GENERAL RELATIVITY AND THE IAU RESOLUTIONS

Report of the IAU WGAS Sub-Working Group on Relativity in Celestial Mechanics and Astrometry (RCMA SWG)

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1. Introduction

RCMA SWG was appointed by the IAU WGAS (Working Group on Astronomical Standards) in accordance with IAU Resolution C6 (1994) with the aim 'to provide definitions of the astronomical units, of the quantities linking these astronomical units to the units of the International System (SI), and of other astronomical quantities, compatible with the theory of General Relativity'. It is evident that the relativistic aspects of units of measurement cannot be isolated from the more general problem of astronomical constants and fundamental astronomy concepts in the relativistic framework. Therefore, along with the problem of units the main topics of discussion of RCMA SWG concerned also the IAU (1991) Resolutions on References Systems (RSs) and Time Scales (TSs) and their interpretation in IERS Standards (1992) and IERS Conventions (1996). In what follows we tried to summarize the results of these discussions.

Nowadays it is common practice to determine from observations the Parameterized Post-Newtonian (PPN) parameters γ and β, which reflect possible violations of Einstein's General Relativity Theory. The IAU (1991) Resolutions, in fact, recommend to use the value $\gamma = 1$ of General Relativity to model the observations. This is justified by the fact that no observational data have shown a statistically significant violation of General Relativity. Therefore this theory, which is the simplest theory of gravitation satisfying all phenomena with the current accuracy of observations, can be recommended for applications. However, the RCMA SWG wishes to stress that it would be against basic scientific principles to stop improving the determination of the PPN parameters or to test General Relativity by other means.

The present status of fundamental astronomy, several years later after the adoption of new IAU (1991) resolutions on reference systems and time scales, leads to the following remarks:

1) the currently employed terminology should be changed to be consistent in the relativistic framework;

2) presently, because of achievable accuracy, relativity has to be taken into account in modeling virtually all kinds of modern astronomical and geodynamical observations. Here, the qualitative difference between the space-time of General Relativity and the Newtonian absolute space and time should never be forgotten. The quasi-Newtonian standpoint of speaking about (Newtonian-theory) + (small correction terms) often leads to misunderstandings or even wrong or ambiguous results. On the other hand, it is much more advantageous to follow basic principles of relativistic modeling which are extremely clear and easy to understand and to use.

J. Andersen (ed.), Highlights of Astronomy, Volume 11A, 194–199.

As an illustration of the present status, we shall consider in Section 3 the IERS Standards (1992) and IERS Conventions (1996).

2. IAU (1991) Resolutions

2.1. NEED OF HIGHER ORDER TERMS

Recommendations I, II and III of IAU (1991) Resolution A4 define the barycentric reference system (BRS) and the geocentric reference system (GRS) in the framework of General Relativity by specifying the corresponding four-dimensional metric forms ds^2. The coordinate time-scales of these forms are TCB and TCG, respectively. Only lowest-order terms of order c^{-2} (c being the light velocity in vacuum) are indicated in the time and spatial components of the metric tensors related to BRS and GRS. These terms are sufficient for present astrometric applications but, in a decade or so, the metric forms of BRS and GRS should be given more accurate by

(1) adding into the time and spatial (c^{-2})-components of the GRS metric a linear (with respect to the geocentric coordinates) term describing the acceleration of the geocenter in the barycentric reference system relative to a fictitious test-particle located in the geocenter and moving under the influence of the gravitational field of the solar system with except for the Earth (such term is taken into account in the high-precision analysis of Earth's satellite motion, e.g., Huang *et al.*, 1990);

(2) specifying a post-Newtonian definition of the geocenter, e.g., by setting the Barycentric Distance (BD) mass-dipole of the Earth to null;

(3) adding the (c^{-3})-order terms for the mixed components of the BRS and GRS metric tensors (this enables one to specify the choice of the space-time coordinates and to distinguish, particularly, between harmonic and PPN standard coordinates; in case of the GRS this permits also to distinguish between dynamically and kinematically nonrotating versions of the GRS; the two versions differ due to relativistic precession including a secular part (precession in astronomical sense) and periodic one (which could be called nutation); the relativistic precession is due to the gravitational field of all the bodies of the solar system, the influence of the Sun being dominating;

(4) indicating the (c^{-4})-order terms in the time components of the BRS and GRS metric tensors (they are necessary to take into account the (c^{-4})-order terms in the time-scale relationships (see, e.g., Fukushima, 1995), i.e. to define TCB and TCG more rigorously than it is done by means of the first approximation expressions in the notes for IAU (1991) recommendations).

