c_0, c_1, c_2, \ldots) \geq U_0(c_1, c_0, c_2, \ldots) \text{ iff } u(c_0) \geq u(c_1),$$

i.e. that having the better bundle at time 0 and the worse at time 1 is preferred to the reverse time order.[5] This entails that if U_0 can at all be represented in the additive form (1), then the discount function d must be strictly decreasing. He finally shows that that a strengthening of his Postulate 3, which he calls *period independence*, indeed entails the additive form (1). And then the stationarity

[2] Be careful in interpreting $U_0(c_1, c_2, \ldots)$. This is the utility at time 0 of evaluation of getting c_1 at time 0, c_2 at time 1, etc. That is, the distance of the time of consuming c_t from the evaluation time is not given by the index t, but by the position of c_t in the sequence $c_1, c_2, \ldots$

[3] Koopman's stationarity differs from the constancy of instantaneous utilities assumed above, which has also been called a form of stationarity.

[4] These are: that there at all exists a continuous utility function U_0 for these infinite consumption profiles (Postulate 1), that U_0 is not trivial in not equally valuing all consumption profiles (Postulate 2), that U_0 can be represented as some function V of $u(c_0), u(c_1), \ldots$ (Postulate 3), and that U_0 is bounded from above and from below (Postulate 5).

[5] Diamond (1965) improves upon Koopman's results by showing that the existence of a continuous utility function U_0 for infinite consumption profiles can be derived from a preference relation $\succcurlyeq$ over these profiles if $u(c_t) \geq$ or $> u(c'_t)$ for all $t \geq 0$, respectively, entails $(c_0, c_1, \ldots) \succcurlyeq$ or $\succ (c'_0, c'_1, \ldots)$ (1965: 173). Moreover, if $\succcurlyeq$ is such that $u(c_t) \geq u(c'_t)$ for all $t \geq 0$ and $u(c_t) > u(c'_t)$ for some $t \geq 0$ already entails $(c_0, c_1, \ldots) \succ (c'_0, c'_1, \ldots)$, then U_0 must be *eventually impatient* in the sense that there is a time t^* such that $U_0(c_0, \ldots, c_t, \ldots) \geq U_0(c_t, \ldots, c_0, \ldots)$ iff $u(c_0) \geq u(c_t)$ for all $t \geq t^*$ (1965: 174). Surprisingly, Koopman's stationarity is not needed for this result.

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postulate forces the discount function d to be of the exponential form $(1 - \delta)^t$. Hence, (1) is rationally compelling insofar these postulates are.

On the other hand, time preferences are treated as a prime example for *preference change*, indeed the one that seems to receive the largest attention in the economic literature. How? Take any commodity. Or perhaps just \$100, because money is useful at any time (while a specific good, e.g. warm clothing, might be useful only at specific times; it would violate Samuelson's invariance). The commodity, i.e. \$100 later are less valued than \$100 now, and the lesser, the later. This sounds like involving utility change; the same thing is valued differently at different times. This is ambiguous, though. What is meant is that the thing valued, the \$100, is placed at different times; the evaluation itself takes place at one and the same time. This is how we have described the matter above. Thus put, this is not yet a case of utility change; \$100 now and \$100 in a year are not the same thing. However, the matter may be expressed more clearly: Let the utility at 1 January 2023, of getting \$100 immediately be 10 and the utility of getting them at 1 January 2024, be 9. Then the utility of the latter raises as the year passes and is 10 at 1 January 2024. The utility of getting this amount at this date has changed during the year. This is genuine preference change, and it is endogenous, since it is not caused by information or other forms of epistemic change.

Indeed, another rationalization of the exponential discount rates in (1), and perhaps the more important one, operates within this picture of preference change. It is that (1) is dictated by time consistency. More precisely: Let U_0 be a utility function at time 0 for consumption profiles for the times 0, 1, …, and U_1 be a utility function at time 1 for such profiles for the times 1, 2, …, and let both be of the form (1) with the same discount function d. Hence, the additive form of (1) is already presupposed. Then d is exponential if and only if U_0 and U_1 are *time-consistent*, i.e.

$$(4)\quad U_0(c_0, c_1, c_2, \ldots) \geq U_0(c_0, c'_1, c'_2, \ldots) \text{ iff } U_1(c_1, c_2, \ldots) \geq U_1(c'_1, c'_2, \ldots$$

for all c_0, $(c_1, c_2, \ldots)$, and $(c'_1, c'_2, \ldots)$.[6] In short, only with an exponentially decreasing discount function there will never be an exploitable preference reversal simply through the passage of time.[7] In this perspective, time preferences involve utility change from U_0 to U_1.

