The Solar Dynamo

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Abstract: The traditional $\alpha\Omega$-dynamo as a model for the solar cycle has been successful in explaining the butterfly diagram, phase relations between poloidal and toroidal field, and polar branch migration features. Observational and theoretical achievements in recent years have however shaken this picture. The current trend is towards dynamos operating in the overshoot region of the convection zone. Nevertheless, there are many open questions and a consistent picture has not been established. In this paper we compare recent approaches and discuss remaining problems.

1. Introduction

Magnetic fields are the engine of solar and stellar activity. The most prominent activity phenomenon is the 11 year sunspot cycle. At the beginning of this century Hale showed that sunspot pairs involve strong magnetic fields which reverse orientation from one cycle to another, and thus the original field orientation is recovered after two cycles, making the magnetic cycle period 22 years. There are variations of activity on a much smaller timescale as well, for example those related to prominences, faculae or flares, which occur in a more irregular fashion. Also, long-term variations are known, such as Grand Minima, which do not seem to occur very regularly either. Of course, the period of the solar cycle is not exactly 11 years, but may vary between 7 and 17 years, and the cycle's amplitude also varies. However, there is a clear peak in the power spectrum computed from the time sequence of the sunspot number. Also, the migration of sunspot activity belts is very systematic.

The overall geometry of the solar cycle magnetic fields has been successfully explained by the $\alpha\Omega$-dynamo models of Steenbeck and Krause (1969). Twenty years ago it was hoped that minor disagreements between models and reality could be ironed out with the development of more realistic turbulence models ($\rightarrow$ α-effect) and with improving observations of solar differential rotation ($\rightarrow$ Ω-effect).

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2. The solar field geometry

Before we discuss different approaches in more detail let us first consider the solar observations that are of direct relevance for mean-field dynamos. Perhaps the most important observation is Hale's polarity law. According to Parker's (1955) interpretation sunspot pairs are formed when toroidal flux ropes rise and break through the solar surface. The two footpoints of an emerging field loop then correspond to a sunspot pair. With remarkably small statistical scatter the orientation of sunspot pairs and bipolar magnetic regions are opposite on opposite sides of the equator, reversing after 11 years (Wang and Sheeley, 1989). From Hale's polarity law we learn that the Sun has a systematic azimuthal magnetic field B_ϕ which is *antisymmetric* about the equator. Measurements of the radial field component B_r can be obtained from the Mt. Wilson and Kitt Peak magnetograms. Yoshimura (1976) and Stix (1976) have shown that B_r and B_ϕ vary approximately in antiphase ($B_r B_\phi < 0$ for most of the time). The radial field at the pole is of special interest, because there the observations of B_r are not contaminated by a B_ϕ-field. The curve $B_r = 0$ corresponds to the location of faculae (see Stix, 1974). For latitudes above 60° this curve shows a poleward migration. This feature is often called this *polar branch*.

3. The traditional $\alpha\Omega$-dynamo

The governing equation in the theory of $\alpha\Omega$-dynamos (Steenbeck and Krause, 1969; Roberts and Stix, 1972) is the induction equation for the mean field $\langle \boldsymbol{B} \rangle$:

$$\frac{\partial}{\partial t}\langle \boldsymbol{B} \rangle = \operatorname{curl}\left(\langle \boldsymbol{u} \rangle \times \langle \boldsymbol{B} \rangle + \alpha \langle \boldsymbol{B} \rangle - \eta_t \operatorname{curl}\langle \boldsymbol{B} \rangle\right). \tag{1}$$

Induction effects are due to the α-effect and to gradients in the angular velocity $\Omega = \langle u_\phi \rangle / r \sin\theta$. In the following, by a *traditional* $\alpha\Omega$-dynamo we mean that

