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MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC WARSAW 1968

A European meeting of the Association for Symbolic Logic was held in Warsaw, Poland, from August 26 to September 1, 1968, in conjunction with the Conference on the Construction of Models for Axiomatic Systems organized by the Institute of Mathematics of the Polish Academy of Sciences.

Below are reproduced abstracts of twenty-six short communications presented at the meeting.

For the Organizing Committee
A. Mostowski
A. Blikle

Gonzalo E. Reyes. *A Galois connection in definability theory.*

Let κ be an infinite cardinal such that $\kappa = \kappa^{\underset{\smile}{\kappa}}$ and let X be a set of power κ. Let C_X^κ be the (polyadic) algebra of all functions from X^κ into 2 which have support of power less than κ. Finally, let $X!$ be the topological group of all permutations of X with the natural κ-topology (i.e., the elements of the canonical basis are of the form $\{\pi \in X! : \pi \supseteq f\}$ for some partial function f having domain of power less than κ). We consider on the one hand relational structures with domain X having at most κ ξ-ary relations ($\xi \in \kappa$) and, on the other, functional polyadic subalgebras of C_X^κ which "correspond" to these structures. A polyadic algebra corresponding to an homogeneous structure is called a *Galois* algebra. (A structure $\mathfrak{A}$ is homogeneous if any isomorphism between substructures of smaller power can be extended to an automorphism of $\mathfrak{A}$.)

Theorem 1. *There is an anti-isomorphism between the lattice of the Galois subalgebras of* C_X^κ *and the lattice of the subgroups of* $X!$ *which are closed* (*in the* κ-*topology*).

Analogues of several theorems of the Galois theory for fields are obtained. Some of these, in turn, imply some definability results. A useful tool in the proofs is the following:

Theorem 2. *Let* $\mathfrak{A}$ *be a homogeneous relational structure with finitary relations and domain* X. *The topological group of the automorphisms of* $\mathfrak{A}$ (*with the* κ-*topology*) *is* κ-*Baire* (*i.e.*, $\varnothing$ *is the only open set which is the union of* κ *nowhere dense sets*).

For finite X, a result similar to Theorem 1 was obtained by A. Daigneault (see ***Transactions of the American Mathematical Society***, vol. 112 (1964), pp. 84–130, where reference to relevant work of M. Krasner can be found).

Máximo A. Dickmann. *Uniform extensions of the downward Löwenheim-Skolem theorem.*

We consider a family of generalizations of the concept of elementary extension (or substructure) of a relational system.

Definition 1. Let $\mathfrak{A}$, $\mathfrak{B}$ be structures, λ a cardinal such that $\omega \leq \lambda \leq \mathfrak{A}$; $\mathfrak{A}$ is a λ-uniform elementary substructure of $\mathfrak{B}$, in symbols $\mathfrak{A} \prec_{\lambda-u} \mathfrak{B}$, if $\mathfrak{A} \subseteq \mathfrak{B}$ and for all $X \subseteq |\mathfrak{A}|$, $\bar{\bar{X}} < \lambda$ all $p \in S_1(X)$ and all $b \in |\mathfrak{B}|$, the following holds: if b realizes p in $\mathfrak{B}$, then there is $a \in |\mathfrak{A}|$ realizing p in $\mathfrak{B}$.

If $\mathfrak{A} \prec_{\lambda-u} \mathfrak{B}$ for $\lambda = \bar{\bar{\mathfrak{A}}}$ we call $\mathfrak{A}$ a uniform elementary substructure of $\mathfrak{B}$ (in symbols $\mathfrak{A} \prec_u \mathfrak{B}$).

In the preceding definition $S_1(X)$ denotes the "Stone space" determined by the subset X of $|\mathfrak{A}|$; the rest is standard notation.

Definition 1 was suggested by recent work by Morley and others on categoricity in power; it resembles Tarski's criterion for elementary substructures. The idea is that if the relation $\mathfrak{A} \prec_{\lambda-u} \mathfrak{B}$ holds, then λ-saturatedness is reflected back from $\mathfrak{B}$ to $\mathfrak{A}$. It is clear that

(i) $\omega \leq \lambda < \kappa$ and $\mathfrak{A} \prec_{\kappa-u} \mathfrak{B}$ implies $\mathfrak{A} \prec_{\lambda-u} \mathfrak{B}$,

(ii) $\mathfrak{A} \prec_{\omega-u} \mathfrak{B}$ implies $\mathfrak{A} \prec \mathfrak{B}$, The reverse implication of (ii) does not hold; this is also true of (i) if λ is regular (I have not been able to decide the case λ singular).

The following generalization of the downward Löwenheim-Skolem theorem is proven using the generalized continuum hypothesis (GCH).

THEOREM 1 (*GCH*). *Let* $\mathfrak{B}$ *be any structure,* $X \subseteq |\mathfrak{B}|$, κ *any cardinal such that* $\max\{\overline{\overline{X}}, \overline{\overline{Sm(\mathfrak{B})}}, \omega\} < \kappa \leq \overline{\overline{\mathfrak{B}}}$; *then there is a structure* $\mathfrak{A}$ *such that* $\overline{\overline{\mathfrak{A}}} = \kappa$, $X \subseteq |\mathfrak{A}|$ *and* $\mathfrak{A} \prec_{\mathrm{cf}(\kappa)-u} \mathfrak{B}$.

It is easy to see that neither the GCH nor the strict inequality $\max\{\cdots\} < \kappa$ can be dispensed with. In fact, the preceding theorem yields "conditional" existence theorems for saturated and λ-saturated structures analogous to (but weaker than) the well-known results by Morley and Vaught. For the denumerable case the following holds.

THEOREM 2. *Let T be a theory satisfying either one of the following conditions*:

(i) *T is totally transcendental,*

(ii) *for some* $\alpha \geq 0$ *T has less than* 2^{ω} *nonisomorphic models of power* ω_{α},

(iii) *T has a denumerable saturated model.*

If $\mathfrak{B} \in \mathrm{Mod}(T)$, $X \subseteq |\mathfrak{B}|$, *and* $\overline{\overline{X}} \leq \omega$, *then there is a structure* $\mathfrak{A}$ *such that* $\overline{\overline{\mathfrak{A}}} = \omega$, $X \subseteq |\mathfrak{A}|$, *and* $\mathfrak{A} \prec_{u} \mathfrak{B}$.

A strong version of Tarski's union theorem holds simultaneously for all "uniform" concepts (for convenience we write $\prec_{1-u}$ for $\prec$).

