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ON THE ASYMPTOTIC BEHAVIOUR OF ASSOCIATED PRIMES OF GENERALIZED LOCAL COHOMOLOGY MODULES

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Abstract

Let M and N be finitely generated and graded modules over a standard positive graded commutative Noetherian ring R, with irrelevant ideal R_+. Let $H^k_{R_+}(M, N)_n$ be the nth component of the graded generalized local cohomology module $H^k_{R_+}(M, N)$. In this paper we study the asymptotic behavior of $\mathrm{Ass}_{R_+}\big(H^k_{R_+}(M, N)_n\big)$ as $n \longrightarrow -\infty$ whenever k is the least integer j for which the ordinary local cohomology module $H^j_{R_+}(N)$ is not finitely generated.

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1. Introduction

There is a lot of current interest in the theory of graded local cohomology modules and in recent years there have appeared many papers concerned with this context and its developments. The main purpose of this paper is to establish an asymptotic behaviour of associated prime ideals of graded components of generalized local cohomology modules.

The concept of generalized local cohomology of R-modules M and N relative to an ideal I of a commutative Noetherian ring R was introduced by Herzog in [10] and studied by Suzuki in [21] (see also [2]) as

$$H^i_I(M, N) = \varinjlim_{n \in \mathbb{N}_0} \mathrm{Ext}^i_R(M/I^n M, N),$$

where $i \in \mathbb{N}_0$, and was studied in [1, 11, 14, 16, 22, 23] (here $\mathbb{N}_0$ and $\mathbb{N}$ denote the set of non-negative and positive integers, respectively; $\mathbb{Z}$ will denote the set of all integers). It is well known that the functor $H^i_I(-,-) : C(R) \times C(R) \longrightarrow C(R)$ is a two variable additive R-linear functor and for $M = R$, it is converted to $H^i_I(-)$, the ith ordinary local cohomology functor (where $C(R)$ denotes the category of all R-modules and R-homomorphisms).

Assume that $R = \bigoplus_{n\in\mathbb{N}_0} R_n$ is a positively graded Noetherian ring which is standard, in the sense $R = R_0[R_1]$, and set $R_+ = \bigoplus_{n\in\mathbb{N}} R_n$, the irrelevant ideal of R. Let $M = \bigoplus_{n\in\mathbb{Z}} M_n$ and $N = \bigoplus_{n\in\mathbb{Z}} N_n$ be non-zero finitely generated graded R-modules. It is well known that $H^i_I(N)$ is equipped with a natural grading for all $i \in \mathbb{N}_0$ (compare [8, Remarks 12.3.6]). The associated primes of graded components of local cohomology modules were studied in a number of papers (compare [3–7, 12, 13, 17, 18]). Brodmann and Hellus, in [6], showed that if k is an integer such that $H^i_{R_+}(N)$ is finitely generated for all $i < k$, then the associated prime ideals of $H^k_{R_+}(N)_n$ is asymptotically stable as $n \longrightarrow -\infty$. Recall that we say $\mathrm{Ass}_{R_0}\left(H^i_{R_+}(N)_n\right)$ are asymptotically stable as $n \longrightarrow -\infty$ if there exists an integer $n_0 \in \mathbb{Z}$ such that $\mathrm{Ass}_{R_0}\left(H^i_{R_+}(N)_n\right) = \mathrm{Ass}_{R_0}\left(H^i_{R_+}(N)_{n_0}\right)$ for all $n \le n_0$. Also, in [14], the first present author established the above result in the context of generalized local cohomology modules (see [14, Theorem 3.5]). We say that $\mathrm{Ass}_{R_0}\left(H^i_{R_+}(M,N)_n\right)$ is asymptotically stable as $n \longrightarrow -\infty$ if there exists an $n_0 \in \mathbb{Z}$ such that $\mathrm{Ass}_{R_0}\left(H^i_{R_+}(M,N)_n\right) = \mathrm{Ass}_{R_0}\left(H^i_{R_+}(M,N)_{n_0}\right)$ for all $n \le n_0$ (see [14, Notation and Remarks 3.3]). In this paper we will show that if $H^i_{R_+}(N)$ is finitely generated for all $i < k$ then $\mathrm{Ass}_{R_0}\left(H^k_{R_+}(M,N)_n\right)$ is asymptotically stable as $n \longrightarrow -\infty$.