The connection between the two considerations is given by the notion of *time invariance*, defined by:

$$(5)\quad U_0(c_1, c_2, \ldots) \geq U_0(c'_1, c'_2, \ldots) \text{ iff } U_1(c_1, c_2, \ldots) \geq U_1(c'_1, c'_2, \ldots)$$

for all $(c_1, c_2, \ldots)$ and $(c'_1, c'_2, \ldots)$.[8] It is obvious from (2), (4) and (5) that stationarity, time consistency and time invariance are independent properties, and,

[6]Note that time consistency differs from Koopman's stationarity above. The mathematical point of the entailment of exponential discounting is, however, the same.

[7]This observation goes back to Strotz (1955/56).

[8]Notice again: $U_0(c_1, c_2, \ldots)$ is the utility at time 0 of getting c_1 at time 0, c_2 at time 1, etc., and $U_1(c_1, c_2, \ldots)$ is the utility at time 1 of getting c_1 at time 1, c_2 at time 2, etc.

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more importantly, that any two of them imply the third one.[9] Halevy (2015: 342) notes that 'time invariance has been an implicit assumption in most of the literature (especially recent behavioral studies)'. That is, given time invariance, time consistency and stationarity are equivalent. This is, roughly, how the two arguments for an exponentially decreasing discount function converge.[10]

So, do time preferences presuppose time invariance or do they involve preference change? The puzzle is not as big as I have made it appear. However, it receives its proper resolution, I think, only when we attend to the phenomenon of the indexicality of utilities and propositional attitudes in general.

3. Indexical Utility

The general point is that we must distinguish two kinds of propositions, indexical and non-indexical or eternal ones. Schematically, indexical propositions are represented by sentences containing indexical or context-dependent expressions such as 'I', 'you', 'now', 'tomorrow', 'here', 'half a mile to the left', etc., while sentences not containing such expressions represent eternal propositions.[11] However, indexical expressions are not a foolproof syntactic criterion for this distinction. 'It's raining' does not contain an indexical expression, but it is usually taken as an indexical sentence meaning that it is raining *now* (and *here*).

Indeed, the relation between sentences and propositions (= sentence meanings) is complicated.[12] One might say that (non-)indexical sentences express (non-) indexical propositions. Utterances of non-indexical sentences express the same eternal proposition in each context. However, an utterance of an indexical sentence, e.g. of 'it's raining today' at 1 November 2022, is ambiguous in this respect. One may say that it expresses the same indexical proposition as the sentence, e.g. *it's raining today*.[13] But one may also say that it expresses the eternal proposition generated by substituting all indexical expressions by the items they refer to, e.g. *it's raining at 1 November 2022*.

This was not always clear. Despite the obvious pragmatic all-importance of indexicality, philosophy tended to neglect the topic. In particular, the existence of indexical propositions was denied for long. Utterances of indexical sentences were taken to only express eternal propositions in the way just indicated. However, if propositions are also to provide mental contents, i.e. the contents of our propositional attitudes, this position is untenable. This became clear with Perry (1979), who convincingly argued that indexical propositions play an ineliminable role in our mental life. I desire to attend the committee meeting at noon. But this

[9]This is Proposition 4 of Halevy (2015: 341).

[10]A difference between the arguments was that Strotz presupposed the additive representation (1), while Koopman derived it from postulates for preferences.

[11]'Eternal' is an odd label that has established in philosophy. It is not intended to suggest something like an eternal existence of such propositions. Neither does it mean that the proposition is about something not located in time, like $2 + 2 = 4$. It rather means that eternal propositions are complete insofar as they contain all times, places and objects they refer to; the latter need not be supplemented by context.

[12]There is no point in broadly unfolding this topic here. Kaplan (1977) was the quantum jump in this field.

[13]Here, italics refer to the corresponding proposition.

