Canad. Math. Bull. Vol. **33** (3), 1990

SAMPLE PATH PROPERTIES OF l^p-VALUED ORNSTEIN-UHLENBECK PROCESSES

BY

B. SCHMULAND

ABSTRACT. We give conditions under which a vector valued Ornstein Uhlenbeck process has continuous sample paths in l^p for $1 \leqq p < \infty$. We also show when the space l^p is not entered at all, i.e., when it has zero capacity.

1. **Introduction.** In [4] D. A. Dawson initiated the study of infinite dimensional stochastic evolution equations which he used to model continuous spacetime population processes. In the linear case these equations reduce to the form

$$\frac{\partial X}{\partial t} = -AX + W \tag{1.1}$$

where W is a white noise and A is (typically) a differential operator acting on an L^2 space over a region in $\mathbb{R}^d$. The first problem we run into is that $X(\cdot)$ will not live in the domain of A and hence cannot be a solution of (1.1) in the strong sense. Next, in order that it even qualifies as a weak solution, we would need to know that the process $X(\cdot)$ lives in L^2. When the invariant measure for the stationary solution is supported by L^2, it is true that $X(\cdot)$ lives in L^2 but it is not easy to prove and requires a detailed analysis of the sample paths of $X(\cdot)$. Therefore the study of such sample path properties of $X(\cdot)$ is, in part, motivated by the desire to understand in what sense (1.1) holds, which in turn should shed light on the process being modelled.

In this paper, we will study the sample path behaviour of the stationary solution $X(\cdot)$ of (1.1). We will make the further simplifying assumption that the operator A and the covariance operator of W share a common discrete spectrum so that (1.1) decouples into the system of stochastic differential equations

$$dX_k(t) = -\lambda_k X_k(t) + \sqrt{2\gamma_k}\, dW_k(t), \qquad k \geqq 1 \tag{1.2}$$

where $\{W_k(t)\}_{k=1}^{\infty}$ are independent Wiener processes and $\{\lambda_k\}$, $\{\gamma_k\}$ are sequences of positive constants. The solution X of (1.1) is the vector valued process whose coordinates are given by the X_k processes, i.e.,

$$X(t) = (X_1(t),\ X_2(t), \ldots, X_k(t), \ldots). \tag{1.3}$$

Received by the editors July 26, 1989 and, in revised form December 9, 1989.
AMS (1980) subject classification: Primary 60H10, 60G17; secondary 60G15.
Research supported by NSERC.

The stationary solution $X_k(\cdot)$ to (1.2) is well known to be an Ornstein-Uhlenbeck process, that is, $X_k(\cdot)$ is a mean zero Gaussian process with covariance

$$E(X_k(s)X_k(t)) = \frac{\gamma_k}{\lambda_k} e^{-\lambda_k|t-s|}.$$

The sample path behaviour of the vector-valued process $X(\cdot)$ and other related processes has been the subject of a number of recent papers [1–3, 6, 8–9, 11–12]. In this paper we will use the theory of Dirichlet forms to obtain conditions under which the process $X(\cdot)$ has continuous sample paths in the space $l^p(1 \leqq p < \infty)$, and also to determine when such spaces are not entered at all. These results extend work done using the same technique for the case $p = 2$ [11].

By focusing on the simple type of equation presented here, where A is linear and has discrete spectrum, we hope to avoid a lot of technical details and make the use of Dirichlet form methods transparent. We emphasize that this approach does not depend in a crucial way on the discreteness of the spectrum of A (cf. [12]), nor on the linearity of A (i.e. the fact that X is Gaussian). In fact, by analogy with the finite dimensional case, it is expected that the theory of Dirichlet forms will prove useful in studying stochastic p.d.e.'s with very singular non-linear drift. This will be the topic of further research.

2. **The Dirichlet form.** In this section we describe the Dirichlet form $\mathcal{E}$ associated with the process $X(\cdot)$ given in (1.3). The general theory of such forms is found in Fukushima's book [7] and for proofs of the closability and the Markovian property, etc. of the form given below the reader is referred to [11].