The grading of the generalized local cohomology module $H^i_{R_+}(M,N)$ has been studied by the first present author in [14]. He found that the grading of the generalized local cohomology modules with respect to the irrelevant ideal of R has some properties similar to the ordinary local cohomology modules. Let us now briefly recall some basic properties of graded generalized local cohomology modules (see [14]).

(i) For every finitely generated graded R-module M, if we forget the grading on the module ${}^*\mathrm{Hom}_R(M,N)$ then we have the module $\mathrm{Hom}_R(M,N)$ (see [9, Pages 32–33]). Hence there is a homogeneous isomorphism $H^0_I(M,N) \cong H^0_I(\mathrm{Hom}_R(M,N))$ for every homogeneous ideal I of R.

(ii) If there is a short exact sequence of graded modules and homogeneous homomorphisms

$$0 \longrightarrow N' \longrightarrow N \longrightarrow N'' \longrightarrow 0$$

(by "homogeneous" here we mean 'homogeneous of degree zero') then there exists the following long exact sequence of graded modules and homogeneous homomorphisms

$$0 \longrightarrow H^0_I\left(M,N'\right) \longrightarrow H^0_I(M,N) \longrightarrow H^0_I\left(M,N''\right) \longrightarrow H^1_I\left(M,N'\right) \longrightarrow \cdots .$$

(iii) Let $R' = \oplus_{n\in\mathbb{N}_0} R'_n$ be a second commutative Noetherian ring and let

$f : R \longrightarrow R'$ be a homogeneous flat ring homomorphism. Let I be a homogeneous ideal of R. Then there exists a homogeneous isomorphism

$$H_I^i(M, N) \otimes_R R' \cong H_{IR'}^i \left(M \otimes_R R', N \otimes_R R'\right),$$

for all $i \in \mathbb{N}_0$.

(iv) If N is a R_+-torsion R-module, then by choosing a *injective R_+-torsion resolution $E^\bullet$ on N in which each term is an R_+-torsion R-module for all $i \in \mathbb{N}_0$, we obtain the following homogeneous isomorphisms (see [16, Lemma 2.2])

$$\begin{aligned} H_{R_+}^i(M, N) &= H^i\left(H_{R_+}^0\left({}^*\operatorname{Hom}_R(M, E^\bullet)\right)\right) = H^i\left(\operatorname{Hom}_R\left(M, H_{R_+}^0(E^\bullet)\right)\right) \\ &\cong H^i\left(\operatorname{Hom}_R(M, E^\bullet)\right) \cong \operatorname{Ext}_R^i(M, N). \end{aligned}$$

Moreover, by [14, Lemma 3.1], for all $i \in \mathbb{N}_0$, $H_{R_+}^i(M, N)_n$ is a finitely generated R_0-module and only finitely many of $H_{R_+}^i(M, N)_n$ can be non-zero.

Throughout the paper, let $R = \bigoplus_{n \in \mathbb{N}_0} R_n$ be a $\mathbb{N}_0$-graded standard commutative Noetherian ring. We will use the notation defined in this section throughout the paper.

2. Asymptotic behavior of associated primes

Let N be a graded R-module and I be a homogeneous ideal of R. A sequence of homogeneous elements $a_1, \ldots, a_k$ of I is said to be a *homogeneous I-filter regular sequence on N* if $a_i \notin \mathfrak{p}$ for all $\mathfrak{p} \in \operatorname{Ass}_R\left(N/(a_1, \ldots, a_{i-1})N\right) \setminus V(I)$ for all $i = 1, \ldots, k$, where $V(I)$ denotes the set of prime ideals of R containing I. Also, a graded R-module $T = \bigoplus_{n \in \mathbb{Z}} T_n$ is said to be *asymptotically gap free*, if the cardinal number of

$$\left\{n \in \mathbb{Z}_{\leq 0} \,\middle|\, T_n \neq 0, T_{n+1} = 0\right\}$$

is finite (compare [6, Definition and Remark 4.1]).

