ANALYTICAL INTEGRATION OF A GENERALIZED EULER-POINSOT PROBLEM: APPLICATIONS

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Abstract. We consider a generalized Euler-Poinsot problem for a stationary gyrostat whose first two components of the gyrostatic momentum are null. The problem is formulated in the Serret-Andoyer canonical variables and analytically integrated by means of the Hamilton-Jacobi equation in terms of elliptic functions and integrals. The obtained solutions are just the same as those for rigid bodies if a specific constant is annulled. Finally, two applications are proposed: 1) to obtain the action-angle variables of this problem, and 2) to the problem of the rotation of the Earth, using a triaxial gyrostat as a model.

The problem of the Earth's rotation, using as a model a symmetric gyrostat is studied in Vigueras (1983) and Cid and Vigueras (1990), by assuming that the first two components of the gyrostatic momentum are null and the third component chosen as a constant, in such a way that the free polar motion has a period of 430 days (Chandler's period). To extend this study to the triaxial case (the effects of triaxiality have been considered in some papers of Kinoshita (1977, 1992)), we generalize, in part, previous papers about the problem of the free rotation of a rigid body, due to, amongst other authors, Deprit (1967), Sadov (1970), Kinoshita (1972,1992), and Deprit and Elipe (1993).

In the Serret-Andoyer variables $(\lambda, \mu, \nu, P_\lambda, P_\mu, P_\nu)$, the Hamiltonian for the free motion of a stationary gyrostat is

$$H = \frac{1}{2}\left[\left(P_\mu^2 - P_\nu^2\right)\left(\frac{\sin^2(\nu)}{A} + \frac{\cos^2(\nu)}{B}\right) + \frac{P_\nu^2}{C}\right] - \sqrt{P_\mu^2 - P_\nu^2}\left(\frac{a_1}{A}\sin(\nu) + \frac{a_2}{B}\cos(\nu)\right) - \frac{a_3}{C}P_\nu \qquad (1)$$

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and the corresponding system is integrable because it has three integrals (P_μ, P_λ, H) which are functionally independent and in involution. When $a_1 = a_2 = 0$, we have a generalized Euler-Poinsot problem that reduces to that of free rotation of a rigid body if the constant a_3 is zero. This problem is separable and a complete solution of the Hamilton-Jacobi equation is given by the formula

$$W(\lambda, \mu, \nu, \alpha_3, \alpha_2, \alpha_1) = \alpha_3\lambda + \alpha_2\mu + \int \frac{a_3 + \sqrt{a_3^2 + C[1 - Cf(\nu)][2\alpha_1 - \alpha_2^2 f(\nu)]}}{1 - Cf(\nu)} d\nu \tag{2}$$

being $f(\nu) = \dfrac{\sin^2\nu}{A} + \dfrac{\cos^2\nu}{B}$.We have used this generating function to: 1) calculate the Serret-Andoyer canonical variables in function of the time t, and 2) obtain the action-angles variables. In both cases, the solutions can be expressed in terms of elliptic functions and integrals. All these calculations and other necessary in order to be able to study the perturbed motion will be given in a next paper.

Finally, the problem of the Earth's rotation when it is attracted by the Sun and the Moon is formulated using as a model a triaxial stationary gyrostat in the canonical variables $(\pi, \zeta, \nu, P_\pi, P_\zeta, P_\nu)$ introduced by Cid and Correas (1973), and referred to the mean ecliptic of an adopted epoch, in a similar way to Kinoshita (1977). The Hamiltonian can be break down into the sum of one unperturbed part $\mathcal{H}_0$ and one perturbed $\mathcal{H}_1$, in this form $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1$, where $\mathcal{H}_0$ is the Hamiltonian of the previous generalized Euler-Poinsot problem, whose solutions we have obtained; and $\mathcal{H}_1$ contains the remaining terms. The Hamiltonian depends on the gyrostatic momentum whose constant components we want to determinate in such a way that the polar motion fits perfectly the observed values.

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