DEFINITION 2. A class **K** of (similar) structures is uniformly directed iff for all $\mathfrak{A}, \mathfrak{B} \in \mathbf{K}$ there are $\mathfrak{C} \in \mathbf{K}$ and cardinals $\lambda, \mu > 0$ such that $\mathfrak{A} \prec_{\lambda-u} \mathfrak{C}$ and $\mathfrak{B} \prec_{\mu-u} \mathfrak{C}$. If $\mathfrak{A}, \mathfrak{B} \in \mathbf{K}$, we define

$$\lambda_{\mathfrak{A},\mathfrak{B}} = \sup\{\lambda \mid \lambda \leq \overline{\overline{\mathfrak{A}}} \text{ and } \mathfrak{A} \prec_{\lambda-u} \mathfrak{B}\} \text{ if there is one such cardinal } \lambda,$$
$$= 0 \text{ otherwise,}$$
$$\kappa_{\mathfrak{A}} = \inf\{\lambda_{\mathfrak{A},\mathfrak{B}} \mid \mathfrak{B} \in \mathbf{K} \text{ and there is } \lambda \geq 1 \text{ such that } \mathfrak{A} \prec_{\lambda-u} \mathfrak{B}\}.$$

THEOREM 3. *If* **K** *is a uniformly directed class of structures, then*

$$\mathfrak{A} \prec_{\kappa_{\mathfrak{A}}-u} \bigcup \mathbf{K} \textit{ for all } \mathfrak{A} \in \mathbf{K}.$$

We also study the connection between the "uniform" concepts and those of prime elementary extension, dimension (in the sense of Marsh), and categoricity in power; in particular we obtain two elegant characterizations of theories categorical in power ω in terms of $\prec_{\omega-u}$ and ω-saturatedness.

MÁXIMO A. DICKMANN. *The upward uniform Löwenheim-Skolem theorem: A generalization of a theorem by Morley.*

The notation is the same as in the preceding abstract. It is trivial to realize that the upward Löwenheim-Skolem theorem does not hold for the "uniform" concepts introduced before. Hence, given a structure $\mathfrak{A}$ and an infinite cardinal $\lambda \leq \overline{\overline{\mathfrak{A}}}$, exactly one of the following situations is possible:

(1) There are λ-uniform elementary extensions of $\mathfrak{A}$ of arbitrarily high powers.

(2) There is a cardinal $\kappa > \overline{\overline{\mathfrak{A}}}$ such that all λ-uniform elementary extensions of $\mathfrak{A}$ are of power less than κ. (Notice that if $\lambda \leq \mathrm{cf}(\kappa)$ and (2) holds, the downward Löwenheim-Skolem theorem of the preceding abstract implies that $\mathfrak{A}$ has λ-uniform elementary extensions, or substructures, in exactly all powers less than κ). If (2) occurs we shall denote by $c_{\lambda}(\mathfrak{A})$ the least such cardinal κ (called the λ-characteristic number of $\mathfrak{A}$); if (1) occurs, we set $c_{\lambda}(\mathfrak{A}) = \mathrm{On}$. It is easily seen that

(i) if $\omega \leq \lambda \leq \mu \leq \overline{\overline{\mathfrak{A}}}$, then $c_{\mu}(\mathfrak{A}) \leq c_{\lambda}(\mathfrak{A})$;

(ii) for every $\kappa < \beth_{\omega_1}$ there is $\mathfrak{A}$ such that $c_{\lambda}(\mathfrak{A}) \leq \kappa$, for all λ, $\omega \leq \lambda \leq \mathfrak{A}$;

(iii) if $\kappa < \beth_{\omega_1}$ is a successor cardinal, there is $\mathfrak{A}$ such that $c_{\lambda}(\mathfrak{A}) = \kappa$ for all λ, $\omega \leq \lambda \leq \overline{\overline{\mathfrak{A}}}$;

(iv) if $\mathfrak{A}$ is λ-saturated, $c_{\lambda}(\mathfrak{A}) = \mathrm{On}$.

(ii) and (iii) are simple consequences of Theorem 2.2 of Morley's *Omitting classes of elements*.

The concept of λ-uniform elementary substructure relates to the problem of omitting types of elements in the following way:

(v) $\mathfrak{A} \prec_{\lambda-u} \mathfrak{B}$ holds iff $\mathfrak{A} \subseteq \mathfrak{B}$ and $\mathfrak{B}$ omits all (partial) types belonging to $S_1(X)$, for all $X \subseteq |\mathfrak{A}|$, $\overline{\overline{X}} < \lambda$, that are already omitted by $\mathfrak{A}$. Thus we define

DEFINITION 3. Let T be a first-order theory in a language L (of any cardinality), $\{\Sigma_i \mid i \in I\}$ be a collection of sets of formulas with one (and always the same) free variable, $\mathfrak{A} \in \mathrm{Mod}(T)$. $\mathfrak{A}$ omits $\{\Sigma_i \mid i \in I\}$ iff $\mathfrak{A}$ omits Σ_i for all $i \in I$.

The following generalization of Morley's theorem on omission of classes (Theorem 3.1, *op. cit.*) can be proved in a straightforward way.

THEOREM 4. *Let T be a theory in a first-order language with ω_β symbols and $\{\Sigma_i \mid i \in I\}$ a collection of sets of sentences indexed by I without repetitions (so $\bar{\bar{I}} \leq 2^{\omega_\beta}$); let ω_γ be ω_β if $\bar{\bar{I}} < \omega_{\mathrm{cf}(\beta)}$, 2^{ω_β} if $\omega_{\mathrm{cf}(\beta)} \leq \bar{\bar{I}} \leq \omega_\beta$, $2^{2^{\omega_\beta}}$ if $\omega_\beta < \bar{\bar{I}} \leq 2^{\omega_\beta}$. If for all α, $2 \leq \alpha \leq \omega_{\gamma+1}$ there is a model of T of power $\geq 2^{\omega_\beta}_\alpha$ omitting $\{\Sigma_i \mid i \in I\}$, then in every infinite power there is a model of T omitting $\{\Sigma_i \mid i \in I\}$.*

As an immediate corollary we obtain

THEOREM 5. *Let $\mathfrak{A}$ be any structure, λ a cardinal such that $\omega \leq \lambda \leq \bar{\bar{\mathfrak{A}}}$, $\kappa = \max(\bar{\bar{\mathfrak{A}}}, \overline{\overline{\mathrm{Sm}(\mathfrak{A})}})$; if $c_\lambda(\mathfrak{A}) \geq 2^{\kappa}_{(2^{\kappa}_2)^+}$, then $c_\lambda(\mathfrak{A}) = \mathrm{On}$.*

A generalization of Theorem 5.4 of Morley's *Categoricity in power* is also studied using the methods of the present abstract; several conjectures that generalize that theorem in several possible ways and use the concept of ω-uniform elementary extension, are proposed.

Added in proof (July 1969). By the methods of these abstracts the author obtained in January 1969 the following result: *Let T be an ω_1-categorical but not ω_0-categorical theory in a countable language; if all denumerable models of T are homogeneous, then there are ω_0 isomorphism types of denumerable models of T.* (More: the members of the tower described in Morley's *Denumerable models of $\aleph_1$-categorical theories* are pairwise nonisomorphic.) Later the author learned from various sources that this result was obtained independently by several people. Most of the results in the paper *On the number of homogeneous models of a given power* by Kiesler and Morley and several other related ones can also be obtained by our methods.