PROPOSITION 2.1. *Assume that I is an ideal of R generated by elements of positive degrees. Then, for every positive integer n, there exists a homogeneous I-filter regular sequence on N of length n.*

PROOF. Let $n \in \mathbb{N}$. By [9, Lemma 1.5.10], since $I \nsubseteq \bigcup_{\mathfrak{p} \in \operatorname{Ass}(N) \setminus V(I)} \mathfrak{p}$, there exists a homogeneous element a_1 in $I \setminus \bigcup_{\mathfrak{p} \in \operatorname{Ass}(N) \setminus V(I)} \mathfrak{p}$. Again, since

$$I \nsubseteq \bigcup_{\mathfrak{p} \in \operatorname{Ass}(N/a_1 N) \setminus V(I)} \mathfrak{p},$$

continuing in this way one can obtain the required homogeneous I-filter regular sequence $a_1, \ldots, a_n$ on N. □

REMARK 1. Note that if $a_1, \ldots, a_k$ is a homogeneous I-filter regular sequence on N then, for all $i = 1, \ldots, k$,

$$\operatorname{Supp}_R\left(\frac{(a_1, \ldots, a_{i-1})N :_N a_i}{(a_1, \ldots, a_{i-1})N}\right) \subseteq V(I).$$

Therefore, by [15, Proposition 1.2] (or [20, Lemma 3.4]), we have the following isomorphisms

$$H_I^i(N) \cong \begin{cases} H^i_{(a_1,\ldots,a_k)}(N) & \text{for } 0 \le i < k, \\ H_I^{i-k}\left(H^k_{(a_1,\ldots,a_k)}(N)\right) & \text{for } k \le i. \end{cases}$$

In the following proposition we show that the above isomorphisms are homogeneous. Let $R' = \bigoplus_{n\in\mathbb{N}_0} R'_n$ be a second $\mathbb{N}_0$-graded commutative Noetherian ring. By [8, Definition 12.2.1], we say that the covariant functor $T : C(R) \longrightarrow C(R')$ has *restriction properties if

(i) whenever M is a graded R-module, the R'-module $T(M)$ is graded, and

(ii) the gradings in (i) are such that if $f : M \longrightarrow N$ is a homogeneous homomorphism of graded R-modules then $T(f) : T(M) \longrightarrow T(N)$ is homogeneous.

PROPOSITION 2.2. *Let I be a homogeneous ideal of a graded ring R. Suppose $k > 1$ and $a_1, \ldots, a_k$ is a homogeneous I-filter regular sequence on N. Then there are homogeneous isomorphisms*

$$H_I^i(N) \cong \begin{cases} H^i_{(a_1,\ldots,a_k)}(N) & \text{for } 0 \le i < k, \\ H_I^{i-k}\left(H^k_{(a_1,\ldots,a_k)}(N)\right) & \text{for } k \le i. \end{cases}$$

PROOF. Let i be an integer such that $0 \le i < k$ and, for every graded R-module N, consider the natural grading on $H_I^i(N)$. Then we can define a grading on $H^i_{(a_1,\ldots,a_k)}(N)$ such that the isomorphism $H_I^i(N) \cong H^i_{(a_1,\ldots,a_k)}(N)$ is homogeneous. In the case $0 \le i < k$, with respect to these gradings, the negative strongly connected sequence of functors $(H^i_{(a_1,\ldots,a_k)})_{0\le i<k}$ has the *restriction property. Similarly, in the case $k \le i$, $(H^k_{(a_1,\ldots,a_k)})_{k\le i}$ and so $(H_I^{i-k}(H^k_{(a_1,\ldots,a_k)}))_{k\le i}$ have the *restriction property. Now, it follows from [8, Theorem 12.3.5] that these gradings coincide with the natural ones. Thus, the desired isomorphisms are homogeneous, as required. □

