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Wave packet scattering and time delay

William Ian Robertson

The scattering of a quantum-mechanical wave packet by a finite range potential may be described by a wave function in which the initial state is specified at time $t = 0$, and the time dependence appears in functions $M(r, k, t)$, first obtained by Moshinsky.

The $t = 0$ wave function is derived, following the treatment of Rosenfeld, but using the Green's function for the radial wave equation to simplify the calculation of the Laplace transform. The treatment is extended to include the case of a packet initially within the potential, and the wave function for a decaying wave packet is derived. This agrees in form with the wave function obtained by Jeukenne in another approach.

By using an alternative expression for the Green's function, it is shown that the $t = 0$ wave function is equivalent to an expansion in scattering states. The relationship between the $t = 0$ wave function and the $t \to -\infty$ wave function of standard scattering theory is examined, and the restrictions on the position and shape of the initial packet in the latter wave function are emphasized. On the question of transients in time-dependent scattering theory, it is pointed out that a distinction should be drawn between plane wave treatments and those using wave packets.

The second part of the thesis is concerned with the calculation of time delay for an arbitrary wave packet interacting with a finite range potential. The $t = 0$ wave function is used, and a new method of calculation developed, in which momentum coefficients in the wave function are written as transforms of the initial packet, and momentum integrals are expressed in terms of Green's functions. General expressions are obtained for the time spent within a sphere of finite radius by the wave packet, in

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the presence of the potential and with the potential removed. The idea of time delay for a scattered packet is extended to include the lifetime of a decaying wave packet, and corresponding expressions are obtained when the packet is initially within the region of the potential.

Previous expressions for time delay have been derived using the $t \to -\infty$ wave function, and it is found that the use of the $t = 0$ wave function gives new terms, which arise from the principal part integral of a $\delta^{(+)}$ function. Terms of the type $\frac{d\alpha}{dk}$, which appear in Ohmura's expression for time delay, are found only in the calculation of lifetime. A discussion in one dimension shows that these terms represent the mean arrival time of the packet at the origin, and that other terms appearing in the various expressions have simple physical interpretations.

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