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desire gets me moving from my office to the meeting room only if I realize that noon is *now*. Any replacement of 'now' by some non-indexical expression can't explain my motion. I am waiting in a waiting hall to be called. The call 'Brad Pitt' makes me move only if I believe that I am Brad Pitt. Without this indexical belief of mine the call is ineffective. The point is a very general one. Here, however, we need only consider indexical references to time. We may neglect other forms of indexicality.[14]

Now we are able to clear up the alleged puzzle about changing or unchanging time preferences. In my view, the economic literature on time preferences assumes that indexical utilities, i.e. utilities for indexical propositions don't change; this was quite clear in my above quote from Samuelson (1937). *I receive 100 $ at time t from now* always receives the same utility, whatever the time referred to by 'now'. This holds also for complex propositions like *I consume bundle* c_0 *at time 0 from now and … and* c_T *at time T from now*.

The point now is that while an indexical belief or utility remains constant, its eternal content still changes simply through the passage of time. For instance, it's 11.55 pm at 1 November 2022. I think: 'Oh, I have worked for 14 hours today, it has been a hard day.' I fall asleep for 10 minutes, it's now 5 minutes after midnight, and I have not changed any of my beliefs. Still, that belief has changed from true to false. It's simply not true that I have worked for 14 hours on 2 November. This explains the point. Similarly with utility: 'My utility for receiving \$100 in a week is 10' uttered on 1 November 2022, expresses that my utility for receiving \$100 on 8 November 2022 is 10. But by 2 November 2 the latter utility has changed to 11, while my utility for receiving \$100 in a week is still 10. This is so because my utility for receiving \$100 in six days is constantly 11.

The moral so far is: We have to distinguish utilities for eternal and for indexical propositions, and the latter may be constant while the former are not. Surely, indexical utilities may change as well, they do so all the time. My utility for getting food *now* changes several times a day, depending on when I eat and how much. But this is not a case of time preference. As said, the literature on time preferences is predicated on constant indexical utilities, which provide an explanation of how eternal utilities can change nevertheless, namely simply through the passage of time.

The economic terminology represented in the previous section is indeed able to capture this point. Time invariance (5) exactly describes the constancy of indexical utility. However, it does so by shifting the contents of evaluation, the eternal propositions, by the same time as the time of evaluation. This is formally equivalent, but it misses the essential point: that there are two kinds of propositions or contents, the relation of which may be unclear or uncertain to me. This will be the crucial point below, but it can become important only by distinguishing the two kinds.

No kind of utility can claim to be more basic. For instance, I have the desire to attend the committee meeting at noon. This is my basic non-indexical desire. I realize that noon is now, and thus I have the desire to attend the meeting now, and I get moving. Reversely, I am hungry now and want to eat something now. Again, I realize that now is noon. Hence, I want to eat something at noon. The derivation

[14]Egan and Titelbaum (2022: sect. 2) nicely summarize the philosophical arguments for the ineliminability of indexical mental contents.

works both ways. But in both cases the crucial link is provided by my belief about when is now.

So, what precisely is the relation between indexical and non-indexical utilities? If I am certain that absolute time t is τ times from now on (i.e. that now is $t - \tau$), then the non-indexical utility of A at t is equal to the indexical utility of A at τ times from now on; so much is clear. The same equation holds for beliefs or subjective probabilities. So, the crucial question is: what is this relation in case I am *not* sure about when is now? Well, in our times, there is hardly any such uncertainty. We are virtually surrounded by synchronized clocks. In earlier centuries this uncertainty was a much bigger problem. But even nowadays, everybody will remember at least some occasions of being confused about which time it is. Of course, this kind of uncertainty is not a pressing economic problem. This might explain why economists saw no need to explicitly address indexical utilities, which are rendered superfluous by the above equation in the absence of such uncertainty.

Still, we should try to state this relation in general, and this requires also treating the case of uncertainty about time. The natural answer is that this kind of uncertainty should be treated in the way any kind of uncertainty is treated in our theoretical context, namely by recourse to expectations. That is, my utility of A at t is just the expectation of my utilities of A at τ times from now on, taken with respect to my probabilities that t is τ times from now on.[15]

Let's spell out this answer in a bit more formal detail. Let $\mathbf{0}$ denote *now*. I use τ as a variable for indexical time, representing the distance from $\mathbf{0}$, and t as a variable for absolute time, the zero point of which is irrelevant. The time unit (seconds, days, years) is irrelevant as well; of course, it is the same for indexical and absolute time.[16] I assume both variables to take integers from $\mathbf{Z}$ as values. Let A be a propositional variable that generates propositions when applied to a certain time; thus, $A(t)$ is the *eternal* proposition that A obtains at t, and $A(\tau)$ is the *indexical* proposition that A obtains at τ (times from now on). For example A may be *it's raining at …* or *I get consumption bundle c at …*

Let us preliminarily take a brief look at how the agent epistemically deals with the two kinds of propositions. She has a probability measure p for all these propositions, but in particular also for *self-locating* propositions of the form $\tau = t$, which represent her uncertainty about when is now. Of course, $\tau = t$ and $\tau - t = \mathbf{0}$ count as the same proposition and must receive the same probability.[17] The basic logical fact is the logical truth of:

(6) $\tau = t \rightarrow (A(t) \leftrightarrow A(\tau))$.