REMARK 2. (See [6, Remark 3.2]) Assume that $(R_0, \mathfrak{m}_0)$ is a local ring. Set $R'_0 = R_0[x]_{\mathfrak{m}_0 R_0[x]}$, where x is an indeterminate. Then R'_0 is a faithfully flat Noetherian local R_0-algebra whose maximal ideal is $\mathfrak{m}'_0 = \mathfrak{m}_0 R'_0$. Put $R' = R'_0 \otimes_{R_0} R = \bigoplus_{n\in\mathbb{N}_0} R'_0 \otimes_{R_0} R_n$. Then R' is a positively standard graded ring, faithfully flat as an R-algebra where $R'_+ = R_+R'$. For every finitely generated graded R-module

$L = \bigoplus_{n\in\mathbb{N}_0} L_n$, set $L' = R' \otimes_R L = R'_0 \otimes_{R_0} L = \bigoplus_{n\in\mathbb{N}_0} R'_0 \otimes_{R_0} L_n$, which is a finitely generated R'-module. Since R'_0 is an R_0-flat, in view of the statement (iii) in Section 1, the faithful R_0-flatness of R'_0 gives rise to an isomorphism of R'_0-modules $H^i_{R'_+}(M', N')_n \cong H^i_{R_+}(M, N)_n \otimes_{R_+} R'_0$ for all $i \in \mathbb{N}_0$ and $n \in \mathbb{Z}$. This, together with [19, Theorem (23.2)(ii)], shows that

$$\mathrm{Ass}_{R_0}\left(H^i_{R_+}(M,N)_n\right) = \left\{ \mathfrak{p}'_0 \cap R_0 \,\middle|\, \mathfrak{p}'_0 \in \mathrm{Ass}_{R'_0}\left(H^i_{R'_+}\left(M',N'\right)_n\right)\right\}.$$

LEMMA 2.3. *Suppose $R = \bigoplus_{n\in\mathbb{N}_0} R_n$ is a positively standard graded Noetherian ring and that the base ring R_0 is local. Let M and N be finitely generated and graded R-modules and let k be a positive integer such that $H^i_{R_+}(N)$ is finitely generated for every $i < k$. Then $\mathrm{Ass}_{R_0}\left(H^k_{R_+}(M,N)_n\right)$ is asymptotically stable as $n \longrightarrow -\infty$.*

PROOF. According to Remark 2, we may replace R, M and N respectively by R', M' and N' and hence assume that the residue field $R_0/\mathfrak{m}_0$ is infinite. By Proposition 2.1, there exists a homogeneous R_+-filter regular sequence $a_1, \dots, a_{k+1}$ on N.

Now, set $S_0 = N$ and $S_i = H^i_{(a_1,\dots,a_i)}(N)$ for $i = 1, \dots, k+1$. In view of [8, Exercise 1.1.2] and Proposition 2.2, we have the homogeneous isomorphisms

$$H^0_{(a_i)}(S_{i-1}) \cong H^0_{(a_1,\dots,a_i)}(S_{i-1}) \cong H^{i-1}_{R_+}(N) \quad \text{and} \quad H^1_{(a_i)}(S_{i-1}) \cong H^i_{(a_1,\dots,a_i)}(N).$$

So, by [8, Exercise 12.4.2], we obtain the following exact sequence of homogeneous homomorphisms

$$0 \longrightarrow H^{i-1}_{R_+}(N) \longrightarrow S_{i-1} \xrightarrow{f_i} (S_{i-1})_{a_i} \longrightarrow S_i \longrightarrow 0$$

for all $i = 1, \dots, k+1$, where f_i is considered as canonical map. We may now obtain two homogeneous short exact sequences as follows

$$(2.1) \qquad 0 \longrightarrow \mathrm{Im}\, f_i \longrightarrow (S_{i-1})_{a_i} \longrightarrow S_i \longrightarrow 0 \quad \text{and}$$

$$(2.2) \qquad 0 \longrightarrow H^{i-1}_{R_+}(N) \longrightarrow S_{i-1} \longrightarrow \mathrm{Im}\, f_i \longrightarrow 0.$$