$$\alpha \cos\theta > 0, \quad \partial\Omega/\partial r < 0. \tag{2}$$

In the model of Steenbeck and Krause (1969), simple profiles for α and Ω were used. If Eq.(1) is solved inside a sphere of radius R, and if a vacuum outside is assumed ($\operatorname{curl}\langle \boldsymbol{B} \rangle = 0$ for $r > R$), then there is a marginal stable oscillatory solution which is antisymmetric about the equator (odd parity, in accordance with Hale's law) with dynamo waves migrating equatorwards. It should be stressed that the preference for an antisymmetric field is sensitive to boundary conditions and to the thickness of the convection zone (see e.g. Table 5 in Roberts, 1972). The magnetic cycle period is $T \approx 0.20\tau_{\text{diff}}$, where $\tau_{\text{diff}} = R^2/\eta_t$ is the global diffusion timescale. For $\eta_t = 10^{13}\text{cm}^2/\text{s}$ we have $\tau_{\text{diff}} \approx 16$ years, i.e. T=3 years, which is too short for the 22 year solar period by a factor of 7. A polar branch is not present in model 1 of Steenbeck and Krause, but models by Köhler (1973) and Yoshimura (1975), which include a gradient $\partial\Omega/\partial\theta$, do indeed show a poleward migration at higher latitudes. Also a more complicated θ-dependence in α (Schmitt, 1987) or in $\partial\Omega/\partial r$ (Makarov *et al.*, 1988; Belvedere, 1990) can produce a polar branch.

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4. The parity problem

We mentioned in the previous section that the parity of the generated magnetic field depends on details of the dynamo model (e.g. boundary conditions and thickness of the convection shell). In addition, the degree of nonlinearity also can have an influence on the parity. The parity of the dynamo can depend critically on the dynamo number D. Models show that there is a range of D, where the odd parity solution loses stability to a mixed parity solution (Brandenburg *et al.*, 1989a). The importance of studying the stability properties of nonlinear dynamos has been stressed by Krause and Meinel (1988). The most detailed investigation has been presented by Jennings (1991).

The parity problem is relevant to understanding details of the solar dynamo. Deviations of the solar field from the pure dipole-type symmetry can be deduced from the observed North-South asymmetry of sunspot numbers. This has been investigated in detail by Vizoso and Ballester (1990). The degree of asymmetry of the toroidal field is an observable quantity which can in principle be predicted from dynamo models (cf. Brandenburg *et al.*, 1989b).

The phenomenon of active longitudes is surprisingly regular (Tuominen, 1962) and may be due to non-axisymmetric contributions to the dynamo generated field (Stix, 1974). It is known that in the presence of strong differential rotation non-axisymmetric fields are much harder to excite than axisymmetric ones (Rädler, 1986). However, in the highly nonlinear regime secondary bifurcations to mixed parity solutions with nonaxisymmetric contributions can occur. In a simplified two-dimensional model Jennings *et al.* (1990) found an example, where the A0-type solution can lose stability to a mixed mode with A0 and S1 contributions. Certainly, more realistic three-dimensional models are required to make conclusive statements concerning active solar longitudes.

5. Transport coefficients

A severe uncertainty in the theory of solar mean-field dynamos concerns transport coefficients such as α and η_t. These coefficients have often been determined analytically using a simple model for isotropic turbulence and considering rotation and stratification as small perturbations. A closure is achieved by taking only second order correlations into account (first order smoothing). In their original paper Steenbeck *et al.* (1966) also obtained the turbulent diamagnetic effect, and anisotropies in α.

In a similar manner coefficients for the eddy viscosity and the turbulent heat conductivity can be obtained. Latitudinal dependences and anisotropies of these coefficients are important here, because they can drive differential rotation. Another, perhaps more important, driver of differential rotation is Rüdiger's (1977) Λ-effect – a hydrodynamic analogue to the α-effect. Similar ideas have been developed by Frisch *et al.* (1987), who called this the AKA-effect (anisotropic kinetic alpha-effect).

A useful complement to analytical theories are numerical simulations. An early example of this type is the two-dimensional model by Moss (1971, unpubl. report), who found $\eta_t \approx 0.2 u_t \ell$, where u_t is the turbulent rms-velocity and ℓ is a correlation length. Kraichnan (1977) found examples where η_t can even be negative. Pulkkinen *et al.* (these Proceedings) determined the latitude dependence of the Λ-effect and found agreement between observations and theory (see Tuominen, 1990). A numerical determination of the eddy heat conductivity and other stellar mixing length parameters has been carried out by Chan and Sofia (1989). Recent simulations of magnetoconvection in the presence of rotation have indicated that the α-effect in the vertical direction can be of opposite sign to the horizontal components of the α-tensor (Brandenburg *et al.*, 1990a).