The process $X(\cdot)$ lives on the state space

$$X = \prod_{k=1}^{\infty} \mathbb{R}_k$$

and has invariant measure

$$m = \prod_{k=1}^{\infty} N\left(0, \frac{\gamma_k}{\lambda_k}\right).$$

A generic point in X will be denoted by $x = (x_1, x_2, \ldots, x_k, \ldots)$. For $1 \leqq p < \infty$, the space l^p can be identified with the subspace

$$\left\{ x \in X : \sum_{k=1}^{\infty} |x_k|^p = \|x\|_p^p < +\infty \right\}$$

of X. Now, in order that l^p serve as a state space for $X(\cdot)$ it is necessary that l^p supports the invariant measure, i.e., $m(l^p) = 1$. Because m is Gaussian, this is the same as requiring that

$$\sum \left(\frac{\gamma_k}{\lambda_k}\right)^{p/2} < +\infty. \tag{2.1}$$

This, in general, will not be sufficient as it is only a fixed time result. Because $X(\cdot)$ is stationary we have, under (2.1),

$$m(l^p) = P(X(t) \in l^p) = 1 \quad \forall t,$$

but we need further conditions on the coefficients to preclude the possibility of exceptional times when $X(t) \notin l^p$. Similarly if $\sum(\gamma_k/\lambda_k)^{p/2} = +\infty$, then

$$m(l^p) = P(X(t) \in l^p) = 0 \quad \forall t$$

and we'd like to know when we can draw the stronger conclusion that

$$P(\exists t \text{ so that } X(t) \in l^p) = 0.$$

The tool we use for getting these results is the Dirichlet form $\mathcal{E}$ associated with the process $X(\cdot)$. This form is given as the closure of the following bilinear form defined over $L^2(X;\ m)$;

$$\mathcal{E}(u,\ v) = \frac{1}{2}\int \sum_{k=1}^{\infty} \gamma_k \left(\frac{\partial}{\partial x_k}u\right)\left(\frac{\partial}{\partial x_k}v\right) dm$$

$$\mathcal{D}(\mathcal{E}) = \{u \in L^2 : u = \phi(x_1, \ldots, x_k)\ \phi \in C_0^\infty(\mathbb{R}^k),\ k \geqq 1\}.$$

For simplicity we will also use $\mathcal{E}$ to denote the closed form. We define the form $\mathcal{E}_1$ on $\mathcal{D}(\mathcal{E})$ by $\mathcal{E}_1(u,\ v) = \mathcal{E}(u,\ v) + (u,\ v)_{L^2}$.

Since the coordinates of $X(\cdot)$ are one dimensional Ornstein-Uhlenbeck processes, it is clear that $X(\cdot)$ is continuous when X is equipped with the product topology. In order to strengthen this to l^p continuity we will rely on the following fundamental lemma of Fukushima [7; Theorems 3.1.4 and 4.3.2.]. This lemma is also used to prove Proposition 2.

LEMMA 1. *Every $u \in \mathcal{D}(\mathcal{E})$ has a quasicontinuous version $\tilde{u}$ so*

$$P(t \longrightarrow \tilde{u}(X(t)) \text{ is continuous}) = 1.$$

Furthermore, if $u_n \longrightarrow u$ in $\mathcal{E}_1$-norm, then for every $T > 0$ there exists a subsequence $\{u_{nk}\}$ so

$$P(\tilde{u}_{nk}(X(t)) \text{ converges to } \tilde{u}(X(t)) \text{ uniformly on } [0,\ T]) = 1.$$

3. **The results.**

PROPOSITION 1. *Fix $1 \leqq p < \infty$ and define*

$$\delta_k = \left(\frac{\gamma_k}{\lambda_k}\right)^{p/2} 4^{p/2} \log^{p/2}\left(\gamma_k \left(\frac{\gamma_k}{\lambda_k}\right)^{(p/2-1)} \vee e\right).$$

If $\sum_k \delta_k < +\infty$, then $X(\cdot)$ has l^p continuous sample paths.

