

MAHLER'S METHOD IN TRANSCENDENCE THEORY

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In 1926 Kurt Mahler introduced functional equation methods into transcendence theory. His work [3] went largely unnoticed until almost fifty years later, when it was taken up by Loxton and van der Poorten and others. The method is particularly suited to investigating the transcendence of numbers whose *decimal* expansions in some base $g \geq 2$ can be produced by a finite automaton (a *computer* with a fixed, finite amount of *memory*). This bears upon the question of whether or not the expansion of an algebraic irrational need be *random*.

In this paper we give a sample transcendence proof in order to illustrate the scope and limitations of a variant of this method. We also demonstrate an algebraic independence theorem which relates the algebraic independence of a collection of power series $f(z)$ which satisfy a certain type of functional equation to the algebraic independence of the numbers $f(\alpha)$ for a suitable algebraic number α . The theorem, while giving a more general result than that of Loxton and van der Poorten [2], is proved by more economical means than those of Kubota [1].

REFERENCES

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- [3] K. Mahler et al., 'Arithmetische Eigenschaften der Lösungen einer Klasse von Funktionsgleichungen', *Math. Ann* **101** (1929), 342-366.

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