

## 35. COMMISSION DE LA CONSTITUTION DES ÉTOILES

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MEMBRES: Biermann, Bondi (C. M.), Bondi (H.), Brownlee, Burbidge (G. R.), Cameron, Chandrasekhar, Cowling, Cox (A. N.), Dingsen, Epstein, Ferraro, Fowler, Gamow, Gjellestad, Haselgrove, Hayashi, Henyey, Hitotuyanagi, Hoyle, Humblet, Kaplan (S. A.), Kippenhahn, Krook, Kushwaha, Lebedinsky, Lehnert, McCrea, Masani, Masevitch, Mestel, Reiz, Rosseland, Salpeter (E. E.), Savedoff, Schatzman, Strömngren, Sweet, Tayler, Tuominen, Wrubel.

### I. NUCLEAR ENERGY GENERATION, NEUTRINOS EMISSION, NUCLEO-SYNTHESIS, SUPER-DENSE STARS AND SUPER-MASSIVE STARS

A very comprehensive review article on nuclear astrophysics attempting to bring to date all of the material on nucleosynthesis and related topics has been published by G. Burbidge (1) and his lectures in Varenna on the same subject (2) should come out of the press in the immediate future. H. Reeves (3) has prepared a detailed review of the stellar energy sources presenting up-to-date rates for the hydrogen, helium, carbon and oxygen burning stages as well as a discussion of the neutrino emission processes.

As far as the C-N-O cycle is concerned, the implications for energy generation in stars of the new experimental results on the significant cross-sections have been discussed by W. A. Fowler and R. Caughlan (4) and R. E. Brown (5) and it seems practically certain now that, apart from  $O_{17}(p, \alpha)N_{14}$ , none of the C-N-O cycle reactions presents resonances in the range 0 to 100 KeV.

Parker and Kavanagh (6) and Trombello and Parker (7) have re-examined the reactions  $(He_3, \alpha)$  and  $(He_3, He_3)$  which are determinant for the termination of the proton-proton chain. On the basis of these results and the latest theoretical information, P. Parker, J. N. Bahcall and W. A. Fowler (8) have assessed the relative importance of the various terminations as a function of density and temperature and J. N. Bahcall and R. A. Wolf (9) have considered especially the case of high density and applications to small mass stars.

As far as the  $3\alpha$  reaction is concerned, P. A. Seeger and R. W. Kavanagh (10) have discussed the rate of energy production using their measure of the width  $\Gamma$  of the  $(C^{12})^*$  nucleus. Various aspects of the helium thermo-nuclear reactions have been studied by H. L. Duorah and R. S. Kushwaha (11) and by M. Thibaudeau, W. Cartledge and H. Reeves (12) while H. Reeves (13) has shown that far less neon should be formed out of these reactions than is actually observed. Rates of thermo-nuclear reactions involving carbon, oxygen and neon nuclei have been calculated by H. Reeves (14) and H. Tsuda (15) has discussed in general the thermo-nuclear reactions involving heavy ions and the secondary protons and  $\alpha$ -particles produced on the way up to the formation of the iron nuclei.

Further study of the  $p$ -process (production of the low abundance proton-rich heavy isotopes) has been carried out by K. Ito (16) who concluded that, at least two temperatures and several densities are required to produce these isotopes in their observed abundances by the  $(p, \gamma)$  and  $(\gamma, n)$  reactions. On the other hand Frank-Kamenetski (17) came to the conclusion that these reactions cannot account for the production of these isotopes which, according to him should rather be formed by  $(p, n)$ ,  $(p, 2n)$  reactions with non-thermal accelerated protons in the very external layers of stars.

H. Tsuji (18) and H. Tsuda and H. Tsuji (19) discussing the formation respectively of the  $4N$ -and neighbouring nuclei ( $A = 20 - 40$ ) and of the Fe-group elements lighter than  $A = 56$  found that their abundances can be explained by proton capturing processes in the general conditions of density and temperatures to be expected in super-novae explosions.

Details of the  $s$ -process (neutron-capture chain on a large time-scale) have been reviewed and discussed by Clayton, Fowler, Hull and Zimmermann (20) while P. A. Seeger (21) is continuing his studies of the nuclear physics basis for the  $r$ -process (neutron-capture on a short time scale) to obtain better theoretical predictions for the resulting abundances. In a series of papers, Bahcall (22, 23) alone or in co-operation has studied the influence of a large range of stellar conditions on beta-decay reactions especially with respect to the electron-capture lifetime of a proton (possibly important in super-novae processes), the stellar beta-decay of  $K^{40}$  (of significance for  $s$ -process theories) and the influence of forbidden transitions on the abundances of elements in the iron peak (with particular reference to the  $e$ -process theory).

Equilibrium abundances have been re-calculated by Clifford and Tayler (24) for the whole range of physical conditions susceptibles to be encountered in stars in which, due to high temperatures and densities, statistical equilibrium is set up between the different nuclides.

The synthesis of D, Li, Be, B and their isotopes (the  $x$ -process) must occur in conditions remote from those permitting hydrogen burning and, generally, an explanation has been looked for in non-thermal processes (magnetic accelerations of protons in flares, etc.) occurring in the external layers of stars especially during the last phases of stellar condensation. Fowler, Greenstein and Hoyle (25) have applied this to the solar system showing that spallation and neutron reactions in the outer layers of the planetesimals could account for the present abundances of these elements as well as for the anomalies of radiogenic  $Ag^{107}$  and  $Xe^{129}$  observed in meteorites. Cameron (26), on the other hand, has taken a completely different point of view assuming that the formation of the solar system was preceded shortly by an active phase of nucleogenesis which enriched the interstellar medium from which it condensed into all the necessary elements including D, Li, Be and B and the needed radioactive elements.

One of the most active fields has been the one concerned with the production of neutrinos and the effects on stellar structure and evolution of the corresponding energy losses when phases of high temperatures are reached. However, much before this, the detection of the neutrino flux from a star could bring unique information on the type of nuclear reactions taking place in the central regions and the physical conditions there. The feasibility of this, in the case of the Sun, once the importance of the  $He^3(\alpha, \gamma) Be^7$  branch of the  $p$ - $p$  chain was realized, was emphasized by Bahcall, Fowler and Iben (27) and has also been discussed by Pochoda and Reeves (28). An actual experiment is, at present, being planned by R. Davis Jr. (Brookhaven) and J. N. Bahcall (Kellogg Rad. Lab.). The latter has recently shown (29) that this experiment could determine the central temperature of the Sun to better than  $\pm 10\%$  and, from a preliminary measurement by Davis (30), he was able to conclude (31) that  $Li^4$  does not play a significant role in the  $p$ - $p$  chain.