PROOF. By supposition, $\sum(\gamma_k/\lambda_k)^{p/2} < +\infty$ which means that $m(l^p) = 1$. Since m is Gaussian, all moments of $\|x\|_p$ are integrable with respect to m [5]. For $n \geqq 1$ we define

$$u_n = \sum_{k=1}^{n} |x_k|^p \ \vee \ \delta_k.$$

The function u_n belongs to the domain of the closed form $\mathcal{E}$ with

$$\frac{\partial}{\partial x_k} u_n(x) = p|x_k|^{p-1} 1_{\{|x_k|^p > \delta_k\}} 1_{\{k \leqq n\}}.$$

Therefore, for $n > m$,

$$\begin{aligned}
&\mathcal{E}(u_n - u_m,\ u_n - u_m) \\
&= \frac{p^2}{2} \int \sum_{k=m+1}^{n} \gamma_k |x_k|^{2(p-1)} 1_{\{|x_k|^p > \delta_k\}} m(dx) \\
&\leqq C \sum_{k=m+1}^{n} \gamma_k \left(\frac{\gamma_k}{\lambda_k}\right)^{p-1} \exp\left(-\frac{1}{4}\delta_k^{2/p} \Big/ \left(\frac{\gamma_k}{\lambda_k}\right)\right) \\
&= C \sum_{u=m+1}^{n} \left(\frac{\gamma_k}{\lambda_k}\right)^{p/2}.
\end{aligned}$$

For the inequality we used the fact that for any power p,

$$\int_x^\infty y^p e^{-y^2/2} dy \leqq e^{-x^2/2} P(x) = O(e^{-x^2/4})$$

as $x \longrightarrow \infty$, where P is some polynomial. Now $u_n \uparrow u = \sum_{k=1}^\infty |x_k|^p \vee \delta_k$ and $u \leqq \|x\|_p^p + \sum_{k=1}^\infty \delta_k$ which belongs to L^2, so $u_n \longrightarrow u$ in L^2. The bound on $\mathcal{E}(u_n - u_m,\ u_n - u_m)$ shows that $\{u_n\}$ is $\mathcal{E}$-Cauchy so $(u - u_n) \longrightarrow 0$ in $\mathcal{E}_1$-norm.

Applying lemma 1 we find that, for any $T > 0$,

$$P\left(\sup_{0 \leqq t \leqq T} \sum_{k=n+1}^{\infty} |X_k(t)|^p \underset{n\to\infty}{\longrightarrow} 0\right)$$

$$\geqq P(\sup_{0 \leqq t \leqq T} (u - u_n)(X(t)) \underset{n\to\infty}{\longrightarrow} 0) = 1$$

which gives the required result. □

COROLLARY 1. *If $\sum(\gamma_k/\lambda_k)^{p/2} \log (\lambda_k \vee e)^{p/2} < +\infty$ for $1 \leqq p < 2$ or $\sum(\gamma_k/\lambda_k)^{p/2} \log (\gamma_k \vee e)^{p/2} < +\infty$ for $2 \leqq p < \infty$, then $X(\cdot)$ has l^p-continuous sample paths.*

COMMENT. Let us consider the case $p = 2$ and for convenience assume that $\{\gamma_k/\lambda_k\}$ is decreasing and $\{\gamma_k\}$ is increasing. It is known ([6], see also [8]) that the condition

$$\sum_k \frac{\gamma_k}{\lambda_k} \log \left(\frac{\gamma_{k+1}}{\gamma_k} \vee e \right) < +\infty$$

is necessary and sufficient for the l^2-continuity of $X(\cdot)$. One sees therefore that our sufficient condition

$$\sum \frac{\gamma_k}{\lambda_k} \log (\gamma_k \vee e) < +\infty$$

is not far off the mark. The extra strength of Fernique's result is a consequence of the assumed independence of the coordinate processes $\{X_k(t)\}$. This independence is not needed for our result, as can be seen by reading the previous proof.

Now let us turn to the opposite problem. Let us suppose that

$$\sum (\gamma_k/\lambda_k)^{p/2} = +\infty$$

so $m(l^p) = 0$. Consequently we have

$$P(X(t) \in l^p) = 0 \; \forall \; t.$$

We seek further conditions under which we can draw the stronger conclusion

$$P(\exists t \; X(t) \in l^p) = 0.$$

PROPOSITION 2. *Suppose that* (γ_k/λ_k) *is bounded but* $\sum_{k=1}^{\infty} (\gamma_k/\lambda_k)^{p/2} = +\infty$ *for some* $1 \leqq p < \infty$. *For convenience, we will assume that the sequence* $\{\gamma_k\}$ *is non-decreasing. Under either of the following conditions we can conclude*

$$P(\exists t \text{ such that } X(t) \in l^p) = 0.$$

(i) For $1 \leqq p < 2$ *it suffices that, for some* $M > 0$,

$$\sum_{k=1}^{n} \gamma_k = O\left(\exp\left\{ M \sum_{k=1}^{n} (\gamma_k/\lambda_k)^{p/2} \right\} \right)$$