Let us note that the terms of order c^{-4} in the time component of the BRS metric tensor and the terms of order c^{-3} in the mixed components are actually taken into account nowadays to derive the post-Newtonian equations of motion of the solar system bodies used to produce modern ephemerides. These terms give herewith effects of the same order of magnitude as the terms explicitly specified by the IAU. The term of order of c^{-4} in the GRS metric is also used to get the equations of motion of Earth's satellites recommended by IERS Standards (1992) and Conventions (1996).

2.2. NEED OF CLARIFICATION IN USING TERRESTRIAL TIME, TT

Returning to the present status of the time-scales it should be noted that the recommendation IV of the IAU (1991) Resolution A4 declares TT as a time-like argument to be used for geocentric ephemerides. This resulted in some confusion about the roles of TCG and TT. IAU (1994) resolution C7 did not remove such a confusion. On the one hand, this resolution defines the epoch J2000.0 and the Julian century in terms of TT. On the other hand, it recommends to develop new ephemerides in terms of the time-like arguments TCB and TCG. Moreover, presently TT is often misunderstood as a time-scale of GRS. This usually leads to altering the spatial coordinates of the GRS (see the following section) in explicit contradiction with the IUGG Resolution 2 (1991) based just on the IAU (1991) Resolution A4.

Hence, even if it is possible to postpone a more accurate exposition of the BRS and GRS metrics one should avoid the confusion in using TCG and TT as soon as possible.

3. IERS Conventions (1996)

IERS maintains two basic reference frames, the International Celestial Reference Frame (ICRF) and the International Terrestrial Reference Frame (ITRF). According to the language used in both IERS Standards (1992) and IERS Conventions (1996), ICRF represents a materialization of the BRS constructed by means of VLBI observations in conformity with IAU (1991) resolutions. However, instead of the BRS coordinate time TCB, spatial coordinates $\vec{x}$ and mass-factors GM, IERS Standards (1992), IERS Conventions (1996), as well as JPL DE/LE, use

$$\mathrm{TDB} = (1 - L_B)\,\mathrm{TCB},\ \ (\vec{x})_{\mathrm{TDB}} = (1 - L_B)\,\vec{x},\ \ (GM)_{\mathrm{TDB}} = (1 - L_B)\,GM \tag{1}$$

with $L_B \approx 1.5505197 \times 10^{-8}$ given in the IAU (1991) resolutions for the relationship between TCB and TDB of JPL DE/LE ephemerides. Actually each version of JPL DE/LE, being a bit different materialization of the BRS, provides slightly different values of L_B. This does not influence the values of $(GM)_{\mathrm{TDB}}$ at a significant level since currently typical precision of the mass factors are $10^{-10} - 10^{-11}$ and only a few leading digits of L_B play a role. However, it is anyway advantageous for many reasons to publish the transformation of the TDB as defined by each version of the DE/LE ephemerides and the TT together with the ephemeris itself. This transformation (which defines in particular L_B) can be considered as a "time part" of a relativistic space-time ephemeris.

Scaling of the spatial coordinates and the mass factors in (1) is performed to keep the value of the light velocity as well as the equations of motion of the solar system bodies invariant under transformation from TCB to TDB.

From the theoretical point of view, the ITRF maintained by IERS is supposed to be a realization of the GRS$^+$, a geocentric system rotating with the Earth and resulting from application of Euclidean rotation of the spatial axes of the GRS envisaged by the IAU/IUGG (1991) resolutions. But instead of the coordinate time TCG, spatial coordinates $\vec{w}$ and mass factors GM related to the GRS, IERS Standards (1992) use actually

$$\mathrm{TT} = (1 - L_G)\,\mathrm{TCG},\ \ (\vec{w})_{\mathrm{TT}} = (1 - L_G)\,\vec{w},\ \ (GM)_{\mathrm{TT}} = (1 - L_G)\,GM \tag{2}$$

with $L_G \approx 6.9629 \times 10^{-10}$ as given by IAU (1991) resolutions. The value of L_G is slightly different for various geoid models. Again, the scaling of the spatial coordinates and the mass factors is aimed to keep the value of the light velocity and the GRS equations of motion (e.g., those of the Earth's artificial satellites) invariant under the transformation from TCG to TT.