The Urca process of Gamow and Schoenberg awoke the interest in the possibility of large energy losses by neutrinos emission but in the last few years, its possible importance has been progressively reduced (32). On the other hand, the direct coupling term ( $e\nu$ ) ( $e\nu$ ) which arises in the development of a theory of universal Fermi interaction has suggested new possibilities. The rates for most of the important neutrino loss processes (in particular pair annihilation:  $e^+ + e^- \rightarrow \nu + \bar{\nu}$ ; photo-neutrinos:  $\gamma + e^- \rightarrow e^- + \nu + \bar{\nu}$ ) have been calculated by now (33, 34) including the most recent one introduced by Adams, Ruderman and Woo (35) who have shown that the collective modes of neutrinos production in a stellar plasma can become important at high densities ( $\rho \simeq 10^7 \text{ gm/cm}^3$ ) but fairly low temperatures ( $T \simeq 10^8 \text{ K}$ ) when the other mechanisms become inactive. This last possibility, as pointed out by Chiu (36) may

be of some importance in red giant stars just prior to the He-flash. However, if this case is more or less marginal, there are many phases in the late stages of stellar evolution where these neutrino processes could have a decisive influence as Chiu in particular [cf. also R. Stothers and H. Y. Chiu (37)] has repeatedly emphasized. Reeves (38) has particularly investigated the effect on the carbon and oxygen burning stages a question which has also been discussed by R. Stothers (39). Masevitch, Dluzhnevskaya, Kotok and Masani (40) in discussing the importance of neutrinos emission in different regions of the H-R diagram, come to the conclusion that the neutrino 'Bremsstrahlung' of electrons in the field of a nucleus may be important in red giants, helium stars and white dwarfs. As far as this last type of stars is concerned, V. D. Granova (41) has discussed three particular cases and found however that the neutrino flux remains substantially smaller than the photon flux.

W. A. Fowler and F. Hoyle (42) are preparing a general discussion of all processes involving neutrinos and their effects especially on the final pre-supernova stage. For massive stars, they reduce the duration of this phase to about one day and this sets the time-scale for the equilibrium process of nucleosynthesis and is the decisive factor in determining the relative abundances of the iron group elements and their isotopes. As the relative abundance of  $e$ -process elements in the solar system suggests that  $\beta$ -decays do not have the time to reach a steady state it may well be that the rate of the star's evolution in the significant phases must have been determined by the neutrino losses confirming thus the direct ( $e\nu$ ) ( $e\nu$ ) electron-neutrino interaction.

A very quick excursion outside theory may perhaps be allowed here to recall a recent suggestion (cf. 43) following the observational results of Greenstein, Helfer, Wallerstein, Sandage, Burbidge, Aller and others, according to which the correlation between metal abundances and the distance from the galactic plane is really the most significant one. If this is confirmed, it will no doubt have considerable influence on our way of thinking about elements formation.

Perhaps this has something to do with nucleogenesis in the super-massive condensations which have been advocated by Fowler and Hoyle to explain the strong radio sources and stars. Dr W. A. Fowler (44) writes 'This nucleosynthesis is probably significant in determining the earliest contributions to element abundances beyond hydrogen in the Galaxy. The early build up of helium and perhaps other elements such as nitrogen, neon and those produced in the  $r$ -process is suggested and is now under intensive investigation'. Fowler and Hoyle have suggested that the enormous energy emitted by these radio sources is related to the evolution of a very massive stellar type object at the centre of a galaxy. In their first two papers on the subject (45) and (46) they adopted essentially a classical treatment for masses up to  $10^8 M_{\odot}$  invoking neutrino emission through annihilation of electron pairs after hydrogen and helium burning to produce a rapid collapse of the star and subsequent explosive burning of oxygen leading to disruption. The latter possibility was however rejected in the second paper as the energy required for the creation of electron-positron pairs in such massive stars overwhelms entirely any form of nuclear energy generation which therefore could not lead to an explosive outburst.

However it appears that for such large masses general relativity corrections to the hydrostatic equilibrium equation must be taken into account and will lead to dynamical collapse. In fact, Zeldovitch (47), in 1962, has already established the existence of static solutions with an excess of energy as compared with the rest mass of the star and thus unstable. He has also discussed the collapse of a star with  $M \cong 10^8 M_{\odot}$  and shown that gravitational curvature of space does not allow any considerable amount of energy to escape as neutrinos (48). In part, similar conclusions were reached by Iben (49) who has shown that the binding energy of quasi-static solutions of the general relativity equations which is positive for large radii, goes through a maximum and then becomes rapidly negative for small enough radii. He concludes that, on approaching the positive maximum, there is a tendency for the star to collapse dynamically. For  $M > 10^7 M_{\odot}$ , nuclear burning is not able to halt this collapse. A general

discussion of this problem can be found in a paper by Hoyle, Fowler, E. M. and G. R. Burbidge (50) in which the authors find no mechanisms susceptible to lead later to an explosion. It is clear that a proper dynamical approach is needed to reach secure conclusions and M. A. Podurets (51) seems to have made progress in that direction by carrying out numerical integrations of the equations of non-stationary contraction of a star including relativistic effects.

Apart from super-massive stars some good reasons have also been advocated to study super-dense stars as well. On one side neutron stars have been re-investigated by Salpeter and Hamada (52) on the basis of the discussion by Salpeter (53) of the equation of state for a high density plasma. It appears that there is always some admixture of other nucleons in a neutron configuration and at about  $10^{15}$  gm/cm<sup>3</sup> a gradual transformation to a gas of protons and hyperons begins. Work on such super-dense matter has been continued by Ambartsumian, Saakian and collaborators (54), (55) and (56) who have studied the relative abundances of elementary particles for density in the range  $7 \cdot 10^{16}$  to  $2 \cdot 5 \cdot 10^{11}$  gm/cm<sup>3</sup>. They have also discussed the equation of state for densities reaching that of nuclei as well as its asymptotic behaviour for  $\rho \rightarrow \infty$ . Models of super-dense stars have been constructed on the basis of the general relativity equations and limits to the mass ( $\cong 1M_{\odot}$ ) and radius ( $R \cong 10$  km) were found for such a 'barionic' star.

T. A. Emin-Zade (57) has discussed the possibility for non stable super-dense configurations of radii smaller than their gravitational radii ( $\rho \cong 10^{14} - 10^{15}$  gm/cm<sup>3</sup>) to break off into separate stellar bodies as a result of expansion.

We may also record here a suggestion by Frank-Kamenetski (58). After discussing possible plasma phenomena in processes leading to supernovae explosions, he concludes that in Supernovae II, before high densities ( $10^{13}$  gm/cm<sup>3</sup>) are reached, a considerable amount of a mixture of matter and antimatter (epiplasma) might be formed.

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## II. OPACITY, EQUATION OF STATE, IONIZATION, CONDUCTIVITY, ETC.