As discussed by Krause (these Proceedings), simulations even at a resolution of 63^3 gridpoints can hardly be expected to represent the circumstances under which mean-field theory is valid. The correlation length in these models is comparable with the size of the simulated domain. Simulations of stellar convection by Stein and Nordlund (1989) do indicate that the correlation length in the vertical direction is much longer than in the horizontal directions (for a review see Spruit *et al.*, 1990). On the other hand, mean-field theory applies only if the correlation length is short compared with global dimensions. In addition, first order smoothing is applicable only if the correlation time is short compared with the turnover time and rotation period. For the Sun this is not the case. It seems therefore that simulations and analytical theories may approach the solar case from opposite directions.

6. Nonlinear feedbacks

An important property of stellar dynamos is nonlinearity. Although the induction equation (1) is at first glance linear, there can be nonlinear feedbacks if the average velocity $\langle \boldsymbol{u} \rangle$ or the α-coefficient depend on $\langle \boldsymbol{B} \rangle$. One may refer to these nonlinearities as micro- and macro-feedback. In both cases the feedback originates from the Lorentz force in the momentum equation and it is the length scale associated with the Lorentz force which leads to this distinction. These aspects have been examined by Gilbert and Sulem (1990), and in the context of $\alpha\Lambda$-dynamos by Brandenburg *et al.* (1990b). Properties of macro-feedback and the relevance of the Elsasser number have been stressed by Roberts (these Proceedings).

Another important feedback is that from magnetic buoyancy (Noyes *et al.* 1984; see also Rädler, 1990; Moss *et al.*, 1990a). Here the magnetic pressure term, as part of the Lorentz force, provides the feedback. The length scale associated with the buoyancy force is usually short compared with the thickness of the convection zone and we may classify this as a micro-feedback. However, convective dynamo simulations indicate that the main feedback is from the magnetic curvature force, rather than from the magnetic pressure gradient (Brandenburg *et al.*, 1990c).

The dynamo has various properties which arise from the presence of nonlinearities. We mentioned already the stability problem (Sect. 4), which makes sense only if nonlinearity is involved. Feedbacks on the mean motions caused by the mean magnetic fields are observed in the form of torsional waves and cyclic variations of the meridional circulation (Tuominen and Virtanen, 1987; 1988). Quite another aspect is chaos and irregularity, which is usually studied using ordinary differential equations derived from the three-dimensional dynamo equations (for reviews see Weiss, 1989, 1990).

7. The solar angular velocity

Contours of constant angular velocity in the Sun are not cylindrical, as early theories of differential rotation predicted (Durney, 1976), but rather are "disk shaped" (Rüdiger, 1989). Realistic numerical models show this property if the Rossby number is of order unity (Tuominen and Rüdiger, 1989) and if the Taylor number is not too large (Ta $\lesssim 10^6$, i.e. $\nu_t \gtrsim 3 \times 10^{13}$). There is a remarkable consequence for $\alpha\Omega$-type dynamo models: oscillatory solutions can only be obtained if C_Ω is large enough ($\gtrsim 10^3$), i.e. $\eta_t \lesssim 10^{12}$. This is a puzzle, because ν_t and η_t are *turbulent* kinematic and magnetic diffusivities which are not expected to differ substantially from each other. Future work will show how much this Taylor number puzzle (Brandenburg *et al.* 1990b) is model dependent.

8. The $\partial\Omega/\partial r > 0$ problem

Differential rotation is a good candidate not only for causing the cyclic behavior, but also the migration of the solar magnetic field pattern. However, strong anisotropies of the α-effect can produce solar-like butterfly diagrams as well (Weisshaar, 1982). One reason why such approaches have not been considered further is that the degree of anisotropy needed is rather large.

Latitudinal differential rotation can also yield oscillatory dynamos, but such models do not exhibit any significant field migration, only periodic field emergence at mid-latitudes followed by diffusion both to higher and lower latitudes (Köhler, 1973). Only with a sufficient amount of radial differential rotation is there equatorward migration at low latitudes and poleward migration at high latitudes (polar branch). Such models have solar-like field geometry if the inductive effects at low latitudes are as in Eq.(2). Parker (1989) suggested that, in addition to a purely latitudinal differential rotation, suitable nonuniform distributions of the α- and Ω-effects might result in an equatorward migration. For example, an enhanced α at low latitudes (cf. Schmitt, 1987) might modify latitudinal diffusion in some way. However, satisfactory models have not been presented yet.

