AN IMPROVEMENT OF THE ORBITAL ELEMENTS OF HYPERION

Yoshizumi Hatanaka
Tokyo Astronomical Observatory
Mitaka, Tokyo 181, Japan

For the improvement of the orbital elements of Hyperion, a program of photographic observations of Saturnian satellites has been made since the opposition of 1966. In this paper are presented the reduction method and results derived from the observations during the oppositions of 1968 and 1977.

The photographic observation has been carried out with the 65-cm refractor of which focal length is about ten meters at the Tokyo Astronomical Observatory, Mitaka. In the earlier stage Fuji FLO plates (16.4 X21.4cm) were used without filter. Kodak 103aO plates (16X16cm) have been used since the opposition of 1970. All of the plates have been taken on or near the meridian. The several minute exposure is necessary to obtain a measurable image of Hyperion, but it is excessively long for those of the inner satellites. In order to obtain the best images of as many satellites as possible, the short time exposure ranging from twenty to forty seconds is made at the middle of the exposure under consideration, on the central area of the plate where the images of Saturn and the inner satellites are produced, though the long time exposure is made on the other parts. On the other hand, a few plates were exposed for scores of seconds on each observable night for the five inner satellites (Enceladus, Tethys, Dione, Rhea and Titan) and Iapetus that are brighter than Hyperion.

The positions of the images of the satellites were measured by the author on the Mann Type 422 comparator of the Tokyo Astronomical Observatory. Each image was bisected five times for the X and Y coordinates in one position of the plate. In order to obtain the Y position with the same accuracy as the X, the Y coordinate was measured in such an artificial posture that the measurer inclined his head by 90°. This method was able to be substituted for the measure in another position of the plate differing by 90°. All bisections were made so carefully that five measured values converged within a range of 7 microns. That is, the internal error of the measurements does not exceed 3.5 microns in the standard deviation, corresponding to 0.07 seconds of arc. When they did not converge due to the badness of the image, it was bisected again until the

R. L. Duncombe (ed.), Dynamics of the Solar System, 167–170.

number of the measured values which converged within the range amounted to five. The measured position was obtained as the average of the five measurements.

The calculated positions of the seven satellites were obtained from the orbital elements which were adopted in the Astronomical Ephemeris. But the elements of Hyperion, given by Woltjer (1928), were used without omitting the smaller perturbations. For their computation, the following original expression (Hatanaka, 1977) was used.

$$\rho\begin{pmatrix}\xi\\ \eta\\ \zeta\end{pmatrix} = R_1(90°-\delta)\ R_3(\alpha+90°)\ R_1(-\epsilon)\ R_3(-\Omega)\ R_1(-i)\ R_3(-f-\Pi+\Omega)\begin{pmatrix}r\\ 0\\ 0\end{pmatrix} \quad (1)$$

where α, δ, ρ, ϵ, r, f, Π, Ω and i are the right ascension, declination and geocentric distance of Saturn, the obliquity of the ecliptic, the distance from Saturn, true anomaly, longitudes of the perisaturnium and node, and inclination of the orbit to the ecliptic of the satellite, respectively. If necessary, the differential refraction correction was made only for the η values of Titan, Hyperion and Iapetus. The apparent Saturnicentric position (ξ, η) with respect to the equator of the Earth is referred to as the C-value.

The reduction of the observed position of Hyperion was as follows. The plate scale was determined from the measures of the angular distance between Rhea and Titan, since our analysis disclosed that the currently adopted orbital elements of Rhea and Titan seemed to be accurate for the purpose. The scale was adopted on all the plates during the opposition. Making use of the plates of which exposures were made on the satellites except for Hyperion, the observed position of Iapetus was obtained from the plate orientation and Saturn's position which were determined by the method of least squares mainly from the positions of Titan and Rhea, and by making complementary use of Enceladus, Tethys and Dione. The orbital elements of Iapetus at each opposition were improved by least squares of the equations described later, from the O-C differences of its position during the opposition. To obtain the observed position of Hyperion from the measures, Titan, Rhea and the improved Iapetus were useful as the reference satellites, that is, the orientation of the plate was derived from the positions of Rhea and Titan and the center of Saturn was determined from the improved position of Iapetus. The reason of the complicated procedure is that there is a discrepancy in the telescope guiding between the partial exposure for the inner satellites and the total for Hyperion and Iapetus. The observed position (X,Y) relative to Saturn, positive towards the east and north of the Earth, is referred to as the O-value. Figure 1 shows the O-C value with the observation date in UTC for all the observations used. An arrow directs from C to O, multiplied 25-fold in size.

The equations of condition for the improvement of the orbital elements are given by the differentiation of (1) and written in the following general forms (Hatanaka, 1977).

$$X - \xi = A da + B d\varepsilon + C d(e \sin\Pi) + D d(e \cos\Pi) + E d\Omega + F di \quad (2)$$
$$Y - \eta = G da + H d\varepsilon + I d(e \sin\Pi) + J d(e \cos\Pi) + K d\Omega + L di \quad (3)$$