The lack of reliable opacity tables which has been a stumbling block for a long time seems to be on the point of being alleviated. Some of the results of Arking and Herring (1) on the relative contribution of absorption lines to the opacity of stellar matter have been published and A. N. Cox has prepared a very complete and up-to-date account of the whole problem including ions line absorption and at least partially molecular absorption to be published soon (2). Apart from tables and graphs of opacities for the mixture ( $X = 0.596$ ,  $Y = 0.384$ ,  $Z = 0.02$ ) this paper contains also a great deal of information on ionization equilibrium, conductivity, etc. for a large range of conditions. Cox reports that work is continuing on

computation of opacities for twenty mixtures ( $X$  varying from 0.9996 to 0.30) in the ranges ( $3000 < T < 10^9\text{K}$ ;  $10^{-12} < \rho < 10^{10}\text{ gm/cm}^3$ ) as well as on monochromatic absorption and scattering coefficients at and below  $10^5\text{K}$ , the results to be published in *Los Alamos Reports* and in the *Astrophysical Journal Supplements*.

T. D. Kusnetsova and D. A. Frank-Kamenetski (3) have evaluated the opacity for completely ionized hydrogen according to the exact formulae of the 'Bremsstrahlung' theory and have given a convenient interpolation formula, while Hayashi, Hôshi and Sugimoto have discussed the corrections to the opacity due to free-free transitions in degenerate matter (4).

One may also mention here the work of D. H. Samson (5) on the effects of the equilibrium concentration of electron-positron pairs on the properties of stars at very high temperatures and the series of papers by C. A. Rouse (6) on ionization equilibrium and equation of state for a large range of stellar conditions.

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#### III. CONVECTION, EXTERNAL CONVECTION ZONES, SURFACE BOUNDARY CONDITIONS.

Thermal convection either laminar or turbulent is of interest for many fields (hydrodynamics, meteorology, oceanography, geophysics) beside our own. Unfortunately, it is impossible to present here a complete survey of all the work accomplished in these different disciplines although much of it may be of interest for our particular problem.

As far as laminar convection is concerned, Chandrasekhar's book (Chap. II-VI) (1) provides an exhaustive and synthetic survey of the associated linear problem including the effects of rotation and magnetic fields when compressibility is negligible. A very short summary of significant results in the same general field together with some remarks on non-linear effects is presented in a paper by Y. Nakagawa (2).

Applications to stellar conditions [cf. general accounts by M. Schwarzschild (3) and E. Böhm-Vitense (4)] requires extensions to take into account:

1. compressibility both in the structure of the layer as well as during the motion of its elements,
2. more realistic boundary conditions,
3. the effects of non-linear terms.

The discussion of the first two points has mainly been approached by the study of convection (usually laminar modes) in superposed polytropic layers (one thermally stable, the other unstable—or of different degrees of instability). In this line, one may report a paper by S. Kato and W. Unno (5) discussing especially the effect of compressibility on the scale of motion at marginal stability, one by S. Kato (6) concerning the influence of the variation of the super-adiabatic temperature gradient on the critical Rayleigh number and the flow pattern, one by P. Souffrin (7) (an unstable polytropic layer bounded on both sides by stable isothermal layers) attention being paid essentially to the penetration of the currents in the stable zone, and one by E. Spiegel and W. Unno (8) on the same general type of questions. One may perhaps consider that this type of approach culminates in a paper by K. H. Böhm (9) in which the same kind of

linear analysis of the unstable laminar modes is carried out for a realistic model of the external layers of a star.

Although their main object is somewhat different, one may add to this series two papers by P. O. Vandervoort (10) devoted to the effects of compressibility, through sound waves and convection, on the Rayleigh instability at the surface of discontinuity separating two compressible media.

As to the difficult problem of the non-linear terms, one may perhaps distinguish the effect of finite amplitudes on purely laminar convection and their effects when turbulence arises, which should be the rule in cosmical bodies. Typical of the first group is the paper of L. L. Segel and J. J. Stuart (11) discussing the influence of non-linear terms on the critical Rayleigh number  $R_c$  and on the preferred mode and its shape in cellular convection. In a similar vein, G. Veronis (12) has shown that, at least in a particular case, sub-critical convection ( $R_a < (R_a)_c$ , linear) can occur for finite perturbations and he has studied the penetration of the currents outside the primitively unstable region. The discussion of the solution of the non-linear equations of cellular convection and heat transfer has also been tackled by H. L. Kuo (13). Although quite different in spirit, we may also include in this group a paper by J. R. Herring (14) in which numerical integrations, retaining only those non-linear terms which affects the mean temperature distribution, allow one to follow the establishment of a steady state which qualitatively agrees fairly well with experimental results and in which dominant modes correspond closely to those maximizing the total heat transport. In this respect, see also the recent paper by L. Howard (15).

As far as properly turbulent convection is concerned, an attempt was made by P. Ledoux, M. Schwarzschild and E. Spiegel (16) to derive the stationary spectrum of turbulent convection by balancing the energy input due to buoyancy evaluated from the linear theory of laminar convection in the Boussinesq approximation against the dissipation into modes of smaller scales due to the non-linear interaction terms evaluated by Heisenberg's heuristic formula. In the second part of Kato's paper (6) one finds an extension of this type of approach to the case of a variable temperature gradient.

Recently other models and hypothesis for the turbulent-energy transfer associated with the non-linear terms have been studied by R. H. Kraichnan and E. A. Spiegel (17) and by I. Dungsstad (18) and useful references on this type of question may be found in (19). Let us note also that a new spectral equation has been recently proposed by S. A. Kaplan (20) for magneto-hydrodynamic convection.

There has appeared also a tendency to generalize the mixing-length treatment so as to cover some of these non-linear effects without recourse to a detailed description of the turbulence spectrum and Kraichnan (21) was able, in this way, to obtain expressions for the heat transport, mean temperature, r.m.s. values of the velocity and temperature fluctuations at any arbitrary Prandtl number  $\sigma$  and which, for  $\sigma$  small, reduce to those found in (16). E. Spiegel (22) has given an expression of the convective heat flux which is not restricted to the case of small mixing-length and which permits to evaluate the penetration of currents in convectively stable regions. On the other hand, W. Unno (23) has shown that the linear theory can be used and is essentially equivalent to the mixing-length theory if one introduces an eddy viscosity reducing the Reynold number to about 30.