JAMES W. GARSON. *A new interpretation of modality.*

It has been common in recent years to interpret modal formulas in the predicate calculus by treating propositional expressions as predicates, with the definitions $(\sim A)y = \sim(Ay)$, $(A \supset B)y = Ay \supset By$, and most characteristically, $(\Box A)y = \forall x(Ryx \supset Ax)$. As is well known, the various modal systems can be captured in predicate calculus by making the proper assumptions about the predicate R. A list of modal systems and the corresponding assumptions on R follows.

OM: $\exists z(Rxz)$
M: Rxx (Reflexivity)
S4: Rxx and $(Rxy \,\&\, Ryz) \supset Rxz$ (Transitivity)
Brouwerische: Rxx and $Rxy \supset Ryx$ (Symmetry)
S5: Rxx, $(Rxy \,\&\, Ryz) \supset Rxz$, and $Rxy \supset Ryx$.

As an alternative to invoking a predicate R, we propose introducing an operation $\circ$, and letting $(\Box A)y = \forall x(A(y \circ x))$. It then turns out that the style of modality captured depends on very standard algebraic properties of the operation, as follows.

OM: No assumptions required
M: $x \circ e = x$ (existence of identity)
S4: $x \circ e = x$ and $x \circ (y \circ z) = (x \circ y) \circ z$ (associativity)
Brouwerische: $x \circ e = x$ and $\exists z(x \circ z = e)$ (existence of inverse)
S5: $x \circ e = x$, $x \circ (y \circ z) = (x \circ y) \circ z$, and $\exists z(x \circ z = e)$.

So S5 is captured by the full theory of groups.

KAZIMIERZ WIŚNIEWSKI. *Weakened forms of the axiom of choice for finite sets.*

A set theory ZF′ differs from ZF by containing a constant 0 for the void set, an axiom stating the existence of infinite set of individuals and by restricting the axiom of extensionality to sets.

Let us consider the following sentences:

$[n]$ For every family of n-element sets there exists a choice function.

$[n]^\circ$ For every linearly ordered family of n-element sets there exists a choice function.

$[n]^*$ For every well-ordered family of n-element sets there exists a choice function.

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THEOREM 1. *If there is a group G such that order of G is divisible by a prime number p and if G has the property that for every sequence $K_1, \cdots, K_r$ of (not necessarily different) proper subgroups of G*

$$(*) \qquad \sum_{i=1}^{r} [G : K_i] \notin S,$$

then non $\vdash_{ZF'} \bigwedge_{m \in S}[m]^\circ \to [kp]$ *for* $k = 1, 2, \cdots$.

THEOREM 2. *If there are a group G and a sequence $H_1, \cdots, H_t$ of proper subgroups of G such that*

(i) $n - \Sigma_{i=1}^{t}[G : H_i]$ *is positive and divisible by p,*

(ii) *for every sequence $K_1, \cdots, K_r$ of proper subgroups of G the formula* (*) *holds,*

then non $\vdash_{ZF'} \bigwedge_{m \in S}[m]^\circ \to [n]$.

THEOREM 3. *If p and q are primes such that $p_1 = (p^q - 1)/(p - 1)$ is prime, then* non $\vdash_{ZF'} [n]^\circ \to [n]$ *for* $n \in (p_1, p)_s - (p_1, p^q)_s$. *($(a, b)_s$ denotes the additive semigroup generated by a and b).*

THEOREM 4. *If for every prime number p there are infinitely many q's such that $(p^q - 1)/(p - 1)$ is prime, then $\bigvee_{n \in T}[n]$ is independent from* ZF′ $\cup$ $\{[n]^\circ : n > 1\}$ *where T is any finite nonempty set of integers* >1.

THEOREM 5. *If for any sequence $K_1, \cdots, K_r$ of proper subgroups of G the formula* (*) *holds, then* non $\vdash_{ZF'} \bigwedge_{m \in S}[m]^\circ \to \bigvee_{n \in T}[n]$, *where every element of T is a sum of indices of proper subgroups of G.*

THEOREM 6. *If p and q are primes such that $p_1 = (p^q - 1)/(p - 1)$ is prime, then* non $\vdash_{ZF'} \bigwedge_{m \in S}[m]^\circ \to \bigvee_{n \in T}[n]^*$, *where S is a subset of $(p_1, p)_s - (p_1, p^q)_s$ and T is any finite nonempty subset of $(p_1, p^q)_s$.*

THEOREM 7. *No Mostowski's model* (***Fundamenta mathematicae***, vol. 33 (1945), pp. 137–168) *which is defined with the help of a group of degree* 15 *can be used to prove independence of* [15] *from* [3], [5] *and* [13].

H. LEBLANC AND R. K. MEYER. *Truth-value semantics for the theory of types.*

Take the (simple) theory of types—T, for short—to be axiomatized as in Quine's ***Set theory and its logic***, p. 331. Where A is a wff of T and X a variable of T of type t, take $\{X: A\}$ to be defined as on p. 259 of Quine's book, and count $\{X: A\}$ as an abstract of T of type $t + 1$. Where A is a wff of T, and X and Y are (not necessarily distinct) variables of T of the same type t, take $A[Y/X]$ to be the result of replacing every free occurrence of X in A by an occurrence of Y if no component of A of the sort $(\forall Y)B$ contains a free occurrence of X; otherwise, take $A[Y/X]$ to be $A^1[Y/X]$, where A^1 is the result of replacing every occurrence of Y in every component of A of the sort $(\forall Y)B$ that contains a free occurrence of X in A by an occurrence of the alphabetically earliest variable of T of type t that is foreign to that component of A. Where A is a wff of T, X a variable of T of nonzero type t, and $\{Y: B\}$ an abstract of T of type t, suppose $A[\{Y: B\}/X]$ to be similarly accounted for. Where α is a function from the set of wffs of T to $\{\top, \bot\}$, count α as a general truth-value function for T if

(a) $\alpha(\sim A) = \top$ if and only if $\alpha(A) = \bot$,

(b) $\alpha(A \supset B) = \top$ if and only if $\alpha(A) = \top$ or $\alpha(B) = \top$,

(c) where X is a variable of T of type 0, $\alpha((\forall X)A) = \top$ if and only if $\alpha(A[Y/X]) = \top$ for every variable Y of T of type 0,

(d) where X is a variable of T of nonzero type t, $\alpha((\forall X)A) = \top$ if and only if $\alpha(A[K/X]) = \top$ for every variable and abstract K of T of type t, and

(e) where X and Y are distinct variables of T of nonzero type t, Z is a variable of T of type $t - 1$, and Z' one of type $t + 1$, $\alpha((\forall Z)(Z \in X \equiv Z \in Y)) = \top$ if and only if $\alpha((\forall Z')(X \in Z' \equiv (Y \in Z'))) = \top$.

And take a wff A of T to be generally valid if $\alpha(A) = \top$ for every general truth-value function α for T. It is readily shown, using results of Henkin, Beth, and Leblanc, that a wff of T is provable in T if and only if generally valid. This account of general validity reminiscent of one of Schütte may be found less *ad hoc* than Henkin's own, and—in that it makes no mention of models—of some philosophical interest. The circumstances under which a wff of T is valid in the standard sense (as opposed to generally valid) can likewise be spelled out without mention of models.