The most promising mechanism for explaining (i) the cyclic behavior and (ii) the migration properties as well as (iii) the strength of the toroidal field relative to the poloidal field seems to be a dynamo of $\alpha\Omega$-type with $\partial\Omega/\partial r$ being important. Recent results of helioseismology (e.g. Goode, these Proceedings) seem to exclude the possibility of a negative radial Ω-gradient at the equator. The consequences for current $\alpha\Omega$-dynamo models are either a poleward migration of dynamo waves (if $\alpha\cos\theta > 0$) or an incorrect phase relation between poloidal and toroidal field ($B_r B_\phi > 0$ if $\alpha\cos\theta < 0$).

In recent years various $\alpha\Omega$-dynamo models with $\partial\Omega/\partial r > 0$ have been presented (e.g. Belvedere *et al.*, 1990a), where the problem with the phase relation has obviously been ignored. These models are mainly designed for the lower convection zone or for the overshoot layer below. The hope is perhaps that the upper convection zone might lead to a systematic phase flip. However, even dynamo models with complicated radial profiles of α and $\partial\Omega/\partial r$ do not give deviations from the expected phase relation. The possibility that the observations are not reliable enough does not seem to be justified, although measurements of B_r and B_ϕ are tricky, cf. Stix (1981).

The model presented by Wilson (1988) tries to explain the dynamo dilemma in terms of a two component fluid with magnetic and non-magnetic constituents. He suggests that the dynamo does not "feel" the average angular velocity $\langle\partial\Omega/\partial r\rangle$, but rather the angular velocity $(\partial\Omega/\partial r)_B$, associated with magnetic flux tubes. A negative $(\partial\Omega/\partial r)_B$ seems plausible, if flux tube motion is considered as being predominantly governed by the Coriolis force. That these gradients are different is supported by the observation that the rotation rate of young sunspots (Tuominen and Virtanen, 1988) and magnetic tracers (Stenflo, 1989a) is larger than the value obtained from helioseismology.

9. Dynamos for the overshoot layer

The overshoot layer of the Sun is of considerable interest for the dynamo for two reasons: (i) magnetic flux losses due to magnetic buoyancy are small here, and (ii) there is evidence that α changes sign at the bottom of the convection zone, which when combined with $\partial\Omega/\partial r > 0$ gives equatorward migration. A change of sign of α in the overshoot layer follows from a formula due to Krause (1967). However, this formula is perhaps invalid at this location where the turbulence is expected to be far from homogeneous. One should note that the argument for a reversal of α, often quoted in the literature, comes from results of Boussinesq convection!

DeLuca and Gilman (1986) considered a hydromagnetic, two-dimensional, Cartesian model for the interface between the convection zone and the radiative interior. An interesting property of their model is that magnetic energy considerably exceeds kinetic energy. The importance of this for understanding the observed magnetic fluxes in the Sun has been stressed by Durney *et al.* (1990).

There has been considerable interest in models without explicit radial dependence, with the idea that the dynamo works in a thin layer. However, it is not at all evident that radial structure is unimportant. A model without vertical extent was presented by Leighton (1969), without, however, attributing it to a "thin shell". Nonlinear dynamics of such models have been investigated by Schmitt and Schüssler (1989), Jennings and Weiss (1990), and Belvedere *et al.* (1990b). There is evidence that such dynamos are of $\alpha^2\Omega$-type (Gilman *et al.*, 1989). In order to produce the correct period and number of field belts the correlation length should be two hundred times shorter than the local pressure scale height (Choudhuri, 1990). This result is based on a Cartesian approximation. Dynamos in thin spherical shells, however, seem to have many belts, or otherwise they become steady (Moss *et al.*, 1990b).