As far as the internal structure of the stars is concerned, refinements of the theory of turbulent convection are mainly needed for the external convective zones which affect the surface boundary conditions which have to be imposed on the interior solution. In this respect, and in the frame of the mixing-length theory, a semi-empirical approach has often been used trying to determine the mixing-length  $l$  in terms of some significant scale-height from a comparison of theoretical models and observational data. Such an attempt by M. Shimoda (24) for the

giant sequence of globular clusters concludes that correct boundary conditions can only be formulated within a theory allowing a complete derivation of the overall structure of the convective flow, but C. Hayashi and R. Hôshi (25), considering the same problem, show that the models lie anyway in a fairly narrow region in the H-R diagram in agreement with the observations. For very small masses (26), they find that the surface boundary conditions become independent of the efficiency parameter in the convective energy transport. K. S. Krishna Swamy and R. S. Kushwaha (27) have discussed, on the basis of atmospheric models, the determination of the constant  $E$  fixing the interior convective solution at different points in the H-R diagram. On the other hand, Faulkner, Griffiths and Hoyle (28) have developed a method allowing integration downwards from the photosphere through any convective zone that may exist without using explicitly the notion of mixing-length  $l$  although, apparently, it is equivalent to adopt a continuously variable  $l$  proportional to the local temperature scale-height. They intend to give the pressure and the depth at which  $T = 10^5\text{K}$  is reached as functions of  $M$ ,  $L$ ,  $T_{\text{eff}}$  and the metal abundance  $A$ .

Finally let us mention a discussion by Ruben (29) which suggests a new version for boundary conditions allowing a direct link from the interior to the atmosphere through the transition region in which  $\bar{\mu}$  and  $\kappa$  change continuously.

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## IV. STELLAR STRUCTURE AND STELLAR EVOLUTION

(a) *General*

A general review of the problem of stellar evolution has been written by C. Hayashi, R. Hôshi and D. Sugimoto (1) which contains also original contributions or suggestions concerning various points which are reviewed in their respective sections. R. L. Sears and R. R. Brownlee have also prepared a general account (2) in which computational methods, evolutionary models and evolutionary tracks and their application to the determination of cluster ages are reviewed. To these we may add the lectures of M. H. Wrubel (3) which present a concise and very clear summary of the main physical and methodological aspects of the problem.

L. Henyey (4) has prepared a general account of the revised numerical methods used in his latest programme to compute evolutionary sequences complying as much as possible with the actual physics of the conditions encountered.

On the basis of a discussion of the ages of hot stars (under different assumptions for their evolution) and cold stars (still in the stage of gravitational contraction) in the Orion Nebula, A. G. Masevitch and E. V. Kotok (5) conclude that the duration of the process of star formation in this cluster is considerable of the order, of the age of the cluster itself.

(b) *Pre-main Sequence Contraction*

In a very important and now well-known paper, Hayashi (6) has shown that contracting stars in the low luminosity, low temperature part of the H-R diagram cannot be in equilibrium. Readjustment of the boundary conditions at the photosphere show that a gravitationally contracting star will be in convective equilibrium throughout its whole mass during a considerable part of this contraction and that the corresponding track in the H-R diagram will be very different from the usual radiative one and nearly vertical offering a possibility of explaining the H-R diagram of very young clusters like NGC 2264. This also reduces very considerably, especially for small masses, the duration of the contraction phases. The consequences for small mass stars are further discussed in (1, §10) ( $M \simeq 2$  to  $0.6 M_{\odot}$ ) and applied to the M-type stars in the Orion Nebula indicating masses of the order of  $0.1$  to  $0.2 M_{\odot}$ . It is also shown that for stars with  $M < 1.3 M_{\odot}$ , lithium but not beryllium may just burn at the bottom of the convection zone as they complete their evolution towards the main sequence. This discussion has been extended by Hayashi and Nakano (7) down to masses as small as  $0.05 M_{\odot}$  taking also into account the dissociation of  $H_2$  in the external layers. In following the contraction, they find that H-burning occurs for  $M > 0.08 M_{\odot}$  but not in stars with  $M < 0.08 M_{\odot}$  which contract directly to degenerate configurations. On reaching the main sequence, stars of the first group which may be compared with actual red dwarfs develop a radiative core if  $M > 0.26 M_{\odot}$  while they remain wholly convective if  $0.25 M_{\odot} > M > 0.08 M_{\odot}$ . The Helmholtz-Kelvin time scale for these small masses is much reduced with respect to the previous radiative gravitational contraction, perhaps by as much as factor of 100 and, as shown by Kumar (8), cannot be much larger than  $10^9$  years.

The case of the Sun has been discussed by Weymann and Moore (9) who confirm Hayashi's result that such a star remains wholly convective as long as  $T_{eff} < 4300^{\circ}$  and show that the question of the depletion of lithium by the implicated mixing down to regions of fairly high temperatures is very delicate. This same problem has been discussed by Cameron and Ezer (10) who find a greater probability for lithium burning. General energy considerations on the dynamical instability arising in the early phases of contraction ( $R > 57R_{\odot}$ ) due to ionization

of H and He and dissociation of  $H_2$  are also presented in (10) while comments on the 'jump' of a proto-star to a state of convective quasi-static equilibrium are contained in (1, §10).

The general problem of the convective phases of gravitational contraction has been re-investigated by Faulkner, Griffiths and Hoyle (11) who find that no factors related to the opacity or non-thermodynamic processes are likely to destroy the convective structure. However, they suggest that the presence of a magnetic field might have reduced very much the efficiency of convection at some phases, particularly when the planetary material separated from the Sun.

### (c) Main Sequence

Using homogeneous models, Iben (12) has critically discussed the assumption of a universal 'age-zero' main sequence independent of composition and its consequences as far as cluster ages are concerned. He finds, in particular, that the helium-content of the Hyades and Pleiades cannot be the same and raises the question of the possible variation with chemical composition of the ratio ( $l/H$ ) of the mixing-length to the scale height. Faulkner, Griffiths and Hoyle (13) also report work in preparation on this question from which it results that with realistic assumptions the more probable compositions yield a fairly narrow main-sequence band.

I. M. Kopylov (14) has evaluated the main parameters ( $S_p^0$ ,  $(B - V)^0$ ,  $R^0$ ,  $T_{eff}^0$ ,  $M_{bol}^0$ ,  $M_v^0$ ) of the initial main sequence by linking through  $M$  the theoretical sequence of homogeneous stellar models with the observed ( $M$ ,  $S_p$ ) relation for spectral classes O5 – M5. The resulting values of  $S_p^0$  and  $M_v^0$  are in good agreement with the initial main sequence evaluated from distances to galactic clusters ( $\Delta M_0 \cong \pm 0^m 15$  except for spectral types O5–O7 and K7–M5 in which they may reach  $1^m$ ). An analogous procedure can be used (15) to yield a new scale of effective temperatures ( $S_p$ ,  $T_{eff}$ ) which is in good agreement with the relation ( $T_{eff}$ ,  $S_p$ ) obtained from models of stellar atmospheres but deviates substantially from Kuiper's scale, especially for types O5–O7 and B9–A5.