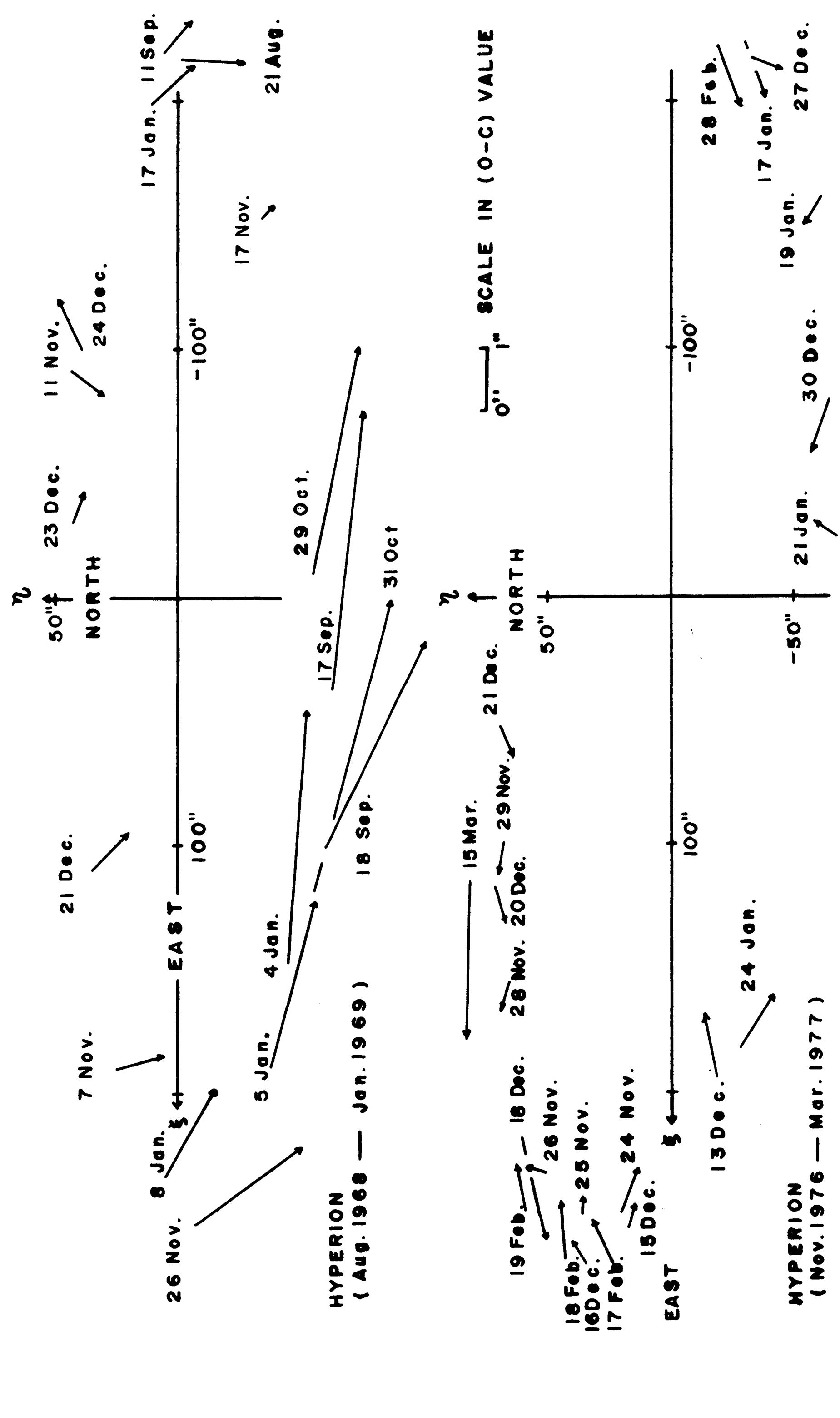

Figure 1. O–C value of the position of Hyperion. An arrow directs from C to O.

in which the twelve coefficients are expressible in terms of the orbital elements of the satellite to be improved and α, δ, ρ, ϵ. A simultaneous least-squares solution was made for corrections to the orbit during each opposition. Thus, the results obtained for Hyperion with the mean error are as follows.

1968 (17 nights)	1977 (22 nights)	
$da = -6''.8 \pm 1''.7$	$da = -5''.4 \pm 1''.2$	(at unit distance)
$d\varepsilon = -0^\circ.42 \pm 0^\circ.05$	$d\varepsilon = -0^\circ.22 \pm 0^\circ.05$	
$de = 0.0038 \pm 0.0011$	$de = 0.0006 \pm 0.0010$	
$d\Pi = -4^\circ.41 \pm 0^\circ.69$	$d\Pi = -1^\circ.36 \pm 0^\circ.58$	
$d\Omega = -0^\circ.16 \pm 0^\circ.11$	$d\Omega = -0^\circ.14 \pm 0^\circ.10$	
$di = 0^\circ.07 \pm 0^\circ.05$	$di = 0^\circ.10 \pm 0^\circ.03$	

The standard deviations of residuals are 0".50 and 0".45, respectively. The corrections of the positions of Iapetus determined by the method of least squares are given below. They were used for obtaining the position of Hyperion mentioned above.

1968 (37 nights)	1977 (32 nights)	
$da = -0''.3 \pm 1''.1$	$da = 5''.1 \pm 0''.6$	(at unit distance)
$d\varepsilon = -0^\circ.040 \pm 0^\circ.013$	$d\varepsilon = -0^\circ.181 \pm 0^\circ.007$	
$de = -0.00093 \pm 0.00030$	$de = 0.00151 \pm 0.00020$	
$d\Pi = -0^\circ.49 \pm 0^\circ.34$	$d\Pi = -4^\circ.19 \pm 0^\circ.38$	
$d\Omega = 0^\circ.22 \pm 0^\circ.04$	$d\Omega = -0^\circ.10 \pm 0^\circ.02$	
$di = -0^\circ.122 \pm 0^\circ.012$	$di = -0^\circ.059 \pm 0^\circ.007$	

The standard deviations of residuals are 0".49 and 0".26, respectively.

The computations were made with the FACOM 230-58 computor at the Tokyo Astronomical Observatory.

References

Hatanaka, Y., 1977. Proceedings of a Symposium on Celestial Mechanics, 1977, p. 42. (Edited by Hori and Yuasa, held in Tokyo)
Woltjer, J., 1928. Annalen van de Sterrewacht te Leiden, 16, part 3.