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10. Flux tubes and magnetic buoyancy

The solar magnetic field is highly intermittent (see review by Stenflo, 1989b). This must have consequences for the traditional $\alpha\Omega$-dynamo. Is it possible, that the mean-field concept is still applicable (Schüssler, 1980), and only that the transport coefficients (e.g. α, η_t) are reduced (Childress, 1979))?

Another consequence is magnetic buoyancy experienced by horizontal field (see Hughes and Proctor, 1988). The idea often presented is that the dynamo generates a diffusive field which gets concentrated into flux tubes by doubly diffusive instabilities (Schmitt and Rosner, 1983), dynamical fragmentation (Schüssler, 1977, 1979), or flux expulsion (Galloway *et al.*, 1977). Magnetic buoyancy acting on these tubes can then rapidly remove field from the dynamo region (Parker, 1975). However, it is unclear how efficient this effect is, or whether it even negates the mechanism of a solar dynamo.

Meanwhile several arguments have been accumulated against the importance of buoyancy, for example turbulent diamagnetism and topological pumping as well as drag forces (Schüssler, 1984). On the other hand, Petrovay (these Proceedings) presented new arguments at this conference as to why topological pumping might actually not work. Many other interesting ideas have been proposed, for example Parker (1987) estimated that thermal shadows could push flux tubes down whereas Choudhuri (1989) argues that flux tube motion is dominated by the Coriolis force and hence tubes rise parallel to the rotation axis. In this case one might expect sunspots to emerge at rather high latitudes. Schüssler (1983) concluded from Hale's polarity law that bipolar regions can only be produced by strong flux tubes which can resist the irregular turbulent motions and prevent the field from being "brainwashed".

11. Fast and convective dynamos

Much attention has been devoted to the investigation of fast dynamos (Vainshtein and Zeldovich, 1972). The relevant timescale here is not the global diffusion time $\tau_{\rm diff} = L^2/\eta$ (where η is the non-turbulent value!), but the convective turnover time $\tau_{\rm conv} = L/u_{\rm turb}$. Thus dissipation acts on the length scale of small magnetic flux concentrations. Fast dynamos are of astrophysical relevance, because the global diffusion timescale for the Sun is comparable to the age of the universe.

Kinematic fast dynamos have been investigated using prescribed ABC-flows, with and without the presence of diffusion (Galloway and Frisch, 1986; Gilbert and Sulem, 1990). Recently there have been a number of *direct simulations* of the dynamo without parametrization of diffusivities and viscosities. Meneguzzi and Pouquet (1989) find dynamo action on a convective timescale, which may be called a fast dynamo. Mean-field dynamos are also fast, because the timescale is essentially independent of the (non-turbulent) diffusivities (cf. Moffatt, 1990). The first dynamo simulations in spherical geometry were those of Gilman and Miller (1981). Valdettaro and Meneguzzi (these Proceedings) presented the first spherical dynamos with compressible flow.