T. C. POTTS. *The logical description of changes which take time.*

Changes which take time can be described in English by means of the auxiliary verb "come to": thus from the schema "$A\ \phi s$" we can obtain the schemas "A is coming to ϕ" (continuous aspect), "A has come to ϕ" (perfective aspect) and "A came to ϕ" (past tense). These three schemas provide the subject-matter of this paper, but to simplify the problem I shall restrict consideration to cases where "ϕ" is of the form "be ψ", e.g. "A is coming to be ψ", "come to be" then contracts to "become".

If "is coming to" is represented by a monadic operator "$\Delta(\ )$", then if "p" stands for "A is ϕ", "Δp" will represent "A is becoming ϕ". With this interpretation of "p", the following system is proposed:

$$\Delta\leftrightarrow \quad (\alpha)\quad(\beta) \qquad\qquad \Delta\wedge+$$

$$\frac{\Delta\alpha \quad \beta \quad \alpha}{\Delta\beta} \qquad\qquad \frac{\Delta\alpha \quad \Delta\beta}{\Delta(\alpha\wedge\beta)}$$

provided that "β" depends on no premises other than "α" and that "α" depends on no premises other than "β"

$$\Delta\neg\ \frac{\Delta\alpha}{\neg\alpha} \qquad\qquad \neg\Delta\ \frac{\neg\Delta\alpha}{\Delta\Delta\alpha}.$$

Problems about combining this system with tense-operators are raised, and a difficulty in the way of regarding "is coming to" as a propositional operator. Two senses of the schemas "A has come to be ϕ" and "A came to be ϕ" are distinguished, and a consequent difficulty about their symbolic representation raised.

S. FAJTLOWICZ. *Birkhoff theorem in category of general algebras.*

By a general algebra or, shortly, algebra we mean any sequence $(A_n)_{n\in\omega}$ where A_0 is an arbitrary, nonempty set, and A_n (for $n \geq 1$) is a family of operations of n variables on the set A_0, such that

(i) the trivial operations $e_i^n(x_1, \cdots, x_n) = x_i$ belong to A_n,

(ii) if $f_1, \cdots, f_k \in A_n$, $f \in A_k$, then the operation

$$f(f_1, \cdots, f_k)(x_1, \cdots, x_n) = f(f_1(x_1, \cdots, x_n), \cdots, f_k(x_1, \cdots, x_n))$$

belongs to A_n.

Let $\mathfrak{A} = (A_n)_{n\in\omega}$ and $\mathfrak{B} = (B_n)_{n\in\omega}$ be general algebras. A sequence of mappings $(h_n)_{n\in\omega}$, such that $h_n\colon A_n \to B_n$ will be called a homomorphism of $\mathfrak{A}$ into $\mathfrak{B}$ if for every $f \in A_n$, $n = 1, 2, \cdots, x_1, \cdots, x_n \in A_0$, the equality

$$(h_n f)(h_0 x_1, \cdots, h_0 x_n) = h_0 f(x_1, \cdots, x_n)$$

holds.

Thus we have defined a category. It will be denoted by $\mathscr{A}$.

In this category there exists the product.

A class of algebras will be called a variety if it is closed with respect to operations of product, homomorphic image, and reduct.

Let $\tau_i(f_1, \cdots, f_k, x_1, \cdots, x_n)$ be terms in which f_i are the symbols of operations of n_i variables.

We say that in an algebra $\mathfrak{A}$ the equality $\tau_1 = \tau_2$ holds if in this algebra the sentence

$$\forall_{f_1\in A_{n_1}} \cdots \forall_{f_k\in A_{n_k}} \forall_{x_1\in A_0} \cdots \forall_{x_n\in A_0} \tau_1(f_1, \cdots, f_k, x_1, \cdots, x_n) = \tau_2(f_1, \cdots, f_k, x_1, \cdots, x_n)$$

holds.

A class of algebras is called equationally defined if it is the class of all algebras satisfying some system of equalities.

In category $\mathscr{A}$ holds the analogue of

BIRKHOFF THEOREM. *The class of algebras is equationally defined iff it is a variety.*

ANGUS MACINTYRE. *Complete theories of topological fields with distinguished dense proper subfields.*

For notation and background one should consult Keisler's paper (***Michigan mathematical journal***, vol. 2 (1964), pp. 71–81).

THEOREM 1. (a) *If A_1, A_2, B_1, B_2, are real-closed fields and B_1, B_2 are dense, proper subfields of A_1, A_2, respectively, then $(A_1, B_1) \equiv (A_2, B_2)$.*

(b) *Suppose p is a prime. If A_1, A_2, B_1, B_2, are p-adically closed fields (i.e. elementarily equivalent to the valued field of p-adic numbers), and B_1, B_2 are dense, proper subfields of A_1, A_2, respectively, then $(A_1, B_1) \equiv (A_2, B_2)$.*

(c) *If A_1, A_2, B_1, B_2, are Henselian-valued fields with residue-class fields of characteristic zero, and $A_1 \equiv A_2 \equiv B_1 \equiv B_2$, and B_1, B_2 are dense, proper subfields of A_1, A_2, respectively, then $(A_1, B_1) \equiv (A_2, B_2)$.*

In the proofs we use the method of saturated models. For (b) and (c) we make use of fundamental work of Ax and Kochen, and Ersov. (a) was already known (A. ROBINSON, ***Fundamenta mathematicae***, vol. 47 (1959), pp. 179–204) but our treatment is much simpler. In particular, no use is made of the concept of linear disjointness.

THEOREM 2. (a) *If A_1, A_2, B_1, B_2 are real-closed fields, and B_1, B_2 are dense, proper subfields of A_1, A_2, respectively, and $(A_1, B_1) \subseteq (A_2, B_2)$, and A_1 and B_2 are linearly disjoint over B_1, then $(A_1, B_1) \prec (A_2, B_2)$.*

(b) *Suppose p is a prime. If A_1, A_2, B_1, B_2 are p-adically closed fields, and B_1, B_2 are dense, proper subfields of A_1, A_2, respectively and $(A_1, B_1) \subseteq (A_2, B_2)$, and A_1 and B_2 are linearly disjoint over B_1, then $(A_1, B_1) \prec (A_2, B_2)$.*

There is an analogue of Theorem 1(c). 2(a) was proved by Robinson (loc. cit.), but our method is simpler.

G. M. WILMERS. *Some problems related to the elementary equivalence of constructible models of set theory.*

The starting point of this work is the paper by A. MOSTOWSKI, ***Acta philosophica fennica***, vol. 18 (1965), pp. 135–144, quoted hereafter as (1).