The lower main-sequence of Population II stars has been studied by P. Demarque (16). Kumar (17) has discussed the structure of wholly convective stars of very low masses ( $0.1$  to  $0.04 M_\odot$ ) for different chemical compositions to determine the limiting masses ( $0.07 M_\odot$  for Pop. I;  $0.09$  for Pop. II) under which H-burning never occurs (cf. also 7). In view of determining the limits on solar neutrino fluxes, Sears (18) has computed an extensive series of solar models in a range covering the uncertainties in the different parameters (composition, opacity, age, nuclear cross-sections) which may affect these fluxes.

Boury (19) has published his results on very massive stars (up to  $6000 M_\odot$ ) formed initially of pure hydrogen. A somewhat similar investigation with comparable results has been carried out by Van der Borcht and Meggit (20).

### (b) Post-main Sequence evolution

Work on the giant branches of clusters is going on and a general discussion embodying various improvements in the physics of the models has been published by Demarque and Geisler (21). Their results which cover the effects of a slight variation in the mass and those of independent variations of the initial metal and helium content reproduce fairly well the observed slope of the giant branch but the helium content remains undetermined. R. L. Sears (22) is constructing evolutionary sequences for stars of appropriate mass and composition to determine the ages of the globular cluster  $M_2$  and NGC188. In this respect, one may recall the restriction on these ages imposed by the fuel supply limit as discussed by Woolf (23).

Faulkner, Griffiths and Hoyle (24), on their side, report that work in progress tends to show that the shape and position of the giant branches are determined solely by surface composition.

Härm and Schwarzschild (25) have continued their investigations of the helium flash in Population II stars but the question of whether or not this leads to a total mixing of the star cannot yet be answered. In discussing these same phases, Hayashi, Hôshi and Sugimoto

(1, § 6) find that stars with  $M < 0.53 M_{\odot}$  never reach the region of the helium-flash their evolutionary tracks turning to the left before that and bringing them down later to the white-dwarfs region.

Considerable success has also been achieved by now in following the post-main sequence evolution of stars much heavier than the Sun. From the study of a star of mass equal to  $47 M_{\odot}$ , Sakashita and Hayashi (26) concluded that the effects of the transition semiconvective zone which, in heavy stars, develops between the convective core of increasing molecular weight and the envelope remain small.

The evolutionary tracks of stars of 4 and  $5 M_{\odot}$  were computed in details up to the onset of helium burning by Hayasho, Nishida and D. Sugimoto (27) and by Pollak (28) respectively.

C. Hayashi and R. C. Cameron (29) have followed the evolution of a Population I star of  $15.6 M_{\odot}$  and initial composition  $X = 0.90$ ,  $Y = 0.08$ ,  $Z = 0.02$  through the stages of hydrogen exhaustion in the convective core, subsequent contraction of the core leading to helium burning and finally the onset of carbon burning. In about  $1.4 \cdot 10^6$  years this leads the star from the main sequence to the region of the M-supergiants, the main gap being crossed in only  $2 \cdot 10^4$  years. According to the same authors (30) these very late stages (C, Ne and O-burning) could account for the giant branches of clusters such as  $h$  and  $\chi$  Persei only if the neutrino loss due to direct electron-neutrino interaction is disregarded.

G. V. Ruben (31) has constructed a homogeneous model for  $M = 5.2 M_{\odot}$ ,  $X = 0.90$ ,  $Z = 0.01$  and four inhomogeneous models with an isothermal core and transition layer (for  $M = 3.8 M_{\odot}$ ,  $X_e = 0.90$ ,  $Z_e = 0.01$ ;  $M = 5.2 M_{\odot}$ ,  $X_e = 0.90$ ,  $Z_e = 0.02$ ,  $0.01$ ,  $0.003$ ) as a start for the study of the evolution of the components of the binary system  $\gamma$  Leo. Similar work has been carried on by U. K. Dservitis (32) to check the sensitivity of the models on different parameters while the evolution from the main sequence of stars of masses greater than  $10 M_{\odot}$  is being tackled.

E. Hofmeister, R. Kippenhahn and A. Weigert (33) have carried the evolution of a Population I star of  $7.0 M_{\odot}$  from the main sequence through the exhaustion of hydrogen in the core, helium burning, exhaustion of helium in the core and the formation, besides the previously established hydrogen burning shell, of a deeper helium burning shell to the point where the carbon burning temperature is reached. They find that this evolutionary track crosses five times the Cepheid strip.

Other work in progress is also reported from Caltech: by A. Boury (34) on the evolution of a  $30 M_{\odot}$  model through  $3\alpha$ -burning to stages of  $C^{12}$ ,  $O^{16}$ , and  $Ne^{20-\alpha}$  burning (the effects of assuming that  $C^{12}$ - $\alpha$ -burning is faster or slower than the  $3\alpha$ -burning will be discussed); by I. Iben, Jr. (35) for a whole range of masses ( $0.1 M_{\odot} < M < 30 M_{\odot}$ ) including the Hayashi contracting phases and, for the small mass stars (no He flash), their cooling to the white dwarfs stage.

P. Pochoda and M. Schwarzschild (36) have investigated the evolution of the Sun under the assumption that the gravitational constant has been steadily decreasing in the past.

#### (e) *Late Stages of Stellar Evolution*

Some of these stages were already encountered in the previous subsection as the natural end of evolutionary sequences but, in some cases, they can also be studied by themselves without a detailed description of their previous history. White-dwarfs provide a good example in this respect and Mestel (37) has prepared a general review of the present state of their theory. On the basis of a discussion by Kirzhnitz (38) of the effects of potential barriers in dense matter on the  $p$ - $p$  chain, Seidov (39) found that this chain would be  $10^5$  to  $10^8$  times slower in white dwarfs than in ordinary stars. This, according to the author, would permit an internal abundance of hydrogen as high as several percents, a conclusion which should however be revised in the light of the corrections introduced by V. P. Kopyshv (40).

Coulomb and beta-decay corrections to the equation of state were calculated for a high-density plasma by Salpeter (41) and used by Hamada and Salpeter (42) to construct white dwarfs models with and without discontinuities in chemical composition. In Paris, A. Baglin (43) is working on white dwarfs models with finite (non zero) temperatures.

Savedoff (44) has evaluated the total energy (gravitational + kinetic energy of electrons) of completely degenerate configurations for a series of values of the degeneracy parameter. He has also discussed (45) the evolution of energy-free gas into white dwarfs with a view to evaluating their age. He finds that the pressure of the non-degenerate nuclei couples with the gravitational field to give a contribution larger by a factor of the order of three than that given by the internal energy only. Further detailed numerical work on this problem is being carried on by S. Vila.

Integrations for highly degenerate isothermal cores ( $\Psi_c \cong 70$  to 200) of collapsed stars have been performed by K. Kaminski (46).