(ii) For $2 \leqq p < +\infty$ *it suffices that, for some* $M > 0$,

$$\gamma_n = O\left(\exp\left\{ M \sum_{k=1}^{n} (\gamma_k/\lambda_k)^{p/2} \right\} \right)$$

PROOF. Fix $N > 0$ and let $u_n(x) = (\sum_{k=1}^{n} |x_k|^p) \wedge N$. Then $u_n(x) \longrightarrow u(x) = (\sum_{i=1}^{\infty} |x_k|^p) \wedge N$ and $m(u = N) = 1$. Hence $u \in \mathcal{D}(\mathcal{E})$ and $\mathcal{E}(u, u) = 0$. We'd like to show that $u_n \longrightarrow u$ in $\mathcal{E}_1$-norm by showing that $\mathcal{E}(u_n, u_n) \longrightarrow 0$.

https://doi.org/10.4153/CMB-1990-060-9 Published online by Cambridge University Press

Now

$$(\partial u_n/\partial x_k)(x) = p|x_k|^{p-1} 1_{\{\sum_{k=1}^n |x_k|^p \leqq N\}} 1_{\{k \leqq n\}}$$

and so

$$\mathcal{E}(u_n, u_n) = p^2/2 \int \sum_{k=1}^n \gamma_k |x_k|^{2(p-1)} 1_{\{\sum_{k=1}^n |x_k|^p \leqq N\}} m(dx).$$

We begin by estimating the integrand. If $p < 2$ so $2(p-1) < p$ we use Hölder's inequality with $p' = p/2(p-1)$ and $q' = p/(2-p)$ to get

$$\sum_{k=1}^n \gamma_k |x_k|^{2(p-1)} \leqq \left(\sum_{k=1}^n |x_k|^p\right)^{1/p'} \left(\sum_{k=1}^n \gamma_k^{q'}\right)^{1/q'}$$
$$\leqq N^{1/p'} \cdot \sum_{k=1}^n \gamma_k,$$

on the set $\{\sum_{k=1}^n |x_k|^p \leqq N\}$. For $p \geqq 2$ we simply use the fact that

$$\sum_{k=1}^n \gamma_k |x_k|^{2(p-1)} \leqq \gamma_n \sum |x_k|^{2(p-1)} \leqq \gamma_n \left(\sum |x_n|^p\right)^{2(p-1)/p}$$
$$\leqq \gamma_n N^2,$$

on this same set. Using Chebyshev's inequality we find

$$m\left(\sum_{k=1}^n |x_k|^p \leqq N\right) \leqq e^{tN} E(e^{-t\sum_{k=1}^n |x_k|^p})$$
$$= e^{tN} \prod_{k=1}^n E(e^{-t(\gamma_k/\lambda_k)^{p/2}|Z|^p})$$

where Z is standard normal. If (γ_k/λ_k) is bounded away from zero we may take t so large that

$$E(e^{-t(\gamma_k/\lambda_k)^{p/2}|Z|^p}) \leqq \rho$$

for any $0 < \rho < 1$. Consequently $m(\sum_{k=1}^n |x_k|^p \leqq N)$ goes to zero faster than any exponential.

If $\gamma_k/\lambda_k \to 0$, but $\sum(\gamma_k/\lambda_k)^{p/2} = +\infty$ this probability may not go to zero as quickly. However, the following computations do give us a rate of convergence. There exists a constant $c(p)$ so that

$$E(e^{-a|Z|^p}) \leqq e^{-c(p)\cdot a}$$

for all $0 \leqq a \leqq 1$. Since $t \cdot (\gamma_k/\lambda_k)^{p/2} \leqq 1$ for large k we get

$$m\left(\sum_{k=1}^n |x_k|^p \leqq N\right) \leqq K(N, t) \exp\left\{-c(p)t \sum_{k=1}^n (\gamma_k/\lambda_k)^{p/2}\right\}.$$

https://doi.org/10.4153/CMB-1990-060-9 Published online by Cambridge University Press

For any $M > 0$, take $t = (M+1)/c(p)$ to conclude that

$$m\left(\sum_{k=1}^{n} |x_k|^p \leqq N\right) = o\left(\exp\left\{-M\sum_{k=1}^{n}(\gamma_k/\lambda_k)^{p/2}\right\}\right)$$

