THE HALF-BAKED METHOD
for
COOKING OF EXPERIMENTAL DATA

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1. Introduction. Students in Mathematics and Physics courses are commonly exposed to a method of treating data known as the "half-table method". The method is presented with encomiums by R. C. Dearle [1] and by Lucius Tuttle and John Satterly [2]. It can best be described by an illustration; since the determination of the period of a pendulum is one of the experiments most commonly used (and occurs in both [1] and [2]), let us borrow a set of data from [2] and some explanatory text from [1].

Number of swing	Time by watch			Number of swing	Time by watch			Time for 30 swings	
	h	m	s		h	m	s	m	s
0	3	5	46	30	3	6	59	1	13
5			59	35		7	12		13
10	3	6	12	40			24		12
15			23	45			37		14
20			36	50			49		13
25			48	55		8	2		14

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We are concerned with "timing of an event which is repeated any desired number of times with a constant interval, for example, the period of a pendulum. The method itself is by no means new and may be found in many laboratory textbooks; nevertheless, it is not as widely known as it should be, nor is it properly understood by many people who know of it. (Underlining added)

"One observer can use the method if he has available an automatic timing device upon which his observations may be recorded by pressing a button or key. The observer begins to count the events in a backward order, beginning at, say, five, and counting four, three, two, one, nought, one, two, three, etc. At the count of nought the time is recorded, and again at five, ten, fifteen, up to fifty-five or any odd multiple of five.

"For a single observer, six time intervals would be measured." (See the table). "The thing that has been accomplished is equivalent to measuring the time required for 180 consecutive events, but it has been accomplished in a peculiar way. Note that in a measurement of this kind, the timing of any given group of consecutive events constitutes a single measurement, whether the group consists of five events or five hundred. The possibility of error arises in noting the time at the beginning and at the end of the group. The sole advantage gained in using a large number of events in a group is that the errors of observation are spread over a broader field so that the possible error in each event is proportionately less.

"In the example quoted above, the time might have been observed at the beginning and at the end of 180 consecutive events but this would have been tedious and would have been, in the end, only a single measurement of the desired quantity, i.e., the time of a single event. During this process, however, observations of time might have been taken at the end of the thirtieth, sixtieth, etc. events, giving six separate measurements, in each of which the errors of observation would be spread over thirty events. This would be an improvement on the single measurement since the respective reliabilities would be in the ratio $\sqrt{6} : 1$.

"Exactly the same result has been achieved in the scheme outlined without the tedium of observing 180 events, it being necessary to observe only 55. (Underlining added)

"One further advantage is that in counting straight ahead

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the same observation serves as the end of one set and the beginning of the next, while in the suggested method separate measurements are made for each group. This is, in effect, the equivalent of doubling the number of measurements and therefore of improving the accuracy nearly one and a half times."

Surely this so-called "half-table method" whereby one can obtain the same results by taking only 55 observations as by taking 180 is a marvellous invention; to a statistician, however, the claim might appear somewhat naive and indeed preposterous. Let us analyze the method a little more closely to see just what the use of the half-table method really accomplishes.

2. Analysis of the Half-Table Method. Using 12 observations, as in the example, let the observed times be y_1, y_2, y_3, ..., y_{12} (the analysis is exactly similar if 12 is replaced by 2n). The successive intervals can be named

$$y_2 - y_1 = d_1,$$

$$y_3 - y_2 = d_2,$$

$$\vdots$$

$$y_{12} - y_{11} = d_{11}.$$

The common sense approach would be to take the average as

$$(y_{12} - y_{11})/11 = (d_1 + d_2 + \ldots + d_{11})/11.$$

In the example, this gives the average time for 5 swings as 136/11 and the time for 1 swing as 136/5.11 = 2·473 seconds.

Let us now analyze the half-table method. The average time for 30 swings is found to be

$$[(y_7 - y_1) + (y_8 - y_2) + (y_9 - y_3) + \ldots + (y_{12} - y_6)]/6 .$$

Expressing the y's in terms of the d's, one obtains

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$$y_7 - y_1 = d_1 + d_2 + \ldots + d_6,$$
$$y_8 - y_2 = d_2 + d_3 + \ldots + d_7,$$
$$y_9 - y_3 = d_3 + d_4 + \ldots + d_8,$$
$$\ldots \qquad\qquad \ldots$$
$$y_{12} - y_6 = d_6 + d_7 + \ldots + d_{11}.$$

Then the average time for 30 swings is

$$(d_1 + 2d_2 + 3d_3 + 4d_4 + 5d_5 + 6d_6 + 5d_7 + 4d_8 + 3d_9 + 2d_{10} + d_{11})/6.$$

We see, from the last equation, that use of the half-table method is presumed to furnish the user with some sort of divine inspiration whereby, instead of weighting all the d_i equally and merely taking

$$(d_1 + d_2 + \ldots + d_{11})/11,$$

he is able to decide that the various d_i merit weights of 1 : 2 : 3 : 4 : 5 : 6 : 5 : 4 : 3 : 2 : 1 . Surely this is an unwarranted piece of legerdemain.

In the illustration, the time for 30 swings is obtained by the half-table method as 439/6 and the time for 1 swing as 439/180 = 2·439 seconds.