Transformations (1) are often considered as resulted from the change of the units of time from the SI unit to the 'TDB unit'. Just similarly, transformations (2) are meant in IERS Standards (1992) as resulted from the change of the units of time from the SI unit to the 'TT unit'. In this report transformations (1) and (2) are regarded as determining new quantities with the same units of measurement. This simplifies the notations and satisfies Recommendation III of the IAU (1991) Resolution A4 demanding the consistency of the units of measurement of the coordinate times with the SI unit of time. The detailed comparison of these two approaches to interpret transformations like (1) and (2) may be found in Guinot (1997).

In such a way, the time scale TT and the quantities related with TT (e.g., $(GM)_{\mathrm{TT}}$ and $(\vec{w})_{\mathrm{TT}}$) should be considered below as other physical quantities related with TCG, GM and $\vec{w}$ by (2).

An important particular question concerns the value of the geocentric constant GM_E. IERS Standards (1992) give two values for the geocentric constant

$$GM_E \text{ in TT units} = 3.986004418 \times 10^{14}\ m^3 s^{-2} \tag{3}$$

and

$$GM_E \text{ in TCG units} = 3.986004415 \times 10^{14}\ m^3 s^{-2}\,. \tag{4}$$

The meaning of these values remains unclear considering that the scaling relation (2) implies $(GM)_{\mathrm{TT}} < (GM)$. IERS Conventions (1996) reproduce the value GM_E coinciding with the IERS Standards (1992) value in 'TT units'. In doing this, IERS Conventions (1996) gives no definite information about the units used herewith. RCMA SWG wishes to stress that the current situation may lead to significant discrepancies between the versions of the ITRF constructed by using different

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observational techniques. This question should be clarified and an unambiguous value of GM_E should be published.

Hence, the GRS$^+$ being adequate to the IAU/IUGG (1991) resolutions differs from ITRF as maintained by IERS Standards (1992) by introducing L_G factor. In spite of the fact that IERS Conventions (1996) claim to achieve a better compatibility of ITRF with the IAU (1991) resolutions, the actual realization of ITRF in IERS Conventions (1996) represents a significant step backward as compared with IERS Standards (1992). The problem is to clarify what kind of spatial coordinates are wanted for ITRF. There are two main options:

(1) spatial coordinates of the GRS corresponding to its coordinate time TCG;

(2) locally measurable distances in the infinitesimal vicinity of an observer's site in analogy to the observer's proper time.

The first option corresponds to the metric adopted by the IAU/IUGG (1991) resolutions. The second option is actually realized by the VLBI formula recommended by IERS Conventions (1996). Implicitly, it makes use of the fact that any metric in General Relativity may be reduced in the infinitesimal vicinity of any point to the Minkowski form where all four coordinates may be regarded as physically measurable time and distances (proper time and proper distances). Coordinate time of such a reference system coincides with the proper time of an observer staying at its origin, while the coordinates correspond to physically measurable distances as far as infinitely small (reasonably small, in practice) space regions are considered. Such a local observer's reference system should be constructed for each observer on the Earth separately. While such an approach is quite suitable for constructing local geodetic networks, it is absolutely wrong for the whole Earth, leading to inconsistencies when sufficiently long distances are to be considered. Let us, however, note that the current level of accuracy allows one to speculate that we could introduce such 'locally measurable' spatial coordinates for the whole Earth. At a level of accuracy the transformations of spatial coordinates can be represented by a scaling $\vec{w}_{\text{locally measurable}} = (1 + L_G)\vec{w}$ (note the factor $1 + L_G$, not $1 - L_G$). Many other terms which could not be represented as a scaling are an order of magnitude smaller and neglected here. An improvement in accuracy by one order of magnitude, which is very likely in the nearest future, will reveal the inconsistency and will make it very clear that such an approach is erroneous not only from the theoretical point of view, but also from the very practical one. Moreover, the procedures of IERS Conventions (1996) for LLR and SLR observations, which still use the scaled GRS coordinates $(\vec{w})_{\text{TT}}$ do not agree with the procedure for VLBI observations (the same is most likely true for the GPS analysis). The discrepancy between the models amounts to 5 mm, at least. Although the present errors of these techniques exceed the 5 mm level, this internal incompatibility of various techniques used in ITRF is not admissible.

For all these reasons RCMA SWG cannot consider IERS Conventions (1996) procedures as satisfying the IAU (1991) resolutions.