On the other hand, since the multiplication by a_i provides an automorphism on $(S_{i-1})_{a_i}$ and $H^t_{R_+}(M, (S_{i-1})_{a_i})$ is R_+-torsion, we have $H^t_{R_+}(M, (S_{i-1})_{a_i}) = 0$ for all $t \in \mathbb{N}_0$ and $i = 1, \dots, k+1$. Hence, by applying the functor $H^t_{R_+}(M, -)$ on (2.1), we obtain the homogeneous isomorphism

$$(2.3) \qquad H^t_{R_+}(M, S_i) \cong H^{t+1}_{R_+}(M, \mathrm{Im}\, f_i) \quad \text{for all } t \in \mathbb{N}_0.$$

Also, by applying the functor $H^t_{R_+}(M, -)$ on (2.2) together with (2.3), we obtain the homogeneous exact sequence

$$H^t_{R_+}\left(M, H^{i-1}_{R_+}(N)\right) \to H^t_{R_+}(M, S_{i-1}) \to H^{t-1}_{R_+}(M, S_i) \to H^{t+1}_{R_+}\left(M, H^{i-1}_{R_+}(N)\right)$$

for all $t \in \mathbb{N}$ and $i = 1, \dots, k+1$. Now, in view of [14, Lemma 3.1], the graded R-module $H^t_{R_+}(M, H^{i-1}_{R_+}(N))$ has only finitely many nonzero components, so there exists $n_1 \in \mathbb{Z}$ such that for all $n \le n_1$,

$$H^t_{R_+}(M, H^{i-1}_{R_+}(N))_n = 0 = H^{t+1}_{R_+}(M, H^{i-1}_{R_+}(N))_n.$$

This implies that there is a homogeneous isomorphism

$$H^t_{R_+}(M, S_{i-1})_n \cong H^{t-1}_{R_+}(M, S_i)_n$$

for all $t \in \mathbb{N}$, $n \le n_1$ and $i = 1, \dots, k+1$. Hence we obtain the following homogeneous isomorphism

$$H^k_{R_+}(M, N)_n \cong H^0_{R_+}(M, S_k)_n$$

for all $n \le n_1$ and so it is enough for us to show that $\mathrm{Ass}_{R_0}(H^0_{R_+}(M, S_k)_n)$ is asymptotically stable as $n \longrightarrow -\infty$. In order to do this, we claim that there is a homogeneous isomorphism $H^0_{R_+}(M, S_k) \cong H^0_{R_+}(M, H^k_{R_+}(N))$. To show this, consider the homogeneous exact sequence

$$0 \longrightarrow H^k_{R_+}(N) \longrightarrow S_k \longrightarrow (S_k)_{a_{k+1}}$$

to deduce the homogeneous exact sequence

$$0 \longrightarrow H^0_{R_+}\left(M, H^k_{R_+}(N)\right) \longrightarrow H^0_{R_+}(M, S_k) \longrightarrow H^0_{R_+}\left(M, (S_k)_{a_{k+1}}\right) = 0.$$

Thus, by [6, Lemma 5.4] in view of [9, Exercise 1.2.28], we conclude that

$$\begin{aligned}\mathrm{Ass}_{R_0}\left(H^k_{R_+}(M, N)_n\right) &= \mathrm{Ass}_{R_0}\left(H^0_{R_+}(M, S_k)_n\right) = \mathrm{Ass}_{R_0}\left(H^0_{R_+}\left(M, H^k_{R_+}(N)\right)_n\right)\\ &= \mathrm{Ass}_{R_0}\left(\mathrm{Hom}_R\left(M, H^k_{R_+}(N)\right)_n\right),\end{aligned}$$

where the last equality holds by statement (iv) of Section 1. Now the last term is asymptotically stable as $n \longrightarrow -\infty$ and the proof is complete. □

In order to state the main theorem, we need the concept of the finiteness dimension of an R-module. Let N be a finitely generated and graded R-module. Recall that the finiteness dimension of N relative to R_+ is defined by

$$f_{R_+}(N) = \inf\left\{i \in \mathbb{N} \,\middle|\, H^i_{R_+}(N) \text{ is not finitely generated}\right\}.$$