Models for pure He-stars have been published by J. B. Oke (47) and J. P. Cox and R. T. Giuli (48). M. Nishida and D. Sugimoto (49) as well as J. P. Cox and E. E. Salpeter (50) have discussed models with a hydrogen-rich envelope and the location of their representative points in the H-R diagram. With W. Deinzer (51) these last authors have now built models for pure helium stars, including the effects of degeneracy and radiation pressure, for a large range of masses and they find that the minimum mass allowing helium-burning is  $0.35 M_{\odot}$ . The build-up of  $C^{12}$  and  $O^{16}$  as the core burns out was also calculated. Some of the problems related to stars with helium-rich cores were reviewed by Kippenhahn in his Varenna lectures (52).

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#### V. STELLAR PULSATIONS AND STABILITY; SHOCK WAVES AND NON-STATIONARY PROCESSES IN STARS

Some aspects of the current interpretation of Cepheids have been reviewed by Ledoux and Whitney (1) and Zhevakin (2) has written a clear and up-to-date account of the pulsation theory of variable stars. The latter has continued his investigations (3) on stellar variability due to the vibrational instability arising, mainly in the second ionization zone of helium assumed in radiative equilibrium, on account of the reduction of the  $I$ 's (and consequently of the temperature variations) and the behaviour of the opacity there. On the basis of the linear nonadiabatic pulsations of his 'discrete' models, Zhevakin has discussed in great detail the phase-shift between the radiation flux and the radial velocities and the factors that affect it in view of explaining the peculiarities of this phase-shift in different types of variable stars.

The de-stabilizing influence of the second helium ionization for stars in the general vicinity of the Cepheids strip has been confirmed by the detailed investigations of Baker and Kippenhahn (4) and J. P. Cox (5) provided that the main transfer of heat there be by radiation and not by convection as detailed investigations seem indeed to indicate.

As to the exact location of the zone of maximum (linear) instability in the H-R diagram one might expect on general ground that it should coincide rather exactly with the actual Cepheid strip providing, at the same time, an explanation of the Period-Luminosity relation. Detailed computations (6) however seem to show that it is shifted to the right by about  $600^\circ$  in effective temperature.

As far as the phase-shift is concerned, Baker and Kippenhahn, contrarily to Zhevakin, failed to recover the observed  $90^\circ$ -value. In this respect there is no absolute necessity to reach an interpretation in the frame of this theory and explanations in terms of linear and non-linear mechanisms outside of it have been proposed at different times, the latest one, resorting to a periodic alternance of radiative and convective equilibria in the ionization zones, being due to E. Böhm-Vitense (7). However, it is also possible that the approximate pressure boundary condition used by Baker and Kippenhahn which forces a peculiar behaviour of the pressure variation close to the surface, is partly responsible for the discrepancy.

All these investigations were perforce limited to the external part of the star since no reasonable models were available for Cepheids. However, with the recent progress in evolutionary tracks for fairly heavy stars, the situation should improve rapidly in that respect. On the other hand, despite the many attractive features of Zhevakin's theory especially as far as providing also a natural mechanism for the limitation of the amplitude to some finite value, the present reviewer (cf. 1) still feels uneasy about the rôle of the higher modes which, on the basis of any theory situating the source of instability far out in the external layers, should also be excited at least as strongly as the fundamental one.

An interesting analysis especially of the possible effects of convection on stellar pulsations is due to R. F. Christy (8) who, since then, has tackled numerically and with encouraging results, the non-linear problem (9) looking for stationary periodic solutions of the general equations of motion in external stellar envelopes of stars of the RR Lyrae type and susceptible to represent their observed finite oscillations. This is, in some respects, quite a formidable problem but perhaps the use of large computers will indeed bring us to the solution much more rapidly than expected although some measure of caution is still indicated. Anyway, work on this problem is also reported by A. N. Cox and K. H. Olsen (10), by J. P. Cox, A. N. Cox and K. H. Olsen (11) as well as by V. I. Aleshin (12), all with some measure of success either in following the establishment of finite self-excited oscillations or (and) in recovering observed characteristics of the light and velocity curves.

As far as RR Lyrae and  $\beta$  Canis Majoris stars are concerned, theoretical work is going on in Oslo and a general critical review of the problems raised by the first group of stars is in preparation by E. Haugen (13). On the other hand, in the course of their general work on the stability and the oscillations of rotating gaseous masses (14), Chandrasekhar and Lebovitz have encountered a possibility of a coupling between two normal modes for a value of  $\Gamma_1$  of the order of 1.6 which has led them to a new suggestion (15) as to the interpretation of the double periods in  $\beta$  Canis Majoris stars.

Accounts of the present state of the linear theory of stellar stability have been prepared by P. Ledoux (16, 17) using for the presentation of the general problem a point of view related to that adopted by Jeans in '*Astronomy and Cosmogony*'. Special attention has been paid to the effects of nuclear processes in particular nuclear equilibrium and, in (17), on the significance of stability considerations for stellar evolution. The remark is also made that the general problem is fundamentally of a higher order in the time than suggested by the usual treatment and that time-scales other than those associated with the classical subdivision into dynamical, vibrational and secular stability might be significant. In fact, in some independent unpublished work by A. S. Thompson (18), an effect of that type is discussed explicitly although partially and perhaps not quite adequately.

Boury (19) has studied the vibrational stability of initially pure hydrogen stars taking into account the thermo-nuclear reactions between elements formed during the contracting phases and has determined the critical mass ( $M_c \cong 300 M_\odot$ ) above which they become unstable. The effects on vibrational stability of convective energy transport and turbulent viscosity in the extensive cores of massive main sequence stars has been investigated in detail by Boury,

Gabriel and Ledoux (20) who find that they affect very little the value of the critical mass derived by Schwarzschild and Härm. On the other hand, Gabriel (21) has shown that the current wholly convective models used to represent small mass stars at the lower end of the main sequence are vibrationally unstable.

The general fourth order problem for the non-radial oscillations of the main sequence models of Schwarzschild and Härm has been solved for a series of modes and spherical harmonics of different degrees by P. Smeyers (22) who is carrying on a discussion of the interaction, through the g-modes, of convective and dynamical stability and also of the vibrational stability of the models.

P. Ledoux has started a general investigation of the asymptotic behaviour of very high modes of radial pulsation (23) which show that the amplitude becomes negligibly small everywhere except close to the centre and in the external layers. This suggests the possibility of a special type of close interaction between these regions which is being checked by a study of the vibrational stability of these high modes. This discussion is being extended to the asymptotic non-radial modes.

Secular stability whose discussion is the least advanced and which is perhaps in need of a new start has been apparently the object of little research, and the only paper of which I am aware is due to Y. Osaki (24) who has developed a criterion of secular stability for degenerate stars on the basis of the relation between the variations of entropy and temperature.