If x is a set, let $D(x)$ denote the set of elements definable in $\langle x, \epsilon\rangle$. In (1) Mostowski defines a function f: Ord $\to$ Ord such that if $\langle F''\alpha, \epsilon\rangle$ is a model of ZF, $f(\alpha)$ is the unique β such that $\langle F''\beta, \epsilon\rangle$ and $\langle D(F''\alpha), \epsilon\rangle$ are isomorphic. He then proves that for such α,

$$\alpha = f(\alpha) \leftrightarrow D(F''\alpha) = F''\alpha.$$

Further to this we show that $f(\alpha)$ can be characterized as the least β such that $\langle F''\beta, \epsilon\rangle \equiv \langle F''\alpha, \epsilon\rangle$.

We define the index of an ordinal α, $I(\alpha)$, as the ordinal corresponding to the order type of the class of all β such that $\langle F''\beta, \epsilon\rangle \equiv \langle F''\alpha, \epsilon\rangle$, under the natural ordering. Our main result is that if λ is a strongly definable ordinal (defined in (1)), and there are less than λ ordinals $\beta < \lambda$ such that $\langle F''\beta, \epsilon\rangle$ is a model of ZF, then there exists an α such that $\langle F''\alpha, \epsilon\rangle$ is a model of ZF and $I(\alpha) = \lambda$.

MAURICE BOFFA. *Sur l'existence d'ensembles niant le Fundierungsaxiom.*

Notons Σ le système axiomatique de la théorie des ensembles comprenant les axiomes A, B et C de Gödel.

Pour toute relation R posons: $x\,R\,y \Leftrightarrow (x, y) \in R$; $x_R = \{y \mid y\,R\,x\}$; $\Delta R = \{x \mid (\exists y)(x\,R\,y \vee y\,R\,x)\}$; R est *extensionnelle* $\Leftrightarrow (\forall xy)((x \in \Delta R \wedge y \in \Delta R \wedge x_R = y_R) \Rightarrow x = y)$; S est une *restriction transitive* de $R \Leftrightarrow S \subset R \wedge (\forall x)(x \in \Delta S \Rightarrow x_S = x_R)$; R est un *graphe* $\Leftrightarrow (\forall x)M(x_R)$. Nous dirons qu'un graphe extensionnel R est *universel* si et seulement si tout graphe extensionnel g est isomorphe à une restriction transitive de R.

THÉORÈME 1. *Dans Σ, on peut construire un graphe extensionnel universel U tel que* $(\forall x)(x \in \Delta U \Rightarrow x_U \notin \Delta U)$.

POSONS. N $\Leftrightarrow$ toute classe propre est équipotente à l'univers; In $\Leftrightarrow$ il existe au moins un cardinal fortement inaccessible; F $\Leftrightarrow$ la relation d'appartenance est un graphe extensionnel universel.

THÉORÈME 2. *Dans* Σ + D + E (+In), *la permutation de l'univers transposant* x *et* x_U *(pour tout* $x \in \Delta U$*) détermine un modèle de Rieger*[1] *du système* Σ + N + F (+In). *Par conséquent, si* Σ (+E + In) *est consistant, alors* Σ + N + F (+In) *l'est aussi.*[2]

POSONS. F* $\Leftrightarrow$ tout graphe extensionnel G non équipotent à l'univers est isomorphe à une restriction transitive de la relation d'appartenance.

THÉORÈME 3. *Dans* Σ + N + F + In, *la classe des ensembles dont la fermeture transitive est de puissance* $< \xi$ *(le plus petit cardinal fortement inaccessible) détermine un modèle supercomplet du système* Σ + E + non N + F*. *Par conséquent, si* Σ + E + In *est consistant, alors* Σ + E + non N + F* *l'est aussi.*

YOSHINDO SUZUKI. *Orbits of denumerable models of complete theories.*

A topology for the denumerable models of a first-order countable theory was introduced and studied by Grzegorczyk, Mostowski and Ryll-Nardzewski. By orbits we mean quotient classes of models with respect to isomorphisms. The orbit consisting of the prime models is called prime. Suitably generalizing results mentioned above and making use of results of Vaught, we can prove the following theorems for complete theories.

THEOREM 1. *Each nonprime orbit is meager.*

THEOREM 2. *The prime orbit is a comeager* G_δ*-set.*

Some examples from β-models of analysis and well-founded models for set theory are mentioned.

A. B. SLOMSON. *An undecidable two-sorted predicate calculus.*

Let L be the first-order predicate language with two sorts of variables and just one dyadic predicate letter whose first place is to be filled by the variables of one sort and whose second place is to be filled by the variables of the other sort. In answer to a question of M. H. Löb we show that there is no decision procedure for determining whether or not a sentence of L is universally valid.

The method used to obtain this result also yields as a consequence that the modal predicate calculus S5* with just *one* monadic predicate letter is undecidable. This strengthens the result of Kripke, ***Zeitschrift für mathematische Logik und Grundlagen der Mathematik***, vol. 8 (1962), pp. 113–116.

CHARLES F. MILLER, III. *On a problem of Graham Higman.*

Higman has shown the existence of a universal finitely presented [f.p.] group U, i.e., a f.p. group U which contains an isomorphic copy of every finitely presented group. In particular, U contains an isomorphic copy of every f.p. group with solvable word problem [WP], but U itself has unsolvable WP of degree $\mathbf{0}'$. We are led to the following question: Does there exist a universal f.p. "solvable WP" group, i.e., a f.p. group S with solvable WP which contains an isomorphic copy of every f.p. group with solvable WP? We show such a group does not exist.

For suppose such an S did exist. Let G be a f.p. group with solvable WP. Using the algorithm $A(S)$ which solves the WP for S, we can enumerate the set of homomorphisms from G into S. Let w be a word of G. If $w \neq 1$ in G, then for some homomorphism $h(w) \neq 1$ in S, which can be tested using $A(S)$. Hence, we can enumerate the set of words $w \neq 1$ in G. Since the set of words $w = 1$ in G can be enumerated, this solves the WP for G. We thus obtain a partial algorithm which, when applied to a f.p. group with solvable WP, solves the WP for that group. This contradicts a theorem of Boone and Rogers, and so the desired S could not exist.

A. WOJCIECHOWSKA. *Generalized limit powers.*

I is any nonempty set and F a filter of subsets of I^2. By $S(I, F)$ we denote a structure $\langle 2^I \mid F, 1, \cap, \cup, \sim, M_\xi \rangle_{\xi < \alpha}$, where $\langle 2^I \mid F, 1, \cap, \cup, \sim \rangle$ is a limit power of a two-element Boolean algebra $\mathbf{2}$. For any set $S \subseteq I$ we denote by S^* the characteristic function of S with values in $\mathbf{2}$. Let $\mathfrak{A} = \langle A, R_\xi \rangle_{\xi < \beta}$ be any relational structure.

[1] Cf. ***Czechoslovak mathematical journal***, vol. 7 (1957), p. 344.

[2] P. Hájek (***Zeitschrift für mathematische Logik und Grundlagen der Mathematik***, vol. 11 (1965), pp. 103–115) a établi, par une autre méthode, la consistance d'une autre version de F.