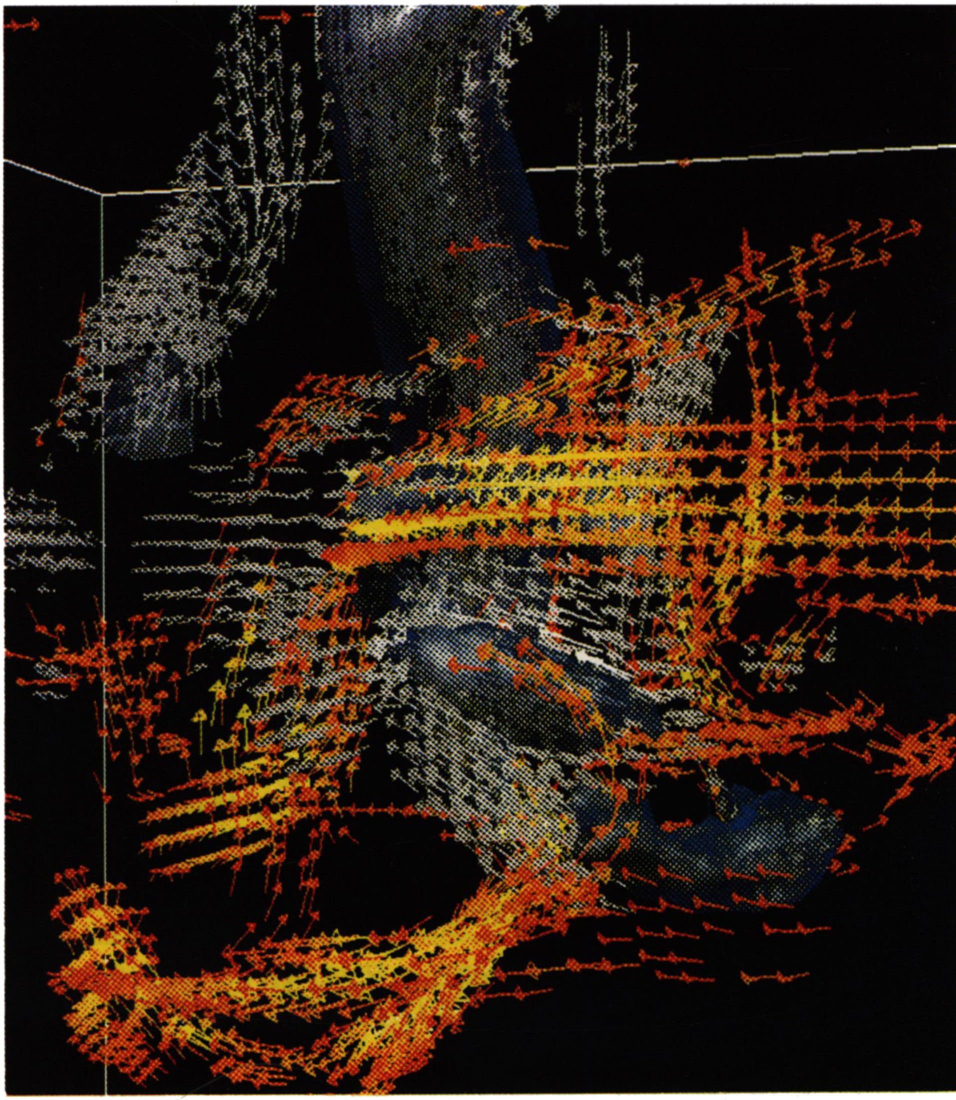

Fig. 1. Snapshot from a video animation showing magnetic field vectors in yellow (the strongest) and red (less strong) and those of vorticity in white. Transparent surfaces of constant negative pressure fluctuation are shown in blue. Note that the vectors of magnetic field form flux tubes which become wound up around a tornado-like vortex.

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Simulations including overshoot (Brandenburg *et al.*, 1990c) give the impression that magnetic buoyancy may not be a problem for the dynamo, because it is overwhelmed by other dynamical forces. Indeed, such dynamical forces are responsible for the formation of flux tubes in the first place. The picture of diffuse field generation is therefore not appropriate.

Figure 1 shows a snapshot from the video animation presented at the conference. This video shows the evolution of magnetic field, generated and maintained by the turbulent motions, in a Cartesian box located at the bottom of the convection zone. There is a strong tendency for field to be sucked by the downdrafts whilst upward motions of buoyant flux tubes were barely detectable.

12. Conclusions

The solar dynamo problem has many aspects – only some of them have been highlighted in this review. It is clear that at present there is no good dynamo model for the Sun! Many details are uncertain. Is the solar dynamo really of $\alpha\Omega$-type? Is the traditional mean-field concept appropriate? Is magnetic buoyancy a problem? Recent simulations indicate that flux tube dynamics are governed by suction of field in the vicinity of strong cyclonic downdrafts. Flux tubes are formed and deformed by the flow, and actively participate in the dynamo process. However, these simulations ignore the global nature of the solar dynamo. It remains a challenge to combine these separate pieces of information into a coherent picture of the solar dynamo, which may perhaps then be describable in terms of mean fields after all.

