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BOOK REVIEWS

BORWEIN, P. and ERD³LYI, T. *Polynomials and polynomial inequalities* (Graduate Texts in Mathematics, Vol. 161, Springer-Verlag, Berlin–Heidelberg–New York–London–Paris–Tokyo–Hong Kong, 1995), x+480 pp., hardcover 0 387 945091, DM 98.00.

T. J. Rivlin states in his well known text, *The Chebyshev polynomials*, that these polynomials are like jewels, revealing different characteristics under illumination from varying positions. Polynomials pervade mathematics and they have benefitted over a long period of time from having many excellent texts written on them, including that by Rivlin. This book by Borwein and Erdélyi is a very welcome and well written addition to the list. If each text on this list, which would be well known to those who have more than a passing interest in any part of the topic, was the face of a hypothetical jewel, then Rivlin's remark could easily be paraphrased with the differences not just those of content. Some of the material in *Polynomials and polynomial inequalities* can be found elsewhere in books, though with a much different style, but there is much that cannot.

There are seven chapters in this book, which is largely self-contained. The introductory one is followed in succession by one each on 'Some Special Polynomials', 'Chebyshev and Descartes Systems', and then 'Denseness Questions'. (Details of the remaining three chapters come shortly.) At the very beginning of the first chapter it is revealed that rational functions are an important inclusion in the content of the book, a fact not evident from the title. The first chapter then covers the fundamental theorem of algebra and, among others, Walsh's two-circle theorem. Further results are then explored in the exercises. This is a strategy the authors continue to use extensively throughout the book and so students using it as a text are firmly invited to attempt these exercises. Chapter 2 covers polynomials and polynomial inequalities, orthogonal functions, orthogonal polynomials and then polynomials with nonnegative coefficients. In addition to the two topics already mentioned in the chapter heading, the next chapter deals with Chebyshev polynomials in Chebyshev spaces, Muntz–Legendre polynomials and Chebyshev polynomials in rational spaces. Chapter 4 covers variations on the Weierstrass theorem and then Muntz's theorem followed by unbounded Bernstein inequalities and Muntz rationals.

The new material, mainly responsible for giving this face of the *jewel* its different characteristics, is found in the later chapters on inequalities. Chapter 5, 'Basic Inequalities', includes classical polynomial inequalities, Markov's inequality for higher derivatives and inequalities for norms of factors. Inequalities in Muntz spaces and a section on nondense Muntz spaces form Chapter 6, while the final chapter deals with inequalities for rational function spaces and for logarithmic derivatives. Then there are five appendices on, respectively, 'Algorithms and Computational Concerns', 'Orthogonality and Irrationality', 'An Interpolation Theorem', 'Inequalities for Generalised Polynomials in L_p' and 'Inequalities for Polynomials with Constraints'. A comprehensive bibliography is then followed by a very useful reference list of the more commonly used spaces.

The material is presented in a very clear and readable manner. The attraction of the subject to the authors is very clear and they have made an excellent job of ensuring that the mathematical beauty of some of the content is apparent to the readers. The typesetting is first class and any errors that may exist in the type escaped the reviewer when preparing this review. I recommend the book as a very good additional reference text to accomplished researchers and as a superb graduate text—which is what it has been published as—for those wishing to learn and appreciate some of the mathematics of polynomials.

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