LEMMA. *For any* $f_1, \cdots, f_k \in A^I \mid F$ *and any formula* $\vartheta(v_1, \cdots, v_k)$ *of the language of* $\mathfrak{A}$ *we have*

$$\{i: \mathfrak{A} \models \vartheta(f_1(i), \cdots, f_k(i))\}^* \in 2^I \mid F.$$

For any formula $\Phi(X_1, \cdots, X_n)$ of the language of $S(I, F)$ and formulas $\vartheta_1, \cdots, \vartheta_n$ of the language of $\mathfrak{A}$ whose free variables are $v_1, \cdots, v_k$, we define a k-ary relation Q on $A^I \mid F$ putting $\langle f_1, \cdots, f_k \rangle \in Q$ iff

(§) $$S(I, F) \models \Phi[\{i: \mathfrak{A} \models \vartheta_1(f_1(i), \cdots, f_k(i))\}^*, \cdots, \{i: \mathfrak{A} \models \vartheta n(f_1(i), \cdots, f_k(i))\}^*].$$

Z will be any set of such relations Q, where Φ, n, $\vartheta_1, \cdots, \vartheta_n$ and k are arbitrary.

THEOREM. *There is an effective procedure whereby to each formula* $\Gamma(x_1, \cdots, x_k)$ *of the language of* $\langle A^I \mid F, Q\rangle_{Q \in Z}$ *can be correlated a sequence* $\langle \Phi, \vartheta_1, \cdots, \vartheta_k \rangle$ *that*

$$\langle A^I \mid F, Q\rangle_{Q \in Z} \models \Gamma(f_1, \cdots, f_k) \quad \textit{iff (§) holds.}$$

This is a natural refinement of a well-known theorem of Feferman and Vaught.

Let us formulate, e.g., the following application of our result.

COROLLARY. *If G is a filter of subsets of* I^2 *including F and if* $2^I \mid F \prec 2^I \mid G$ *then for any structure* $\mathfrak{A}$ *we have* $\mathfrak{A}^I \mid F \prec \mathfrak{A}^I \mid G$. *($\prec$ denotes elementary inclusion.)*

JAN WASZKIEWICZ AND B. WĘGLORZ. *Remarks on theories of reduced powers.*

Let T be a complete theory, I a nonempty set and D a filter of subsets of I. Then for any model $\mathfrak{A}$ of T the theory $\mathrm{Th}(\mathfrak{A}^I_D)$ of the reduced power is the same and will be denoted by T^I_D By **2** we denote a two-element Boolean algebra.

THEOREM 1. *If T is a countable theory which is* $\aleph_0$*-categorical then* T^I_D *has the same property provided the set of atoms of the Boolean algebra* $\mathbf{2}^I_D$ *is finite.*

The supposition that $\mathbf{2}^I_D$ has finitely many atoms is essential. Easy examples show that no result of that kind holds for $\aleph_1$-categorical theories.

THEOREM 2. *If* $\mathfrak{A}$ *is the countable model of an* $\aleph_0$*-categorical theory T, then each countable model of* T^{ω}_F*, where F is the filter of all cofinite subsets of* ω*, is isomorphic to* $\mathfrak{A}^{\mathfrak{c}}$*, which is a substructure of a direct power of* $\mathfrak{A}$ *consisting of all continuous functions from the Cantor set* $\mathfrak{c}$ *to* $\mathfrak{A}$ *endowed with a discrete topology.*

These results can be applied to obtain a simple construction of an $\aleph_0$-universal partial ordering (Mostowski, 1938).

THEOREM 3. $\langle \eta, < \rangle^{\mathfrak{c}}$ *is an* $\aleph_0$*-universal partial ordering.*

Proofs and other applications are prepared for publication.

B. WĘGLORZ. *On models of theories of reduced powers.*

We use the notation and terminology of H. J. Keisler's *Limit ultrapowers*, ***Transactions of the American Mathematical Society***, vol. 107 (1963), pp. 382–408. **2** denotes a two-element Boolean algebra, I and J denote nonempty sets and D a filter of subsets of I.

THEOREM 1. *If* $\mathbf{2}^I_D \equiv \mathbf{2}^J \mid F$ *then for every structure* $\mathfrak{A}$, $\mathfrak{A}^I_D \equiv \mathfrak{A}^J \mid F$. *(F is any filter of subsets of* J^2.*)*

COROLLARY 1. *If* $\mathfrak{A}$ *is a countable structure (which may have uncountably many relations) then* $\mathrm{Th}(\mathfrak{A}^I_D)$ *has a countable model.*

THEOREM 2. *If F is a filter containing all equivalence relations* $\sim$ *over I such that* $I/\sim$ *is finite then* $\mathfrak{A}^I_D \mid F \prec \mathfrak{A}^I_D$.

This result also follows from a theorem of A. Wojciechowska—see her abstract.

REMARKS. Let now F be the filter of finite partitions of I. Then

(1) If $\mathbf{2}^I_D$ is finite and has 2^n elements then $\mathfrak{A}^I_D \mid F \cong \mathfrak{A}^n$.

(2) If $\mathbf{2}^I_D$ is infinite then $\mathfrak{A}^I_D \mid F$ is a union of a directed system of finite powers of $\mathfrak{A}$.

COROLLARY 2. (a) *If an* $\forall\exists$*-class is closed under finite direct powers then it is closed under reduced powers.*

(b) *If an* $\forall\exists$*-class is closed under finite direct products then it is a Horn class.*

Stronger results on $\forall\exists$-classes were obtained by Weinstein, ***Notices of the American Mathematical Society***, vol. 11 (1964), p. 391.

GABOR T. HERMAN AND STEPHEN D. ISARD. *Computability over arbitrary fields.*

The usual practice in attempts to make precise the concept of a computable function over a field F is to require that the elements of F should be in some sense effectively describable, and hence that F itself should be countable (see Rabin [1]). From some points of view, this is an awkward restriction. For instance, we may wish to consider a function f on the reals "computable" if an engineer can produce a device whose output wire will carry current whose measure is $f(x)$ when its input wire carries current whose measure is x.

In this paper we propose a definition of computability over arbitrary fields F. We base it on the Shepherdson-Sturgis [2] concept of an unlimited register machine. The registers are to contain elements of F. If Σ is a set of functions over F, a Σ-program for the machine is a sequence of instructions which either change the contents of registers by applying functions of Σ to them, or cause the program to "jump" if a certain register contains 0. A function is Σ-computable if there is a Σ-program which computes it.

As an example we show that if Σ is the set of real functions $\langle +, -, \times, ^{-1}, \lambda\rangle$, where λ is the characteristic function of $\leq$, the set of $n \times n$ matrices A with real entries such that $A^j = I$, for some j, is Σ-computable if and only if $n = 1$. For the complex numbers and a natural extension of Σ, the corresponding predicate is not computable even if $n = 1$. This result has significance for the problem of state accessibility for infinite linear sequential machines [3].

REFERENCES. [1] M. O. RABIN, *Computable algebra, general theory and theory of computable fields*, ***Transactions of the American Mathematical Society***, vol. 87 (1960), pp. 341–360.

[2] J. C. SHEPHERDSON and H. E. STURGIS, *Computability of recursive functions*, ***Journal of the Association for Computing Machinery***, vol. 10 (1963), pp. 217–255.