As, by [8, Proposition 15.1.5], $H^i_{R_+}(N)_n$ is finitely generated for all $i \in \mathbb{N}_0$ and all $n \in \mathbb{Z}$ and vanishes for all $n \gg 0$, we may write

$$\begin{aligned}f_{R_+}(N) &= \inf\left\{i \in \mathbb{N}_0 \,\middle|\, \#\left\{n \le 0 \,\middle|\, H^i_{R_+}(N)_n \ne 0\right\} = \infty\right\}\\ &= \inf\left\{i \in \mathbb{N}_0 \,\middle|\, \#\left\{n \in \mathbb{Z} \,\middle|\, H^i_{R_+}(N)_n \ne 0\right\} = \infty\right\}.\end{aligned}$$

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THEOREM 2.4. *Let M and N be finitely generated and graded R-modules and let $f = f_{R_+}(N) \in \mathbb{N}$. Then $\operatorname{Ass}_{R_0}\left(H^f_{R_+}(M, N)_n\right)$ is asymptotically stable as $n \longrightarrow -\infty$. Moreover, $H^f_{R_+}(M, N)$ is an asymptotically gap free R-module.*

PROOF. It is well known that, for all $i \in \mathbb{N}_0$,

$$\mathfrak{p} \in \operatorname{Ass}_R\left(H^i_{R_+}(M, N)\right) \quad \text{if and only if} \quad \mathfrak{p} \cap R_0 \in \operatorname{Ass}_{R_0}\left(H^i_{R_+}(M, N)\right),$$

and that

$$\operatorname{Ass}_{R_0}\left(H^i_{R_+}(M, N)\right) = \bigcup_{n \in \mathbb{Z}} \operatorname{Ass}_{R_0}\left(H^i_{R_+}(M, N)_n\right).$$

Also, in view of [16, Theorem 2.3], $\operatorname{Ass}_R\left(H^f_{R_+}(M, N)\right)$ is finite. So,

$$\mathscr{A} = \bigcup_{n \in \mathbb{Z}} \operatorname{Ass}_{R_0}\left(H^f_{R_+}(M, N)_n\right)$$

is finite. Suppose that $\mathfrak{p}_0 \in \mathscr{A}$. Then $H^i_{(R_{\mathfrak{p}_0})_+}(N_{\mathfrak{p}_0})$ is a finitely generated $R_{\mathfrak{p}_0}$-module for all $i < f$. Now, Lemma 2.3 implies that

$$\operatorname{Ass}_{(R_0)_{\mathfrak{p}_0}}\left(H^f_{(R_{\mathfrak{p}_0})_+}\left(M_{\mathfrak{p}_0}, N_{\mathfrak{p}_0}\right)_n\right)$$

is asymptotically stable as $n \longrightarrow -\infty$. As $\mathfrak{p}_0 \in \operatorname{Ass}_{R_0}\left(H^f_{R_+}(M, N)_n\right)$ if and only if

$$\mathfrak{p}_0(R_0)_{\mathfrak{p}_0} \in \operatorname{Ass}_{(R_0)_{\mathfrak{p}_0}}\left(H^f_{(R_{\mathfrak{p}_0})_+}\left(M_{\mathfrak{p}_0}, N_{\mathfrak{p}_0}\right)_n\right),$$

it follows that $\operatorname{Ass}_{R_0}\left(H^f_{R_+}(M, N)_n\right)$ is asymptotically stable as $n \longrightarrow -\infty$, as required in the first statement. The second statement is now obvious. □

COROLLARY 2.5. *(See [6, Proposition 5.6].) Suppose that N is a finitely generated and graded R-module and let $f = f_{R_+}(N) \in \mathbb{N}$. Then $\operatorname{Ass}_{R_0}\left(H^f_{R_+}(N)_n\right)$ is asymptotically stable as $n \longrightarrow -\infty$ and $H^f_{R_+}(N)$ is an asymptotically gap free R-module.*

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References

[1] J. Asadollahi, K. Khashyarmanesh and Sh. Salarian, 'On the finiteness properties of the generalized local cohomology modules', *Comm. Algebra* **30** (2002), 859–867.