[15]I have elaborated this idea with respect to indexical belief, i.e. the relation of indexical and non-indexical probabilities in Spohn (2017: sect. 3). I should mention, though, that there is quite an open philosophical debate about how to best account for that relation concerning probabilities. See for example Titelbaum (2016) and Egan and Titelbaum (2022).

[16]Note that the distinction between indexical and absolute time corresponds to McTaggart's (1908) distinction between the A-series and the B-series.

[17]p should be defined on a full propositional algebra. We may conceive of it as generated from the eternal, indexical and self-locating propositions just mentioned. There is no need to formally introduce it. For details see Spohn (2017: 364ff.).

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And hence:

(7) (a) $A(t) \leftrightarrow \exists\tau\ (\tau = t\ \&\ A(\tau))$ and

(b) $A(\tau) \leftrightarrow \exists t\ (\tau = t\ \&\ A(t))$.

(6) entails the basic connection between indexical and non-indexical probabilities:

(8) $p(A(t) \leftrightarrow A(\tau) \mid \tau = t) = p(A(t) \mid A(\tau)\ \&\ \tau = t) = p(A(\tau) \mid A(t)\ \&\ \tau = t) = 1$.

And (7a+b) translates into:

(9) (a) $p(A(t)) = \sum_{\tau\in\mathbf{Z}} p(A(\tau)|\tau = t)p(\tau = t)$ and

(b) $p(A(\tau)) = \sum_{t\in\mathbf{Z}} p(A(t)|\tau = t)p(\tau = t)$.[18]

This is just the formula of total probability with $A(t)$ and $A(\tau)$ replacing each other, as licensed by (7). (9a) says that the probability of $A(t)$ is the expectation of the probabilities of the $A(\tau)$ given $\tau = t$, the weights being the probabilities of $\tau = t$, and (9b) makes the reverse claim. If $A(\tau)$ or $A(t)$ would be probabilistically independent from $\tau = t$, (9) would simplify further. However, this independence cannot be generally assumed.

Which direction of (9) is more useful depends on the example. Sometimes the indexical probabilities are more easily accessible (e.g. when you are only told the weather forecast for tomorrow), sometimes you can proceed from the non-indexical probabilities (e.g. when you are told the weather forecast for 2 November 2022).

The point now is that these relations similarly hold for utilities as well. This is best explained in terms of the version of decision theory of Jeffrey (1965) in which *all* propositions receive probabilities as well as utilities. Let w denote the agent's utility function for all propositions. According to Jeffrey (1965: 5.4-5) w is governed by the desirability axiom, which states for any proposition B and a partition $(B_i)_{i\leq n}$ of B (i.e. B is true iff exactly one of the B_i is true) that:

(10) $w(B) = \sum_{i\leq n} w(B_i)p(B_i|B)$.

That is, the utility of each proposition is the expected utility of the ways in which it may come true. In this picture, only possible worlds (the atoms of the propositional algebra) have intrinsic, non-expected utility.[19]

How does this carry over to our set-up with eternal and indexical propositions? For the sake of vividness, let u be the restriction of the agent's utility function w to indexical propositions of the form $A(\tau)$ and v the restriction of w to eternal propositions of the form $A(t)$ (plus, respectively, self-locating propositions).

[18]This is formula (4) of Spohn (2017: 374), where I deal extensively with the dynamics of these probabilities.

[19]I am grateful to a reviewer for the suggestion to base the following discussion on Jeffrey's desirability axiom.

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We may ignore the utility of the many mixed propositions for the rest of the paper. Then, given the agent is sure that $\tau = t$, her utility of $A(t)$ must be the same as her utility of $A(\tau)$, i.e.:

(11) if $p(\tau = t) = 1$, then $v(A(t)) = u(A(\tau))$.

This follows immediately from (6) and (10).