[3] H. GALLAIRE, J. N. GRAY, M. A. HARRISON, and G. T. HERMAN, *Infinite linear sequential machines*, ***Journal of computer and system sciences***, vol. 2 (1968) pp. 381–419.

ALAN ROSS ANDERSON. *Completeness and decidability of a fragment of the system E of entailment.*

Definitions of *positive* and *negative part* are as in L. L. Maksimova, *Nékotoryé voprosy isčisléniá Akkérmana*, ***Doklady Akadémii Nauk SSSR***, vol. 175 (1967), pp. 1222–1224; English transl., ***Soviet mathematics***, vol. 8, no. 4 (1967), pp. 997–999. We also define *truth-functional part* (tfp): A is a tfp of A, and if $\bar{B}\{B \,\&\, C, B \vee C\}$ is a tfp of A, so is {are} B {B and C}.

LEMMA. *Every wff of* E *has an intensional conjunctive normal form* $B_1 \,\&\, B_2 \,\&\, \cdots \,\&\, B_m$ *each* B_i *having the form* $C_1 \vee C_2 \vee \cdots \vee C_n$, *where each* C_j *is either* (*a variable*) p, *or* $\bar{p}$, *or of the form* $D \to E$ *or* $\overline{D \to E}$. *Moreover* C_j *is a positive* {*negative*} *part of at least one of the* B_i *iff it is a positive* {*negative*} *part of* A.

By a *positive-entailment-free* ("pef") formula A of E, we mean a formula containing no positive parts of the form $D \to E$. Any such formula A may be rewritten equivalently as $B_1 \,\&\, B_2 \,\&\, \cdots \,\&\, B_m$ in the lemma above, so provability of A reduces to provability of B_i for each i. We then use techniques of Anderson and Belnap (this JOURNAL, vol. 24 (1959), pp. 301–302), supplemented by a Rule 3: from $\phi(F)$ and $\phi(\bar{G})$ to infer $\phi(\overline{F \to G})$, to construct trees in an attempt to prove each B_i. If the tree-construction terminates in axioms (as in the paper just cited), A is provable in E; if on the other hand the tree has a branch which terminates in a nonaxiom, all formulas in this bad branch can be falsified in a domain of two (intensional) propositions (one of which is true, the other false). It follows that the pef fragment of E is both decidable and complete.

Decision procedures and completeness results are known for several fragments of E (zero-degree formulas, first-degree entailments, first-degree formulas; see Belnap in this JOURNAL, vol. 32 (1967), pp. 1–22). Such interest as the present paper has stems from the fact that it gives, for a well-defined fragment of E, the first decidability and completeness results which do *not* depend in any way on the amount of nesting of arrows within arrows in the formulas under consideration.

ALBERT CHAUBARD. *Remarques sur la définissabilité explicite.*

Soit T une théorie du calcul des prédicats du premier ordre avec égalité renfermant les prédicats $r_0, r_1, \cdots, r_k$. On désigne par T_0 la théorie dont les théorèmes sont ceux de T où

r_0 n'occure pas. Soient $\mathcal{M}$ la classe des modèles de T et $\mathcal{M}_0$ celle des modèles de T_0. Désignons par F l'application de $\mathcal{M}$ vers $\mathcal{M}_0$ qui à $M = \langle U, R_0, R_1, \cdots, R_k\rangle$ associe $F(M) = \langle U, R_1, \cdots, R_k\rangle$. Les résultats intéressent la surjectivité de F.

THÉORÈME 1. *Si la théorie T_0 est complète et si la fonction F n'est pas surjective il existe un modèle M_0 de T_0 tels que*

(i) $M_0 \notin F(\mathcal{M})$.

(ii) *M_0 à une extension élémentaire M_0', isomorphe à une ultralimite de M_0, qui appartient à $F(\mathcal{M})$.*

Une théorie T est existentiellement close si, lorsque $T \vdash \exists x\beta(x)$ il existe une constante d'individu a de T tel que $T \vdash \beta(a)$.

Une extension T' de T obtenue à partir de T en ajoutant un ensemble U de constantes d'individus est une description d'un modèle M de T si T' est compatible, complète et existentiellement close.

THÉORÈME 2. *Soit $M_0 \in \mathcal{M}_0$. Pour que $M \in F(\mathcal{M})$ il faut et il suffit qu'il existe une extension T' de $T \cup D(M_0)$ compatible, complète et existentiellement close sur son langage, telle que T' soit une description d'un modèle M de T telle que $M_0 = F(M)$.*

Un énoncé α est sémantiquement achevé dans une théorie T si T est une description d'un modèle de α.

THÉORÈME 3. *Une extension compatible, complète T' de $T \cup D(M_0)$ telle que T_0' est une description de M_0 est existentiellement close si et seulement si les théorèmes de T' en r_0 seul sont sémantiquement achevés dans T'.*

J. BARZDIN. *On the complexity of the initial segments of recursively enumerable sets.*

The concept of the complexity of words was introduced by A. Kolmogorov. Let $\phi(p)$ be a recursive function (x and $\phi(p)$ are words over the alphabet $\{0, 1\}$). Let $l(p)$ be the length of the word p. For every word x over the alphabet $\{0, 1\}$ let us define

$$K_\phi(x) = \min_{\phi(p)=x} l(p).$$

It is known that there exists a recursive function $A(x)$ such that for any recursive function $\phi(x)$

$$K_A(x) \leq K_\phi(x) + C_\phi,$$

where C_ϕ is a constant not dependent on x. The complexity $K(x)$ of the word x is defined as $K_A(x)$ where $A(x)$ is a fixed function of the type given above.

Let M be a set of natural numbers. The initial segment M_n of the set M is the collection of all elements s such that $s \in M$ and $s \leq n$ Let $\overline{M}_n$ be a word $a_1a_2 \cdots a_n$, such that $a_i = 1$ if $i \in M_n$ and $a_i = 0$ if $i \notin M_n$. By the complexity $K(M_n)$ of the initial segment M_n we mean the complexity $K(\overline{M}_n)$.

It is shown by P. Martin-Löf that for almost all binary sequences or what is the same—for almost all sets M of natural numbers there is $K(M_n) \geq n$.

I consider the complexity of the initial segments of recursively enumerable sets.

THEOREM 1. *For any recursively enumerable set M and for any natural n there holds*

$$K(M_n) \leq 2 \log_2 n + C_M,$$

where C_M is a constant not dependent on n.

For any recursively enumerable set M there exists an infinite amount of natural n such that

$$K(M_n) \leq \log_2 n + C_M,$$

where C_M is a constant not dependent on n.

THEOREM 2. *There exists a recursively enumerable set M (the set of maximal complexity) such that for any natural n*

$$K(M_n) \geq \log_2 n - C,$$

where C is a constant not dependent on n.

Further the complexity of the initial segments of recursively enumerable sets is investigated with restrictions on the allowed difficulty of transforming the "program" p into the word x.