It is impossible to review here the extensive work of Chandrasekhar and Lebovitz on the stability of rotating fluid configurations using high order virials. Among the papers which have appeared in the *Astrophysical Journal* these last three years, we have already referred to a few (14) and (15) which are of special interest for this Commission and we may add another one by Chandrasekhar and Lebovitz (25) devoted to the interaction between dynamical and convective instability in gaseous stars and one by Chandrasekhar (26) concerning a general variational principle applicable to both radial and non-radial oscillations of compressible masses.

Applications of the generalized virial to the hydromagnetic oscillations of a self-gravitating fluid have been developed by D. G. Wentzel (27) and Woltjer (28) has emphasized the point that for non-radial oscillations of reasonable configurations the direct effect of the presence of a magnetic field is to increase the frequency, thus to reinforce the stability. Kristian (29) has studied the hydromagnetic oscillations of a fluid sphere about a stationary (non-static) state. In presence of a small field  $H$ , he finds modes with frequencies directly proportional to  $H$  as well as the more usual ones whose frequencies are corrected by a term proportional to  $H^2$ .

Considerable developments are to be noted in the application of the theory of shock waves to stars mainly to explain either peculiarities in the spectrum of variable stars or the characteristics of novae and super-novae. With respect to the latter a general review by Schatzman (30) should appear shortly which contains some new suggestions concerning the increase of radius of novae during the exploding phase and the mechanism of mass ejection.

Many of the detailed investigations rest on extensions of Chisnell's and Whitham's methods to stellar configurations. Ono and Sakashita (31) have discussed super-novae explosions, considering that the gravitational energy released by the collapse of the core of a far evolved star and by the nuclear reactions induced in the envelope generates a shock. The propagation of the latter through the envelope can then be followed using their previous results giving, in a physical star, the rate of variation of the strength of the shock and the material velocity behind it as a function of the distribution of the state variables in front of it. This allows one to compute the point where the material velocity reaches the escape velocity and the mass above it which is ejected. Again Ohyana (32) using the same general picture but including more explicitly the energy released by reactions between light elements such as C and O following the collapse of the core and the effects of the escape of radiation from the shock as it approaches the stellar

surface finds general agreement with the observational data of type II super-novae. Another variant of this is used by Sakashita and Tanaka (33) in an attempt to relate the origin of Planetary Nebulae to the helium flash in red giants.

The propagation of strong shock waves in polytropic gaseous spheres has also been studied by Nadezhin and Frank-Kamenetski (34) who discussed the fraction  $\Delta M$  of the mass loss in term of the initial energy  $E$  of the disturbance ( $E \cong GM^2/R$ ,  $\Delta M \cong M$ ;  $E = 10^{-3} GM^2/R$ ,  $\Delta M \cong 10^{-6} M$ ). On this picture, taking radiation and the ionization of hydrogen in the external layers into account, Imshenik and Nadezhin (35) have constructed theoretical light curves of novae and super-novae in reasonable agreement with the observations.

Colgate and his group (36 a) have continued to work on the problem using a more direct numerical approach but they seem to have had difficulties in making the collapsing configuration bounce back especially if the emission of neutrinos is taken into account. However a non-negligible interaction of the envelope with the flux of neutrinos has been suggested as a possible way out (36 b). Using his work reported in Sect. IV, Emin-Zade (37) assumes that, before outbursts, novae are white-dwarfs containing a residual abundance of hydrogen of several percents, the explosions being attributed to the strong increase of the rate of thermonuclear reactions due to expansion and transition from a condensed state to normal stellar plasma.

In two papers (38), Masani, relying especially on Witham's method, gives approximate formulae and tables for the evolution of the strength of a weak adiabatic shock as it propagates in a spherical compressible mass and which bring out the different behaviour in the central regions where the curvature effect is dominant and the external half where the gradient of density is the main factor. Kaplan (39) has also investigated the propagation of ordinary and shock waves in stellar interiors and envelopes paying special attention to the transformation of Riemann waves into shock waves.

With a view to applications to surface nuclear reactions and the explanation of peculiar abundances, Schatzman (40) has discussed the accelerations of particles in a magneto hydrodynamic shock in presence of heterogeneities in the magnetic field.

The effects of precursor radiation in the Lyman continuum on the structure of a shock front in a stellar atmosphere have been studied by Whitney and Skalafuris (41). The general effects of radiation and conductivity on shock waves have also been discussed by V. S. Imshenik (42). Frank-Kamenetski (43) has investigated radiative transfer of energy in the continuum in a medium which is not in thermal equilibrium and Kaplan (44) has considered the scattering of light in a non-stationary medium. I. A. Klimishin (45) has also discussed the influence of emission and absorption of energy on the characteristics of a shock wave moving in a stellar envelope. He has analyzed (46) the possibility for a medium shock wave to throw off an external envelope in a giant star and has discussed the possibility of stationary supersonic outflow of material from stellar atmospheres (47). Attention may also be drawn to the general discussion on shock-waves in stellar envelopes and their astrophysical implications in the Proceedings of the Varena Symposium on aerodynamic phenomena in stellar atmospheres (48).

The same volume contains also a chapter on non-catastrophic mass-loss from stars with a very elegant presentation of the relevant fundamental hydrodynamics by Germain.

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VI. EFFECTS OF ROTATION, MAGNETIC FIELDS AND EXTERNAL  
GRAVITATIONAL FIELD

The progress of our knowledge of actual stars, of their formation and of their evolution is somewhat hampered by the complexities raised by the effects of rotation, magnetic fields and external gravitational fields (binaries) at least when the intensity of the corresponding forces becomes appreciable.

These complexities are of two main types. Even if these fields of forces are reduced to their simplest models, quite serious difficulties are encountered in trying to build a hydrostatic configuration as soon as the density is allowed to vary with depth as is well illustrated by the considerable and not too successful efforts devoted to the classical problem of the heterogeneous incompressible fluids in uniform rotation. The second type of difficulties of a more physical character arises when the thermodynamical properties of an actual gaseous star are taken into account allowing the establishment of thermal baroclinic fields generating secondary circulations.