In the case of uncertainty about time the one kind of utility is the expectation of the other kind. More precisely, (10) entails, again with the help of (6):

(12) (a) $v(A(t)) = \sum_{\tau \in \mathbf{Z}} u(A(\tau) \ \& \ \tau = t) p(\tau = t | A(t))$, and

(b) $u(A(\tau)) = \sum_{t \in \mathbf{Z}} v(A(t) \ \& \ \tau = t) p(\tau = t | A(\tau))$.

One cannot generally say which kind of utility is the more basic one. Sometimes it is important that certain things happen at fixed times (like submitting your tax declaration); what you decide to do depends on how you think you now relate to those fixed times. Sometimes it is important that things are as they should be from now on (like having your drugs on disposal); what this means in terms of absolute times is only derived. In the context of this paper, it is clear, however, that only the direction (12a) is relevant. The indexical utilities u are given and constant, and (12a) answers what this implies for the 'eternal' utilities v in case of uncertainty about when is now.

Still, there are two unexpected items in (12a), as in (12b), that require discussion. First, the expression $u(A(\tau) \ \& \ \tau = t)$ was formally required – whence my remark that u is explained not only for indexical, but also for self-locating propositions. But what does it mean to evaluate a self-locating proposition? I don't see how $\tau = t$ could carry intrinsic utility. Still, valuing $\tau = t$ may well make sense. At least, if one thinks of utility in terms of news value, as proposed by Jeffrey (1965), one may imagine contexts in which learning that it is currently a particular day would be good news.[20] In such a way, $\tau = t$ might have extrinsic utility. However, this makes clear at the same time that this possibility is irrelevant in our context. The only utilities in play here are those of possible consumption profiles. There is nothing in the picture from which $\tau = t$ could derive expected utility. If so, we can reduce $u(A(\tau) \ \& \ \tau = t)$ to $u(A(\tau))$.

Second, (12a) refers to the probabilities $p(\tau = t \mid A(t))$. But how can the probabilities of self-locating propositions depend on other propositions? Well, they must. We continuously learn through observations what time it presently is. I look at my watch and see that its hands presently show 10 o'clock. Trusting my watch, I infer that it is 10 o'clock now. Without this dependence, this learning would be impossible.[21] However, this suggests that the dependence obtains between self-locating propositions of the form $\tau = t$ and indexical propositions of the form $A(\tau)$. It is hard to see how the eternal proposition $A(t)$ could inform us about what time it

[20]This is a rough quote of a reviewer who pointed this possibility out to me.

[21]In Spohn (2017: 375ff.) I describe in detail how this learning works.

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is. If we may assume independence here, then the probabilities $p(\tau = t \mid A(t))$ reduce to $p(\tau = t)$.

If we accept both arguments, then (12a) simplifies to:

$$(13) \quad (a) \quad v(A(t)) = \sum_{\tau \in \mathbf{Z}} u(A(\tau)) p(\tau = t).$$

That is, the utilities of the eternal propositions are the expectations of the utilities of the corresponding indexical propositions with respect to the probabilities of possible self-locations. This corresponds to (9a) and may thus have appeared plausible from the outset. Note, however, that its derivation from (10) depends on the further assumptions required for reducing (12a) to (13a).

Arguing only with this plausible correspondence would have been problematic. The equation

$$(13) \quad (b) \quad u(A(\tau)) = \sum_{t \in \mathbf{Z}} v(A(t)) p(\tau = t)$$

corresponding to (9b) would have appeared equally plausible. But we cannot simultaneously have (13a) and (13b)—except in the case of $p(\tau = t) = 1$ for some t (where $\tau = t$ is independent of all propositions) and in the case where $u(A(\tau))$ and thus $v(A(t))$ are constant functions (so that it is irrelevant with which probabilities they are mixed). In other words, when we accept the independence that reduces (12a) to (13a), we cannot generally accept the independence that reduces (12b) to (13b). If eternal propositions don't inform us about self-location, then, generally, indexical propositions must do so.

In any case, my further argument will only build on (13a). More than this first general take on the topic of indexical utilities will not be required.