[2] M. H. Bijan-Zadeh, 'A common generalization of local cohomology theories', *Glasg. Math. J.* **21** (1980), 173–181.

[3] M. Brodmann, 'Asymptotic behaviour of cohomology: tameness, supports and associated primes', in: *Commutative algebra and algebraic geometry, Contemp. Math.* **390** (Amer. Math. Soc., Providence, RI, 2005) pp. 31–61.

[4] M. Brodmann, S. Fumasoli and C. S. Lim, 'Low-codimensional associated primes of graded components of local cohomology modules', *J. Algebra* **275** (2004), 867–882.

[5] M. Brodmann, S. Fumasoli and R. Tajarod, 'Local cohomology over homogeneous rings with one-dimensional local base ring', *Proc. Amer. Math. Soc.* **131** (2003), 2977–2985.

[6] M. Brodmann and M. Hellus, 'Cohomological patterns of coherent sheaves over projective schemes', *J. Pure Appl. Algebra* **172** (2002), 165–182.

[7] M. Brodmann, M. Katzman and R. Y. Sharp, 'Associated primes of graded components of local cohomology modules', *Trans. Amer. Math. Soc.* **354** (2002), 4261–4283.

[8] M. Brodmann and R. Y. Sharp, *Local cohomology - an algebraic introduction with geometric applications*, volume 60 of *Cambridge studies in advanced mathematics* (Cambridge University Press, 1998).

[9] W. Bruns and J. Herzog, *Cohen-Macaulay rings*, volume 39 of *Cambridge studies in advanced mathematics* (Cambridge University Press, 1993).

[10] J. Herzog, 'Komplexe, auflösungen und dualität in der lokalen algebra', **Preprint** ???? , 1974.

[11] J. Herzog and N. Zamani, 'Duality and vanishing of generalized local cohomology', *Arch. Math. (Basel)* **81** (2003), 512–519.

[12] S. Huckaba and T. Marley, 'On associated graded rings of normal ideals', *J. Algebra* **222** (1999), 146–163.

[13] M. Katzman and R. Y. Sharp, 'Some properties of top graded local cohomology modules', *J. Algebra* **259** (2003), 599–612.

[14] K. Khashyarmanesh, 'Associated primes of graded components of gneralized local cohomology modules', *Comm. Algebra* **33** (2005), 3081–3090.

[15] K. Khashyarmanesh and Sh. Salarian, 'Filter regular sequences and the finiteness of local cohomology modules', *Comm. Algebra* **26** (1998), 2483–2490.

[16] K. Khashyarmanesh, M. Yassi and A. Abbasi, 'Filter regular sequences and generalized local cohomology modules', *Comm. Algebra* **32** (2004), 253–259.

[17] C. S. Lim, 'Graded local cohomology modules and their associated primes: the Cohen-Macaulay case', *J. Pure Appl. Algebra* **185** (2003), 225–238.

[18] ———, 'Graded local cohomology modules and their associated primes', *Comm. Algebra* **32** (2004), 727–745.

[19] H. Matsumura, *Commutative Ring Therory*, volume 8 of *Cambridge studies in advanced mathematics* (Cambridge University Press, 1986).

[20] U. Nagel and P. Schenzel, 'Cohomological annihilators and Castelnuovo-Mumford regularity', in: *Commutative algebra: syzygies, multiplicities, and birational algebra (South Hadley, MA, 1992), Contemp. Math.* **159** (Amer. Math. Soc., Providence, RI, 1994) pp. 307–328.

[21] N. Suzuki, 'On the generalized local cohomology and its duality', *J. Math. Kyoto Univ. (JAKYAZ)* **18-1** (1978), 71–85.

[22] S. Yassemi, 'Generalized section functor', *J. Pure Appl. Algebra* **95** (1994), 103–119.

[23] S. Yassemi, L. Khatami and T. Sharif, 'Associated primes of generalized local cohomology modules', *Comm. Algebra* **30** (2002), 327-330.

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