A renewal of interest in the first type of problems seems to have developed recently. Chandrasekhar and Lebovitz (1) have discussed the integral relations yielded by the second-order virial equations for uniformly rotating configurations and Chandrasekhar and Roberts (2) have used them, in the case of a slow rotation, to set up upper and lower bounds for  $m/\epsilon_R$  where  $m$  is the ratio, at the equator, of the centrifugal to the gravitational accelerations and  $\epsilon_R$  the ellipticity of the external surface. Roberts (3) has built approximate polytropic configurations in high uniform rotation assuming equidensity surfaces to be similar oblate spheroids and determining, by variational methods, the excentricity of these spheroids and the distribution of density. A detailed discussion of the polytrope of index  $n = 1$  led him to the conclusion that bifurcation to a Jacobi form will occur for a critical compressibility corresponding to a value slightly less than  $n = 1$  ( $\gamma = 2$ ) which is only slightly larger than that of Jeans ( $n = 0.83$ ,  $\gamma = 2.2$ ). This discussion is generalized in another paper (4) in which the excentricity of the equidensity surfaces is allowed to vary. This type of problem allowing however for non-uniform rotation has also been tackled by R. Stoeckly (5) in Princeton. Kalitzin (6) has discussed some consequences of a cylindrical law of rotation ( $\omega$ , function of the distance to the axis) both for the homogeneous and Roche's models and generalized some classical propositions. On his side, V. V. Porfiriev (7) has continued his studies of stellar models in rotation looking for stable stationary situations and evaluating the resulting circulations for different models.

Strong anisotropic viscosity such as may arise due to turbulence in the hydrogen convection zone can also produce meridional circulation and differential rotation as it has been shown by Kippenhahn (8) and Roxburgh (9) who finds that this gives rise to large scale motion towards the equator at low latitudes and towards the poles at high latitudes. Here we touch the second aspect of the problem referred to in the introduction and which has been the object of a detailed review by Mestel (10) which covers not only rotation but also the other factors susceptible to create such circulations. However before coming to this point, let us note first the articles of Wentzel (11) and Woltjer (12) in which the effects of a magnetic field on the shape of stars in static equilibrium are discussed.

The combined effect of rotation and magnetic fields have been further discussed by Mestel (13) who finds that in a rotating star, an initially poloidal field acquires a toroidal component which will lead to a steady increase of angular velocities towards the equator. In another paper with Roxburgh (14), it is shown that the Biermann effect (i.e. the building up of an appreciable toroidal magnetic field due to the electron partial pressure in a rotating star devoid initially of any magnetic field) is much reduced by the presence of a primeval magnetic field with a weak poloidal component. Roxburgh (15) finds that in radiative zones in presence of arbitrary angular velocity and magnetic fields there are only two steady solutions: (1) uniform rotation maintained by a weak magnetic field and Sweet circulation currents, (2) dominant toroidal field and the splitting of the circulation into two zones allowing mixing between the convective

core and the envelope. In absence of a poloidal field, one must distinguish two cases depending on whether the centrifugal force is larger or smaller than the magnetic force and the corresponding solutions are presented in different papers (16).

The effects of a magnetic field in a contracting star have been considered by Chvojková and Kohoutek (17) who find that a thick equatorial shell could be detached from the star. Schatzman (18) has pursued his studies of the consequences of conservation of angular momentum and mass loss in presence of strong magnetic activity during star formation. In his lectures in Varenna (19), he has also discussed the effects of mass loss from the equator of a fast rotating star and made a first attempt to find the resulting meridian circulation.

A short review of the rotational, magnetic and tidal effects on the shape of a star and of the influence of the resulting circulations on the mixing between different parts of the star as their chemical composition varies in the course of evolution was presented by Kippenhahn (20) during the same course in Varenna.

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## VII. GRAVITATIONAL INSTABILITY — ORIGIN OF STARS

A short but very pertinent review of the classical problem of gravitational instability as introduced by Jeans has been presented by S. Chandrasekhar (1) including the effects of a uniform rotation and a uniform magnetic field alone or in combination. Quite a number of papers have been devoted to generalizations of this problem to somewhat more complex situations. For

instance, Kossacki (2) has considered the gravitational instability of a homogeneous, infinite viscous rotating medium with finite electrical conductivity and many aspects of this generalized problem have been reviewed by Pacholczyk (3).

Other discussions especially directed at the stability of spiral arms are due to Stodolkiewicz (4). R. K. Jaggi (5) has also rediscussed the stabilizing effect of a magnetic field on the perturbations of a cylindrical plasma and J. L. Tassoul (6) has extended the discussion to include the effect of rotation as well. The problem has also been the object of a critical analysis by R. Simon (7) emphasizing the importance of correct boundary conditions. If compressibility is taken into account, Simon (8) finds that the notion of critical wavelength and marginal mode vanishes. He has also reconsidered quite generally the influence of a non-uniform rotation (9) on the usual Jeans's problem and has brought some precisions on the circumstances in which Bell and Schatzman's result applies. The effects of non-uniformity either in rotation or magnetic fields or of finite conductivity have also been the object of other papers (10) some of which have been rather uncritical in their use of 'local analysis' (cf. 11).

Hruska (12) has considered the problem of the stability of a cylindrically symmetrical gas undergoing a radial stationary expansion from the axis where mass is supplied. Under certain assumptions, he finds that strong instabilities should occur at a critical distance which recalls some characteristics of the central region of the galaxy.

But even the classical Jeans's problem is not without ambiguities and its application to actual astrophysical conditions raises some difficult points. Simon has devoted a series of papers (13) to a critical discussion of the formal problem considering in particular the instability associated with the growth of a single local perturbation. Attempts on more realistic problems, concerning the fragmentation of gas clouds already in contraction or expansion are due to Hunter (14) and Savedoff and Villa (15), the latter concluding that the instabilities encountered in this case do not seem to have much to do with Jeans's criterion (except in an isolated critical case without probably much physical significance); it is not obvious either that they will really lead to the wanted fragmentation.

On the other hand, Hatanaka, Unno and Takebe (16) trying to introduce the actual physics of interstellar matter in the relation between the perturbation of density and pressure, have obtained a correcting factor to Jeans's criterion which may reduce the critical mass appreciably.

A penetrating analysis of the difficulties presented by the problem of the fragmentation of a more or less homogeneous cloud can be found in a paper by D. Layzer (17) and he has begun to develop his own ideas on the formation of self-gravitating systems through gravitational clustering in an expanding cosmic distribution in another paper (18). Clustering but of small 'flocules' plays also a major role in McCrea's suggestion (19) aimed especially at getting rid of the angular momentum difficulties. Cameron (20) and Cameron and Ezer (21) have emphasized the importance of the phases of dynamical instability and dynamical collapse which must arise in a contracting cloud when its internal temperature is able to bring about dissociation of molecular hydrogen and ionization of hydrogen and helium.

Mestel (22) has discussed in detail the evolution of the magnetic field of a gravitationally-bound isothermal magnetic cloud formed by the non-homologous compression of an external uniform medium permeated by a uniform magnetic field and shown that it is able to relax to a dipole-type structure with most of the cloud field-lines detached from the background galactic field.

Papers (23) attempting to account for the observed mass function in stellar system on the basis of gravitational instability may be of some help in confirming or otherwise the real significance of the corresponding criteria.

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