References

Belvedere, G.: 1990, in *Inside the Sun*, eds. G. Berthomieu and M. Cribier, Kluwer, 371

Belvedere, G., Proctor, M. R. E., Lanzafame, G.: 1990a, in *Progress of Seismology of the Sun and stars*, eds. Y. Osaki and H. Shibahashi, Lecture Notes in Physics

Belvedere, G., Pidatella, R. M., Proctor, M. R. E.: 1990b, *Geophys. Astrophys. Fluid Dyn.* **51**, 263

Brandenburg, A., Tuominen, I., Moss, D.: 1989a, *Geophys. Astrophys. Fluid Dyn.* **49**, 129

Brandenburg, A., Krause, F., Tuominen, I.: 1989b, in *Turbulence and Nonlinear Dynamics in MHD Flows*, eds. M. Meneguzzi *et al.*, Elsevier Science Publ., p. 35

Brandenburg, A., Nordlund, Å., Pulkkinen, P., Stein, R.F., Tuominen, I.: 1990a, *Astron. Astrophys.* **232**, 277

Brandenburg, A., Moss, D., Rüdiger G., Tuominen, I.: 1990b, *Solar Phys.* **128**, 243

Brandenburg, A., Jennings, R. L., Nordlund Å., Rieutord, M., Ruokolainen, J., Stein, R. F., Tuominen, I.: 1990c, *Nordita preprint* 90/67

Chan, K. L., Sofia, S.: 1989, *Astrophys. J.* **336**, 1022

Childress, S.: 1979, *Phys. Earth Pl. Int.* **20**, 172

Choudhuri, A. R.: 1989, *Solar Phys.* **123**, 217

Choudhuri, A. R.: 1990, *Astrophys. J.* **355**, 733

DeLuca, E. E., Gilman, P. A.: 1986, *Geophys. Astrophys. Fluid Dyn.* **37**, 85
Durney, B. R.: 1976, *Astrophys. J.* **204**, 589
Durney, B. R., De Young, D. S., Passot, T. P.: 1990, *Astrophys. J.* **362**, 709
Frisch, U., She, Z. S., Sulem, P. L.: 1987, *Physica* **28D**, 382
Galloway, D. J., Proctor, M. R. E. and Weiss, N. O.: 1977, *Nature* **266**, 686
Galloway, D. J., Frisch, U.: 1986, *Geophys. Astrophys. Fluid Dyn.* **36**, 53
Gilbert, A. D., Childress, S.: 1990, *Phys. Rev. Lett.* (in press)
Gilbert, A. D., Sulem, P.-L.: 1990, *Geophys. Astrophys. Fluid Dyn.* **51**, 243
Gilman, P. A., Miller, J.: 1981, *Astrophys. J. Suppl.* **46**, 211
Gilman, P. A., Morrow, C. A., DeLuca, E. E.: 1989, *Astrophys. J.* **338**, 528
Hughes, D. W., Proctor, M. R. E.: 1988, *Ann. Rev. Fluid Mech.* **20**, 187
Jennings, R.: 1991, *Geophys. Astrophys. Fluid Dyn.* (in press)
Jennings, R., Weiss, N.O.: 1990, in *Solar photosphere: structure, convection and magnetic fields*, ed. J. O. Stenflo, Kluwer Acad. Publ., Dordrecht, p. 355
Jennings, R., Brandenburg, A., Moss, D., Tuominen, I.: 1990, *Astron. Astrophys.* **230**, 463
Köhler, H.: 1973, *Astron. Astrophys.* **25**, 467
Kraichnan, R. H.: 1976, *J. Fluid Mech.* **77**, 753
Krause, F.: 1967, Habilationsschrift., Univ. Jena
Krause, F., Meinel, R.: 1988, *Geophys. Astrophys. Fluid Dyn.* **43**, 95
Leighton, R. B.: 1969, *Astrophys. J.* **156**, 1
Makarov, V. I., Ruzmaikin, A. A., Starchenko, S. V.: 1988, *Solar Phys.* **111**, 267
Meneguzzi, M., Pouquet, A.: 1989, *J. Fluid Mech.* **205**, 297
Moffatt, H. K.: 1989, *Nature* **341**, 285
Moss, D., Tuominen, I., Brandenburg, A.: 1990a, *Astron. Astrophys.* **228**, 284
Moss, D., Tuominen, I., Brandenburg, A.: 1990b, *Astron. Astrophys.* **240**, 142
Noyes, R. W., Weiss, N. O., Vaughan, A. H.: 1984, *Astrophys. J.* **287**, 769
Parker, E. N.: 1955, *Astrophys. J.* **121**,
Parker, E. N.: 1975, *Astrophys. J.* **198**, 205
Parker, E. N.: 1987, *Astrophys. J.* **321**, 984
Parker, E. N.: 1989, *Solar Phys.* **121**, 271
Rädler, K.-H.: 1986, *Plasma Physics*, ESA SP-251, 569
Rädler, K.-H.: 1990, in *Inside the Sun*, eds. G. Berthomieu and M. Cribier, Kluwer, 385
Roberts, P. H.: 1972, *Phil. Trans. Roy. Soc.* **A274**, 663
Roberts, P. H., Stix, M.: 1972, *Astron. Astrophys.* **18**, 453
Rüdiger, G.: 1977, *Astron. Nachr.* **298**, 245
Rüdiger, G.: 1989, *Differential rotation and stellar convection: Sun and solar-type stars*, Gordon and Breach, New York
Schmitt, D.: 1987, *Astron. Astrophys.* **174**, 281
Schmitt, D., Schüssler, M.: 1989, *Astron. Astrophys.* **223**, 343
Schmitt, J. H. M. M., Rosner, R.: 1983, *Astrophys. J.* **265**, 901
Schüssler, M.: 1977, *Astron. Astrophys.* **56**, 439
Schüssler, M.: 1979, *Astron. Astrophys.* **71**, 79
Schüssler, M.: 1980, *Nature* **288**, 150
Schüssler, M.: 1983, in *Solar and stellar magnetic fields: origins and coronal effects*, ed. J. O. Stenflo, Reidel, p. 213
Schüssler, M.: 1984, in *The hydromagnetics of the Sun*, eds. T. D. Guyenne and J. J. Hunt, ESA SP-220, p. 67
Spruit, H. C., Nordlund, Å, Title, A. M.: 1990, *Ann. Rev. Astron. Astrophys.* **28**, 263