Combining these two estimates and using the hypotheses on $\{\gamma_k\}$ we find that $\mathcal{E}(u_n, u_n) \longrightarrow 0$. Applying lemma 1 we see that $u(X(\cdot))$ is a continuous process, so

$$P(u(X(t)) = N \;\forall\, t) = 1.$$

As this is true for all N, we conclude that

$$P(\|X(t)\|_p^p = \infty \;\forall\, t) = 1. \qquad \square$$

4. **Examples.** 1. Let us consider the case $\gamma_k = 1$, $\lambda_k = k(\log\, k)^\alpha$ $\alpha > 1$. In this case the conditional sum $S(t) = \sum_{k=1}^\infty X_k(t)$ exists at fixed times because

$$E(S^2(t)) = \sum_{k=1}^{\infty} E(X_k^2(t)) = \sum_{k=1}^{\infty} 1/k(\log\, k)^\alpha < +\infty.$$

However it is known that when $\alpha \leqq 3/2$, the process $S(t)$ has discontinuous sample paths [9]. For $\alpha > 5/2$ the sample paths are continuous and a modulus of continuity is given in [2].

As far as the l^p spaces go, we look first to the invariant measure to find that $m(l^p) = 0$ for $1 \leqq p < 2$ and $m(l^p) = 1$ for $p \geqq 2$. This gives the fixed time result, what can we say about the process at all times?

First of all, by Corollary 1 we see that $X(\cdot)$ is continuous in l^2.

Secondly, for $1 \leqq p < 2$ we note that $\sum_{k=1}^n \gamma_k = n$ and

$$\sum_{k=1}^{n}(\gamma_k/\lambda_k)^{p/2} = \sum_{k=1}^{n}(k(\log\, k)^\alpha)^{-p/2} \geqq \sum_{k=1}^{n} k^{-1} \sim \log\,(n)$$

so condition (i) in Proposition 2 holds with $M = 1$. Therefore

$$P(\exists t \text{ so that } X(t) \in l^p \text{ for some } 1 \leqq p < 2) = 0.$$

For $\alpha > 5/2$ we get a process where $\sum_{k=1}^\infty X_k(t)$ is finite and continuous but $\sum_{k=1}^\infty |X_k(t)|$ is identically infinite, in the sense that there are no exceptional times when $\sum_{k=1}^\infty |X_k(t)| < +\infty$. In fact, there aren't even exceptional times when $\sum_{k=1}^\infty |X_k(t)|^{2-\epsilon} < +\infty$ for any $\epsilon > 0$.

2. Let us consider the stationary version of Walsh's stochastic model of neural response ([13], [14]) on the interval [0, 1]. This is a process $\{\hat{V}(t)\}_{t\geqq 0}$ which satisfies (1.1), where W is a white noise on $L^2([0, 1]; dx)$, and A is the operator $(I - \Delta)$ where

the Laplacian is given reflecting boundary conditions at the endpoints. This operator has a complete system of eigenfunctions and eigenvalues as follows:

$$\phi_0(x) \equiv 1,\ \phi_k(x) = 2^{1/2} \cos(\pi k x)$$

$$\lambda_k = 1 + \pi^2 k^2;\ k = 0,\ 1,\ 2,\ \ldots.$$

If we let $X_k(t) = \langle \hat{V}(t),\ \phi_k \rangle$, then we get a sequence valued process $X(t)$ as in (1.3), where

$$\gamma_k \equiv 1/2 \text{ and } \lambda_k = 1 + \pi^2 k^2;\ k = 0,\ 1,\ 2,\ \ldots.$$

Using the results of the previous section we can get some information on how $\hat{V}$ behaves as a function of x. Because of the decomposition used here, the method of Dirichlet forms is an effective way of studying these properties of $\hat{V}$ which are easily described in terms of its sequence $\{X_k\}$ of Fourier coefficients. Three examples of such properties are given below.