4. Discounting

Let us apply now these considerations to our case of time preferences. To begin with, let us simplify the case by looking at only one commodity c, e.g. \$100 or a piece of cloth or whatever, at various times. c corresponds to our propositional variable A. So, $c(\tau)$ means 'I get c at τ times from now on', $c(\mathbf{0})$ means 'I get c *now*', and $c(t)$ means 'I get c at absolute time t', e.g. on 8 November 2022. For notational ease, we may set the zero point 0 of absolute time at *now* $=\mathbf{0}$. We may also assume that $\tau \geq 0$; only present or future consumption is at issue. Presently, I have indexical utility $u(c(\tau))$, or $u(\tau)$ for short, for $c(\tau)$, and non-indexical utility $v(c(t))$, or $v(t)$ for short, for $c(t)$. We need to assume that both u and v are bounded, but we may take this for granted.

As argued above, the indexical utilities are the ones that are the constant part concerning time preferences, invariant 'throughout the time of consideration', in Samuelson's terms. So, u remains the same; I have it at 1 November, at 2 November, etc. At the same time, this makes clear that the indexical utilities are the basic ones, while the non-indexical ones are derived. If there should be uncertainty about time, it is reflected in the non-indexical utilities. In short, only (13a) applies in the case of time preferences.

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Let now $d_u(\tau) = u(\tau)/u(0)$ be the discount function governing u. So far it may have any bounded shape. And let the function $d'_u(\tau) = d_u(\tau+1)/d_u(\tau) = u(\tau+1)/u(\tau)$ measure the proportional increase or decrease of d_u from τ to $\tau+1$. In the same way, let the discount function governing v be defined as $d_v(t) = v(t)/v(0)$ and $d'_v(t) = d_v(t+1)/d_v(t)$. My initial announcement was that our indexical perspective helps specifying d_u and the other functions. Let us see how.

Given I am certain that $\tau = t$, (11) applies, and we have $v(t) = u(\tau)$ and hence also $d_v(t) = d_u(\tau)$. This does not constrain d_u. However, I may also be uncertain about when is now. Then, we said, the relation between u and v is described by (13a). Let us further simplify by reducing uncertainty. I am only unsure whether t is τ or $\tau+1$. Hence, $p(\tau = t) = x > 0$ and $p(\tau+1 = t) = 1 - x > 0$. Then (13a) reduces to

(14) $v(t) = x \cdot u(\tau) + (1 - x) \cdot u(\tau+1) = u(\tau) \cdot [x + (1 - x) \cdot d'_u(\tau)]$.

This allows to compute $v(t+1)$ in two ways, either by discounting $v(t)$ by $d'_v(t)$ or by applying (14) to $t+1$ instead of t. Thus:

(15) $v(t+1) = d'_v(t) \cdot v(t) = d'_v(t) \cdot u(\tau) \cdot [x + (1 - x) \cdot d'_u(\tau)]$, and

$v(t+1) = u(\tau+1) \cdot [x + (1 - x) \cdot d'_u(\tau+1)] = d'_u(\tau) \cdot u(\tau) \cdot [x + (1 - x) \cdot d'_u(\tau+1)]$.

Now, what is $d'_v(t)$? On the one hand, one could say that $d'_v(t)$ is simply calculated from $d'_u(\tau)$ and $d'_u(\tau+1)$ according to (14). This provides no further constraint on the discount function. On the other hand, one could reason as follows: If I knew that $t = \tau$, then $v(t)$ would equal $u(\tau)$, and hence $d'_v(t) = d'_u(\tau)$. If I knew that $t = \tau+1$, then $v(t)$ would equal $u(\tau+1)$, and hence $d'_v(t) = d'_u(\tau+1)$. Now, I don't know either and I must evaluate the proposition to get $c(t)$ in view of this uncertainty. This is why $v(t)$ is a mixture of $u(\tau)$ and $u(\tau+1)$. Still, I do know that $d'_v(t) = d'_u(\tau)$ or $= d'_u(\tau+1)$. And there is no reason why the discount rate $d'_v(t)$ at t should vary with my uncertainty, with my probabilities about when is t. Hence, it seems legitimate to use this knowledge about $d'_v(t)$.

If we accept this reasoning, we can assume that

(16) either $d'_v(t) = d'_u(\tau)$ or $d'_v(t) = d'_u(\tau+1)$

This is an additional assumption; it does not follow from (14). However, it helps to complete our reasoning. The two equations in (15) reduce to

(17) $d'_v(t) \cdot [x + (1 - x) \cdot d'_u(\tau)] = d'_u(\tau) \cdot [x + (1 - x) \cdot d'_u(\tau+1)]$,

And hence either horn of the assumption (16) entails

(18) $d'_u(\tau+1) = d'_u(\tau)$.