4. Astronomical Units

Along with the SI units of time (s), mass (kg) and length (m), defined as proper quantities, astronomical units (AS) may be used when appropriate. The astronomical unit of time $d = 86400\,s$ is defined directly by its relationship to the SI unit of time. As for AS units of mass and length are concerned, one should distinguish between their definitions and their relationships to the corresponding SI units. The astronomical unit of mass is defined as the mass of the Sun M_S. In order to find its relationship to the SI unit of mass, one may use the laboratory determined value of G ($= 6.672 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$ in the IAU (1976) system of constants) and the value of GM_S ($= 1.32712438 \times 10^{20}\ m^3\ s^{-2}$ in the IAU (1976) system of constants) determined from the relativistic equations of motion of planets and planetary radar observations. In such a way one gets M_S ($= 1.9891 \times 10^{30}\ kg$ from the values above) in terms of the SI units of mass.

As far as the astronomical unit of length A is concerned, it is defined now by the condition $k = 0.01720209895$ provided that the units of measurement are d, M_S and A. Its relationship to the SI unit of length resulting from this definition is

$$A = (GM_S/k^2/d^2)^{1/3}\,, \tag{5}$$

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GM_S being a value in SI units discussed above. In such manner one gets the numerical value of A ($= 1.49597870 \times 10^{11}$ m for the values above). Although Eq. (5) originally related with the third Kepler law is 'Newtonian' by its internal philosophy, it can be considered also in the relativistic framework as simply a numerical definition to be retained for the reasons of continuity. Note also, that the accuracy of M_S expressed in the SI units is much lower than that of A and d.

Hence, General Relativity affects not the definitions of the astronomical units of mass and length, but only the numerical values of M_S and A in the SI units. These numerical values are not fixed. On the contrary, they are in permanent improvement with the increasing observation precision and by addition of new terms (Newtonian and relativistic contributions) into the model of the planetary motion. Thus, the values of GM_S and A have been significantly improved since 1976 and, for example, the values corresponding to each version of the JPL DE/LE ephemerides can be calculated using the lists of constants delivered together with each ephemeris.

The only principal question here is to choose some explicit relativistic definition of the mass of the Sun. First, there exist many reasonable definitions of the mass of a body in General Relativity (which do not differ quantitatively for the Sun at current level of accuracy). Second, the mass of a body cannot, generally speaking, be considered as constant, while the present definitions of the astronomical units of mass and length assume M_S to be constant. Let us note that the latter problem is by no mean specific to General Relativity: the Solar mass is variable also due to, say, solar wind and emission, and these effects (giving a change of about 10^{-13} per year) are several orders of magnitude larger than possible relativistic variability. However, within the present accuracy one may forget about this problem.

5. Recommendation

The foregoing analysis enables us to formulate the following draft of the recommendation to be considered at the XXIIIrd IAU General Assembly.

The XXIIIrd General Assembly of the IAU

considering that

- a relativistic solar system barycentric four-dimensional coordinate system with its coordinate time scale TCB was defined by IAU Resolution A4 (1991),

- a relativistic geocentric four-dimensional coordinate system with its coordinate time scale TCG was defined by IAU Resolution A4 (1991) and IUGG Resolution 2 (1991),

- the basic physical units of space-time in all coordinate systems were recommended by IAU Resolution A4 (1991) to be the SI second for proper time and the SI meter for proper length,

noting that

- practical realization of barycentric and geocentric coordinate systems in many groups (see IERS Standards, 1992) is based on time scales TDB and TT instead of TCB and TCG, respectively, and involves the scaling factors $1 - L_B$ and $1 - L_G$ for the spatial coordinates and mass factors GM in barycentric and geocentric systems, respectively, L_B and L_G being given in IAU Resolution A4 (1991),

- even more complicated scaling factors are introduced in the VLBI model of IERS Conventions (1996),

- currently employed definitions of fundamental astronomy concepts and astronomical constants are based on Newtonian mechanics with its absolute space and absolute time leading to ambiguities in dealing with relativistic effects,

recommends that

- the spatial coordinates of the Barycentric and Geocentric Reference Systems as defined the IAU (1991) resolutions be used for celestial and terrestrial reference frames, respectively, without any scaling factors,

- the final practical realizations of the coordinate systems for use in astronomy and geodesy should be consistent with the systems defined by IAU/IUGG (1991) resolutions,

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- the use of TT for convenience of observational data analysis should not be accompanied by scaling of the spatial geocentric coordinates,

- algorithms for determination of astronomical constants and definitions of fundamental astronomy concepts should be explicitly given within the basic reference systems envisaged by IAU/IUGG (1991) resolutions,

- IAU WGAS continue the consideration of relativistic aspects of the concepts, algorithms and the constants of fundamental astronomy.

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This list of references in no way claims to be complete. It enables one to get a more detailed information about the topics considered above and to find further references on general relativity applications in fundamental astronomy.