Let us take the number of steps $A^+(p)$ of the Turing machine which computes $A(p)$ as the measure of difficulty of such transforming. Let $t(x)$ be a general recursive function (where x is a word and $t(x)$ a number). Let us define

$$K^t(x) = \min_{A(p)=x;\, A^+(p) \le t(x)} l(p).$$

THEOREM 3. *There exists a recursively enumerable set M such that for any general recursive function $t(x)$ there holds*

$$K^t(M_n) \geq C_t n,$$

where C_t is a constant not dependent on n.

This estimate cannot be essentially improved.

Further the question is investigated which of the sets naturally appearing in the theory of algorithms are of maximal complexity. The complexity of the initial segments of some real problems (recursively enumerable sets) is also investigated.

CHARLES F. MILLER, III. *Transfers of word and conjugacy problems.*

A technique is given for passing from a finitely presented group H with a word problem [WP] of r.e. degree D to a finitely presented group G with solvable WP but conjugacy problem [CP] of degree D. This gives a new proof of a theorem of Collins—our proof uses a mild generalization of his methods. Conversely, under certain circumstances, a method is given for passing from an unsolvable CP to an unsolvable WP. The construction can be modified to establish the following

THEOREM 1. *The free product of two free groups with finitely generated amalgamation has solvable WP, but may have unsolvable CP.*

THEOREM 2. *The automorphism group A of a free group of finite rank ≥ 5 has, for each r.e. degree D, a finitely generated subgroup B_D such that the membership problem for B_D has degree D (so-called generalized WP). A is known to be residually finite and to have solvable WP.*

RAIMO TUOMELA. *On eliminability and definability of auxiliary concepts in first-order theories.*

The present paper is mainly an outgrowth of some results concerning deductive interpolation in the theory of distributive normal forms by Jaakko Hintikka (*Distributive normal forms and deductive interpolation, Zeitschrift für mathematische Logik und Grundlagen der Mathematik*, vol. 10 (1964), pp. 185–191). Consider a first-order theory with $\lambda + \mu$ as its set of extra-logical parameters. Denote its distributive normal form at depth d by $T^{(d)}(\lambda + \mu)$ and by $T^{(d)}(\lambda + \mu)(\lambda)$ the subtheory obtained from it by omitting all the members of μ. The depth of a sentence is here defined to be the maximal length of sequences of nested quantifiers occurring in it. It can be shown by a separation result in the theory of distributive normal forms (a generalization of Craig's interpolation theorem) that the elimination of the members of the set μ (interpreted as theoretical concepts) does not preserve deductive power with respect to formulas containing concepts of λ only. This gives rise to measure the possible gains obtained by using auxiliary concepts in the following ways:

(1) gain in deductive power measured by

$$\lim_{e\to\infty} \operatorname{cont}(T^{(d+e)}(\lambda + \mu)(\lambda)) - \operatorname{cont}(T^{(d)}(\lambda + \mu)(\lambda)),$$

where cont is the usual measure of semantic information;

(2) gain in simplicity measured by $\operatorname{cont}((T^{(d)}(\lambda + \mu)(\lambda))) - \operatorname{cont}((T^{(d_0)}(\lambda + \mu)(\lambda)))$, $d_0 < d$.

In the case when the members of μ are explicitly definable no gain of the first kind is obtained (the subtheory is finitely axiomatizable by $T^{(d+e)}(\lambda + \mu)(\lambda)$, $(d + e)$ being the maximal depth of the definientia. But gain of the second kind is obtained. The greater the maximal depth of the definientia is, the greater this gain.

L. PACHOLSKI. *Elementary substructures of direct powers.*

THEOREM 1. *If $\mathfrak{A} = \langle \omega, +, \cdot \rangle$, then the substructure of $\mathfrak{A}^\omega$ consisting of all arithmetical functions is an elementary substructure of $\mathfrak{A}^\omega$.*

THEOREM 2. *If $\mathfrak{A} = \langle \omega, R_i \rangle_{i<\omega}$ is a structure such that all relations over ω defined by elementary formulas of the language of $\mathfrak{A}$ are recursive then the substructure of $\mathfrak{A}^\omega$ consisting of all recursive functions is an elementary substructure of $\mathfrak{A}^\omega$.*

THEOREM 3. *For any relational structure $\mathfrak{A}$, any infinite set I and any infinite cardinal number $\mathfrak{M} \leq |I|$ the substructure of the direct power $\mathfrak{A}^I$ consisting of all functions f such that $|f(I)| < \mathfrak{M}$ is an elementary substructure of $\mathfrak{A}^I$.*

This result follows also from Theorem 2 of B. Węglorz—see his abstract.

THEOREM 4. *For any $\mathfrak{A}$, I and $\mathfrak{M}$ as above the substructure of $\mathfrak{A}^I$ consisting of all functions f for which there exists an element $a_f \in \mathfrak{A}$ such that $|I - f^{-1}(a_f)| < \mathfrak{M}$ is an elementary substructure of $\mathfrak{A}^I$.*

This result follows also from the result of A. Wojciechowska—see her abstract.

JONATHAN P. SELDIN. *General models for type theory based on combinatory logic.*

In [2] and [3], Sanchis showed how systems of type theory can be formulated on the basis of combinatory logic using the theory of functionality and how general models can be constructed for these theories in terms of which total valuations can be defined. But the results of Sanchis hold only for simple (or finite) type theory; they do not apply to transfinite systems such as that of Andrews [1]. In this paper, the results of Sanchis are extended to transfinite type theory and to still more general systems.

REFERENCES. [1] PETER B. ANDREWS, *A transfinite type theory with type variables*, North-Holland, Amsterdam, 1965.

[2] LUIS E. SANCHIS, *Normal combinations and the theory of types*, Ph.D. thesis, The Pennsylvania State University, University Park, Pennsylvania, 1963.

[3] LUIS E. SANCHIS, *Types in combinatory logic*, *Notre Dame journal of formal logic*, vol. 5 (1964), pp. 161–180.

KAREL L. DE BOUVÈRE. *Logical and ontological models.*

1. Let L be a standard language, S the set of all its sentences (of finite length). Let T be a theory in L and $\mathfrak{T}_T$ the set of all extensions of T in L. The algebra

$$A_T = \langle \mathfrak{T}_T, \cap, \cup, \dot{-}, T, S \rangle$$

is a Brouwerian algebra (Tarski, 1935).

2. Let T be a theory as described above with exactly four complete extensions (the inconsistent extension is not considered to be complete). All extensions of T are unions of four building stones: $\dot{\bar{T}}_1$, $\dot{\bar{T}}_2$, $\dot{\bar{T}}_3$ and $\dot{\bar{T}}_4$. We can exhibit them as vertices of a simplex (in this case a tetrahedron).

3. The notion of a model is used in a different sense by logicians on one hand and natural and social scientists on the other hand. The situation is confusing, especially for philosophers. The example of 2 may clarify the differences and interrelationships of the notions involved when applied to some philosophical considerations. Moreover, the example is apt to serve as a "mathematical model" to explain some qualitative properties of some physical worlds built from some elementary particles.