This analysis of the half-table method shows that it is a completely arbitrary scheme of assigning unjustified weights to the various differences d_i; it weights some differences much more highly than others with no valid reason whatever; in this way the user is (unwittingly) just as unscrupulous as any student who attempts to cook the results of his experiments. Indeed, the half-table method is worse than ordinary cookery since, by disguising the cookery involved and assuming a cloak of plausibility, it deceives the unwary student (or professor).

Using the analogy of the half-table method, an ingenious student might proceed one step further and argue: "In this experiment I happen to be doing, it seems to me that d_6 is just about the sort of answer I need in order to satisfy the prof; why don't I obtain my average time for 5 swings by using a geometric weighting? That would give me a period of

$$(d_1+2d_2+4d_3+8d_4+16d_5+32d_6+16d_7+8d_8+4d_9+2d_{10}+d_{11})/470."$$

His scheme would be just as valid as use of the half-table method; indeed, use of powers of 3, or of a completely random selection of coefficients, would be equally valid. Why should we discriminate against a non-symmetric scheme such as

$$(2d_1+d_2+8d_3+3d_4+7d_5+6d_6+d_7+28d_8+2d_9+2d_{10}+13d_{11})/365?$$

This has the advantage that the denominator is the number of days in the year!

3. The "Gain in Accuracy". Something must now be said about the claim in [1], quoted in section 1, that the half-table method, in the version given by the example, produces the same accuracy as would have been attained by counting 180 swings. This claim is equivalent to the belief that mathematical analysis can replace measurement, and falls in with the idea (unfortunately far too common among experimental scientists) that statisticians can take bad measurements and semi-worthless results, perform some clever mathematical tricks, and come out with a good result. This is, of course, a fairy tale. No statistical analysis can save bad data, and likewise no statistical analysis can convert 55 readings into 180.

First let us deal with the claim (Section 1), that recording readings 0, 30, 60, ... , 180, instead of only the first and the last, increases the accuracy in the ratio of $\sqrt{6}$: 1. This is simply not so. By taking the readings at 0 and 180, we are really taking an average of 180 values of the time of 1 swing (without recording the 180 separate times) and the standard error of the mean is $\sigma/\sqrt{180}$ where σ is the standard error of one value. In the case of 6 groups of 30, the standard error of the mean of a group is $\sigma/\sqrt{30}$, and the standard error of the over-all mean is $\sigma/\sqrt{30}\sqrt{6} = \sigma/\sqrt{180}$, as before. This is merely a particular case of the fundamental fact that _no grouping or manipulation of a set of data can increase the amount of information given by the data_. The amusing thing about the claim (Section 1) that recording readings at 0, 30, 60, ... , 180, instead of just recording readings at 0, 180, will increase accuracy in the ratio $\sqrt{6}$: 1 is that exactly the same numerical value will be obtained by either method. Supposedly, recording readings 0, 1, 2, ... , 180, would similarly increase the accuracy of the computed period in the ratio $\sqrt{180}$: 1 (even though the numerical values of these intermediate readings have no effect whatever on the period!)

The claim for a gain in accuracy by the half-table method is like-wise spurious. The difference is that the half-table method is itself spurious, whereas the method of recording readings at 0, 30, . . . , 180, is unobjectionable. In the latter method, 6 independent observations for the time of 30 swings are made. In the half-table method, 6 values for the time of 30 swings are indeed obtained, but these 6 values are not independent. They are intertwined one with another and posses a high degree of correlation.

4. Proper Procedure. In [2], the method of least squares is also suggested as an alternative procedure. We have seen that the half-table method is invalid; let us now consider whether least-squares can be employed.

The line of "best-fit" to $(0, y_1)$, $(5, y_2)$, . . . , $(55, y_{12})$ can easily be shown to have slope

$$b = (-11y_1 - 9y_2 - 7y_3 - 5y_4 - 3y_5 - y_6 + y_7 + 3y_8 + 5y_9 + 7y_{10} + 9y_{11} + 11y_{12})/1430$$

$$= (11d_1 + 20d_2 + 27d_3 + 32d_4 + 35d_5 + 36d_6 + 35d_7 + \ldots + 11d_{11})/1430.$$

The least-squares weighting coefficients thus stand in the ratio 11 : 20 : 27 : 32 : 35 : 36 : 35 : 32 : 27 : 20 : 11. If we are prepared to accept an arbitrary set of weights, these are perfectly satisfactory (and, of course, the arbitrary weights suggested by the half-table method are perfectly satisfactory if we take them as just arbitrary weights and do not try to fool ourselves into thinking we have obtained something for nothing and justified our work mathematically).

However, we can not justify any application of least squares here, because we are not dealing with a regression line. The various y_i are definitely correlated; they do not vary randomly and independently about a theoretical regression line. Indeed, they are effectively time values read to the nearest integral second and any random error in making the readings, if existent, is of minor importance compared to the round-off error. The true line is $y = a + bx$ and we are, in the experiment, determining the nearest points to the ends of the line which have integral co-ordinates (that is, we are finding lattice points).

This last fact indicates our best procedure. It is clear that the best slope for the line will be found (barring unpredictable and unknowable cases) by having the longest x-range. This will give us two lattice points near the line, but separated by the maximum distance. So we estimate the period by (in this example)

$$T = (y_{12} - y_1)/11.5 = (d_1 + \ldots + d_{11})/55.$$

A very simple and obvious result after all this discussion!! But we have had to reject the superficial but counterfeit charms of the half-table method and of the method of least-squares in order to end up with this estimate.

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