Now $\lambda_k \sim ck^{-2}$ as $k \longrightarrow \infty$, so that the invariant measure m satisfies $m(l^p) = 1$ for $p > 1$ and $m(l^1) = 0$. It is easy to see that the conditions of Corollary 1 and Proposition 2 are met, so

$$P\{X(t) \text{ is } l^p\text{-continuous for } p > 1\} = 1 \tag{4.1}$$

and

$$P\{X(t) \in l^1 \text{ for some } t\} = 0. \tag{4.2}$$

Equation (4.1) combined with the Hausdorff-Young theorem [10; p. 99] now tells us that $\{\hat{V}(t)\}$ is continuous in $L^q([0,\ 1];\ dx)$ for any $q \geqq 2$. However an even stronger result is already known since Walsh has shown that, with probability one, $\{\hat{V}(t,\ x)\}_{t \geqq 0, x \in [0,1]}$ is jointly Hölder continuous.

Equation (4.2) along with Bernstein's theorem [10; p. 32] does give us a new result. It tells us that there are no exceptional times t when $\hat{V}(t,\ x)$ has a Hölder modulus greater than 1/2 as a function of x.

Finally, we mention without proof another result along these lines. If we let

$$u_n(x) = \left(\sup_{k=1}^{n} |kx_k|\right) \wedge N$$

in Proposition 2, we can prove for Walsh's process that

$$P\{|X_k(t)| \text{ is not } O(k^{-1}) \text{ for all } t\} = 1.$$

This fact, combined with [10; p. 24] tells us that there are also no exceptional times when $\hat{V}(t,\ x)$ has bounded variation as a function of x.

It is known that, for fixed t, $\hat{V}(t,\ x)$ looks like a Brownian motion path plus a C^2 function [13; Prop. 6.1]. The latter two results suggest that this is not valid only at fixed times, but for all times, by implying a certain uniformity in t of the roughness in x of the function $\hat{V}(t, x)$.

References

1. E. Csáki, M. Csörgo, and P. Révész, *On infinite series of independent Ornstein-Uhlenbeck processes*, Tech. Rep. Ser. Lab. Res. Stat. Probab. No. 125, Carleton University – University of Ottawa (1989).

2. M. Csörgo, and Z. Y. Lin, *On moduli of continuity for Gaussian and l^2-norm squared processes generated by Ornstein-Uhlenbeck processes*. Tech. Rep. Ser. Lab. Res. Stat. Probab. No. 125, Carleton University – University of Ottawa. (1989)

3. ———, *On moduli of continuity for Gaussian and χ^2 processes generated by Ornstein-Uhlenbeck processes*, C.R. Math. Rep. Acad. Sci. Canada **10** (1988), 203–207

4. D. A. Dawson, *Stochastic Evolution Equations*. Math. Biosci. **15** (1972), 287–316.

5. X. Fernique, *Gaussian random vectors and their reproducing kernel Hilbert Spaces*, Tech. Rep. Ser. Lab. Res. Stat. Probab. No. 34, Carleton University – University of Ottawa. (1985)

6. ———, *La régularité des fonctions aléatoires d'Ornstein-Uhlenbeck à valuers dans l^2; le cas diagonal*. (1989) Unpublished note.

7. M. Fukushima, *Dirichlet Forms and Markov Processes*, Tokyo Kodansha and Amsterdam North-Holland. (1980).

8. I. Iscoe, M. B. Marcus, D. McDonald, M. Talagrand and J. Zinn. *Continuity of l^2-valued Ornstein-Uhlenbeck processes*. To appear Ann. Probab. (1990).

9. I. Iscoe and D. McDonald, *Continuity of l^2-valued Ornstein-Uhlenbeck processes*, Tech. Rep. Ser. Lab. Res. Stat. Probab. No. 58, Carleton University – University of Ottawa. (1986).

10. Y. Katznelson, *An introduction to harmonic analysis*, Dover New York (1976).

11. B. Schmuland, *Some regularity results on infinite dimensional diffusions via Dirichlet forms*. Stochastic Analysis and Applications., **6(3)** (1988), 327–348.

12. ———, *An energy approach to reversible infinite dimensional Ornstein-Uhlenbeck processes*. Manuscript. (1989)

13. J. Walsh, *A stochastic model of neural response*. Adv. Appl. Prob. **13** (1981), 231–281.

14. ———, *Regularity properties of a stochastic partial differential equation*. *In*: E. Cinlar, K. L. Chung, R. K. Getoor, eds. Proc. Seminar on Stochastic Processes. Birkhäuser Boston (1983).

Department of Statistics and Applied Probability
University of Alberta
Edmonton, Alberta, Canada
T6G 2G1

https://doi.org/10.4153/CMB-1990-060-9 Published online by Cambridge University Press