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Thus, the bounded u must have exponential shape of the form $u(\tau) = (1 - \delta)^{\tau}\, u(\mathbf{0})$ for some δ with $0 < \delta < 1$, i.e. u is decreasing at a constant discount rate $1 - \delta$, as we have it in (1).

Mathematically, this is trivial. One might say that the progress of (18) over (16) just consists in replacing the 'or' in (16) by 'and'. So, is the assumption (16) question-begging? Certainly not completely; after all, (14) does crucial work as well. I think it is not at all. (14) seems correct in deriving non-indexical utilities from the indexical ones and thus making them dependent on the agent's uncertainty about time. This seems unavoidable. By contrast, (16) assumes that at least the rate of change of non-indexical utilities from $v(t)$ to $v(t+1)$ is not afflicted by this uncertainty, but fixed in advance; it may be $d'_u(\tau)$ or $d'_u(\tau+1)$. In short: *The discount under uncertainty about time is the same as a discount given certainty about time*, whichever it is. The argument then shows that there is no difference between $d'_u(\tau)$ and $d'_u(\tau+1)$. (16) does not directly make assumptions about the discount rates of indexical utilities.

The argument generalizes to the case where uncertainty about when is now spreads not only over two, but many times. However, I don't think that it is necessary to demonstrate this. If the shape of the discount function for indexical utilities can be determined for some probability distribution, then this result must be accepted for all distributions. Basic or intrinsic utilities and their discounting are independent of the beliefs or probabilities held along with them.

The argument also generalizes to Samuelson's equation (1). First of all, it is irrelevant whether c represents a single commodity as above or an entire bundle of goods. We may also consider a commodity bundle $c_{\tau'}$ for each time τ' from $\mathbf{0}$ up to some time T (this is a capital tau). By the above argument we have $u(c_{\tau'}(\tau)) = (1 - \delta)^{\tau}\, u(c_{\tau'}(\mathbf{0}))$ for each time τ. And if we share Samuelson's assumption (ii) (after (1)), then we finally get

$$(19) \quad u(c_{\mathbf{0}}(\mathbf{0}), \ldots, c_{\mathrm{T}}(\mathrm{T}) = \sum_{\tau=0}^{\mathrm{T}} (1-\delta)^{\tau} u(c_{\tau}(\tau)).$$

This is the same as (1) in our notation. Obviously, we have relied here also on Samuelson's other assumptions (i), (iii) and (iv).

How does this compare with the traditional rationalizations of exponential discount functions via stationarity or time consistency and the exploitability of preference reversals? Well, it is a completely different argument via uncertainty about when is now, with all the assumptions I have made explicit. I don't see even a subliminal connection between the two types of argument.

So, one might hope that the novel argument relevantly adds to the old ones? I am unsure whether it does. Surely, uncertainty about absolute time is not a relevant condition for economic decisions. We are all familiar with it, but usually we quickly resolve it, and we don't fix things like consumption plans under such uncertainty. On the other hand, one could plead that the novel argument does not require actual uncertainty about absolute time. It suffices for the agent to merely imagine that she might be uncertain about time and then to ask herself how she would decide. This suffices for running the argument in favour of constant discount rates.

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Even if this hope should be dim, is the result an interesting one? Well, at least insofar as it opens a new perspective on the phenomenon of time preferences. In this perspective it is not a dynamic phenomenon which we may rationally constrain by postulates such as time consistency. It is rather generated by the gap between indexical and non-indexical attitudes. By all means, this perspective is worth developing, even independently of time preferences, and this paper is at least a start.

A final question: Do our considerations throw new light on the empirical findings disconfirming exponential and rather confirming hyperbolic discount rates? I don't see how. For instance, Halevy (2015) has run careful experiments testing for all three conditions, stationarity (2), time consistency (4) and time invariance (5). The last condition, he says, was simply presupposed by earlier experiments. He found that only 45–60% of the subjects had time invariant preferences (= constant indexical utilities) and would thus fall within the scope of the present paper, and he offers various explanations for the violation of time invariance. Around 80% of the time-invariant subjects conformed to stationarity and hence to time consistency. He does not address the other 20%. Frederick *et al.* (2002) review the experimental literature up to their time. Lots of experiments have been added. However, I don't know of any experiment that checks for uncertainty about when is now. They would be difficult to design for sure. So, in particular I don't know how to comment on the experimentally favoured hyperbolic discount functions from the perspective of this paper. If this perspective is interesting, it may arouse the interest of the experimenters as well.

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