I have considerable doubts whether the distinction between $f:\mathbb{R} \to \mathbb{R}$ with $f(x) = e^x$ and $f: \mathbb{R} \to \mathbb{R}^+$ with $f(x) = e^x$ is helpful to the student at the stage of real variable. The situation is not a complicated one. It is evident from the graph of $y = e^x$ that it lies above $y = 0$, that it could be used to give the natural logarithm of any positive number and that it does not help in any way to derive a logarithm of -1. If you wanted to emphasise that you were interested in the mapping $\mathbb{R} \to \mathbb{R}^+$ I suppose you could cut off the graph paper on or below the x-axis. I would be interested to see a lecturer dealing with a persistent student who wanted to know why such considerations were relevant to mathematics *at this level.*

On the other hand, it would be both useful and instructive to collect examples of mathematics at levels where such distinctions are important and even vital. Such collections would be particularly helpful to future teachers, who need both to understand research mathematicians and also to know when not to copy them.

Yours sincerely,

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Symbols for science

DEAR SIR,

It was most interesting to read Mr. John Bausor's recent article *Symbols and how scientists use them* in the *Gazette*, June 1975 issue. It is becoming clear that letters should be allowed to stand for quantities as well as for numbers. What does not seem to be realised is that there is a simple device by which the needs of the scientists and of the mathematicians could be reconciled, i.e. the use of a symbol to stand for a physical quantity and/or the use of *another* symbol to stand for its numerical value. In the example given in Mr. Bausor's article, the setting out might be as follows:

Find g if $v = u + gt$, $u = 4{\cdot}9 \text{ m s}^{-1}$, $v = 19{\cdot}6 \text{ m s}^{-1}$ and $t = 1{\cdot}5$ s.
Let $g = z \text{ m s}^{-2}$. (Or, let $g/\text{m s}^{-2} = z$.)
Then $19{\cdot}6 \text{ m s}^{-1} = 4{\cdot}9 \text{ m s}^{-1} + z \text{ m s}^{-2} \times 1{\cdot}5$ s,
so that $19{\cdot}6 = 4{\cdot}9 + z \times 1{\cdot}5$.
(Or, straight away, $19{\cdot}6 = 4{\cdot}9 + z \times 1{\cdot}5$.)
Solving, $1{\cdot}5z = 19{\cdot}6 - 4{\cdot}9 = 14{\cdot}7$,

$$z = \frac{14{\cdot}7}{1{\cdot}5} = 9{\cdot}8,$$

and $g = 9{\cdot}8 \text{ m s}^{-2}$.

In this presentation, g represents acceleration and z represents its numerical value. It is, indeed, tedious and possibly confusing to retain physical units throughout the sometimes lengthy solution of an equation; anyone who has tried to calculate a final temperature in elementary calorimetry, while faithfully keeping in all the proper units, will have realised this.

The answer, I believe, is not to be found in the early introduction of transposition of formulae (so that in the above example you would find that $g = (v - u)/t$ and then solve by substitution and calculation). For there *is* an inherent difficulty in solving the literal equation $v = u + gt$, for g, compared with solving the numerical equation $19{\cdot}6 = 4{\cdot}9 + z \times 1{\cdot}5$, for z. Transposition of a formula needs a greater degree of abstraction than substituting in the formula for the known values and then finding the unknown value by solving a numerical equation. The learning process should proceed from the concrete to the abstract; learning how to solve $v = u + gt$, for g, should come *after* learning how to

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solve $19{\cdot}6 = 4{\cdot}9 + z \times 1{\cdot}5$. When, eventually, manipulation in the literal form has been established, it should of course be used.

Incidentally, with regard to graphs, the requirement that the scales should represent numbers rather than quantities has a serious consequence that perhaps needs to be realised: the slope of the graph and the area under the graph will *also* represent pure numbers rather than physical quantities. Thus, for the expansion of a gas, in the graph of $y = p/\text{N m}^{-2}$ as a function of $x = V/\text{m}^3$, the area under the graph for the interval (x_1, x_2) does *not* represent the work done by the gas when its volume changes from $V_1 = x_1\ \text{m}^3$ to $V_2 = x_2\ \text{m}^3$; this area only represents the *numerical value* of the work done. Similarly, to come back to Mr. Bausor's example, in the graph of $y = v/\text{m s}^{-1}$ as a function of $x = t/\text{s}$, the slope of the graph does *not* represent the acceleration g, the slope only represents the *numerical value* of g.

To recapitulate, I believe that the needs of the mathematicians and scientists could be met by using a symbol to represent a physical quantity and/or a different symbol to represent its numerical value, and relating the two symbols by means of the principle:

physical quantity = numerical value × unit.

Finally, I would suggest that the last four small letters of the alphabet, w, x, y, z, do not have great claims laid on them by the scientists for representing physical quantities. The symbols w, x, y, z are therefore particularly suitable for representing pure numbers, and there is the added advantage that the meaning of the symbols x and y in the context of graphs is widely understood.

Yours truly,

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Reviews

Transcendental number theory, by Alan Baker. Pp. x, 147. £4·90. 1975. SBN 0 521 20461 5 (Cambridge University Press)

Within the space of a mere 130 pages the author gives a panoramic account of modern transcendence theory, based on his own Adams Prize essay. The fact that this is now "a fertile and extensive theory, enriching wide-spread branches of mathematics" is due in large measure to the author himself, who was awarded in 1970 a Fields Medal (the Nobel Prize of mathematics) for his contributions. The prose is clear and economical yet interspersed with flashes of colour that convey a sense of personality; and each chapter begins with a helpful summary of the subsequent matter. The mathematical argument at all stages is highly condensed, as, indeed, is inevitable in a short research monograph covering so much ground. One might reproach the author for not having been more merciful to the beginner; but even a beginner can gain from the book a clear impression of what are the major achievements to date in this profoundly difficult field and which are the outstanding problems, while for others there is here a wealth of material for numerous fruitful study-groups. Each of the twelve chapters is, in effect, the account of a mathematical epic; an adequate description here is impossible, but the following remarks may be found helpful.

Almost all numbers are irrational, in the sense of Cantor. The earliest instances of irrational numbers were found among algebraic numbers, that is, among zeros of irreducible polynomials with integer coefficients (a necessary and sufficient condition for irrationality being that the degree of the polynomial exceeds 1). However, the set of all algebraic numbers is denumerable, so that almost all numbers are in fact non-algebraic, that is transcendental. Liouville observed that algebraic numbers are not too well