Steenbeck, M., Krause, F., Rädler, K.-H.: 1966, *Z. Naturforsch.* **21a**, 369
Steenbeck, M., Krause, F.: 1969, *Astron. Nachr.* **291**, 49
Stein, R. F., Nordlund, Å.: 1989, *Astrophys. J. Letters* **342**, L95
Stenflo, J.O.: 1989a, *Astron. Astrophys.* **210**, 403
Stenflo, J.O.: 1989b, *Ann. Rev. Astron. Astrophys.* **1**, 3
Stix, M.: 1974, *Astron. Astrophys.* **37**, 121
Stix, M.: 1976, *Astron. Astrophys.* **47**, 243
Stix, M.: 1981, *Solar Phys.* **74**, 79
Tuominen, I.: 1990, in *The dynamic Sun*, ed. Dezsõ, Publ. Debr. Heliophys. Obs., p. 27
Tuominen, I., Virtanen, H.: 1987, in *The internal solar angular velocity*, eds. B. R. Durney and S. Sofia, D. Reidel, Dordrecht, p. 83
Tuominen, I., Virtanen, H.: 1988, *Adv. Space Sci.* **8**, (7)141
Tuominen, I., Rüdiger, G.: 1989, *Astron. Astrophys.* **217**, 217
Tuominen, J.: 1962, *Z. Astrophys.* **55**, 110
Vainshtein, S. I., Zeldovich, Ya. B.: 1972, *Sov. Phys. Usp.*, **15**, 159
Van Ballegooijen, A. A., Choudhuri, A.,R.: 1988, *Astrophys. J.* **333**, 965
Vizoso, G., Ballester, J. L.: 1990, *Astron. Astrophys.* **229**, 540
Wang, Y.-M., Sheeley, Jr., N. R.: 1989, *Solar Phys.* **124**, 81
Weisshaar, E.: 1982, *Geophys. Astrophys. Fluid Dyn.* **21**, 285
Weiss, N. O.: 1989, in *Accretion disks and magnetic fields in astrophysics*, ed. G. Belvedere, Kluwer Acad. Publ., Dordrecht, p. 11
Weiss, N. O.: 1990, *Phil. Trans. Roy. Soc.* **A 330**, 617
Wilson, P. R.: 1988, *Solar Phys.* **117**, 217
Yoshimura, H.: 1975, *Astrophys. J. Suppl.* **29**, 467
Yoshimura, H.: 1976, *Solar Phys.* **50**, 3

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