PART IV

THEORETICAL CONSIDERATIONS OF POPULATION II VARIABLES

VARIABLE STARS AND EVOLUTION IN GLOBULAR CLUSTERS

PIERRE DEMARQUE
Yale University Observatory, U.S.A.

1. Introduction

Traditionally, cluster variables have been used as distance indicators and have in this sense played an important role in our understanding of stellar evolution. In particular, the determination of the distance moduli of globular clusters and of the absolute magnitude of the main sequence turnoff, thus yielding the ages of the cluster, have relied heavily in the past on observations of RR Lyrae stars.

During the last two decades, it has become possible to follow in some detail the evolution of stars of the halo population from the main sequence to the tip of the giant branch. The last few years have further seen a very rapid advance in our understanding of those phases of evolution which follow the red giant phase, i.e. evolution on the horizontal branch, on the asymptotic branch and directly following the asymptotic branch. We are thus now able to discuss with some reliability the place of the RR Lyrae variables in the evolution of halo stars, and even to discuss the evolutionary status of the long period variables such as the W Virginis stars and the red variables, although perhaps in a somewhat more tentative fashion.

The theory of stellar evolution tells us something about the ages of stars in globular clusters, and about their masses and chemical compositions, these last two parameters to be tested in turn against atmospheric data and the results of detailed calculations of pulsation theory. Theory should eventually explain the existence of the two Oosterhoff (1939) period groups among short period variables. It should also enable us to understand the division recently discussed by Kraft (1972) of the W Virginis stars into two distinct groups, one with periods less than eight days, characterized by the BL Her stars, and the other with longer periods.

Variable stars provide further tests of the theory of stellar evolution. From the calculated rates of evolution it is possible to derive rates of period changes. At the same time, the direction of evolution provides a test of the sense of the period changes, i.e. whether the period is increasing or decreasing. It also gives an explanation for more subtle effects such as that on the relative numbers of Bailey types ab and c variables (Christy, 1966; van Albada and Baker, 1972). This last point will no doubt be discussed at this colloquium by the experts in pulsation theory.

Finally, one should mention the possibility that mass loss is of importance during the RR Lyrae phase, and that it may affect the subsequent evolution of the star. Work on this subject has recently been done by Laskarides (1972) at the University of Victoria.

J. D. Fernie (ed.), Variable Stars in Globular Clusters and in Related Systems, 187–195. *All Rights Reserved.*

2. The RR Lyrae Variables

The RR Lyrae variables occupy the center of the horizontal branch. Since the work of Schwarzschild (1940), who showed that all RR Lyrae variables in M3 are located in a well defined color range, and that this color range includes only variables, variability is generally regarded as a property of all horizontal branch stars whose evolutionary tracks take them through this color range. We shall assume here that, in Schwarzschild's words, 'stars which can pulsate, do pulsate', and therefore discuss briefly our present understanding of the horizontal branch.

Although a number of alternative models have been proposed (Hayashi *et al.*, 1962; Larson, 1965; Petersen, 1972), there are good reasons to believe that the structure of horizontal branch stars is the following: a helium burning core surrounded by a hydrogen rich envelope at the bottom of which lies a hydrogen burning shell. Such double energy source models were first constructed by Hoyle and Schwarzschild (1955), but the study of the systematics of the horizontal branch started with the work of Faulkner (1966), since followed by Giannone (1967), by Rood (1970), and by Gross (1972).

A. SYSTEMATICS OF THE ZERO-AGE HORIZONTAL BRANCH (ZAHB)

The systematics of the ZAHB have most recently been studied by Gross (1972), who considered a wide range in heavy element abundances. The four figures have been taken from his work.

Figure 1 shows the effect of adding to a helium configuration of fixed mass a hydrogen rich envelope. If q denotes the ratio of helium core mass to total mass, it is

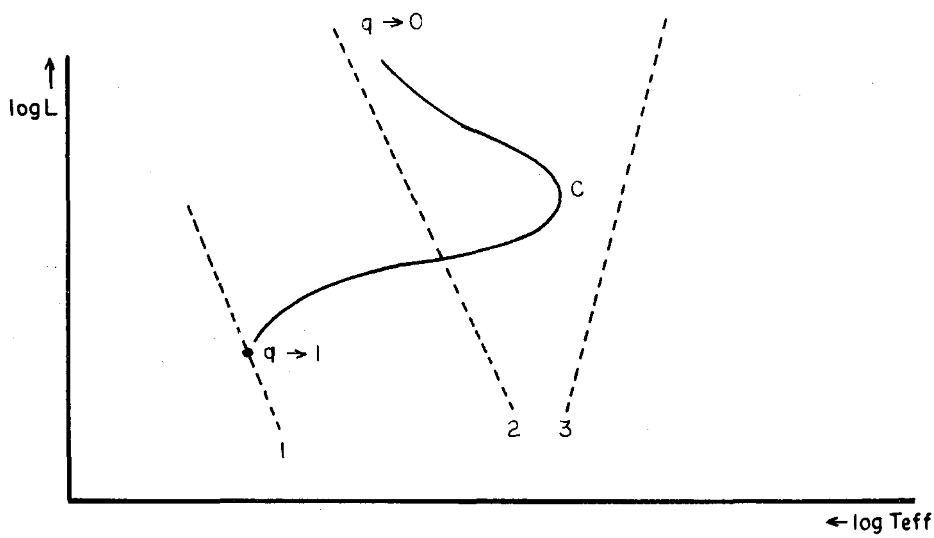

Fig. 1. Locus in the HR diagram of stellar models with constant helium core mass as the ratio q of core mass to total mass varies from 1 to 0.

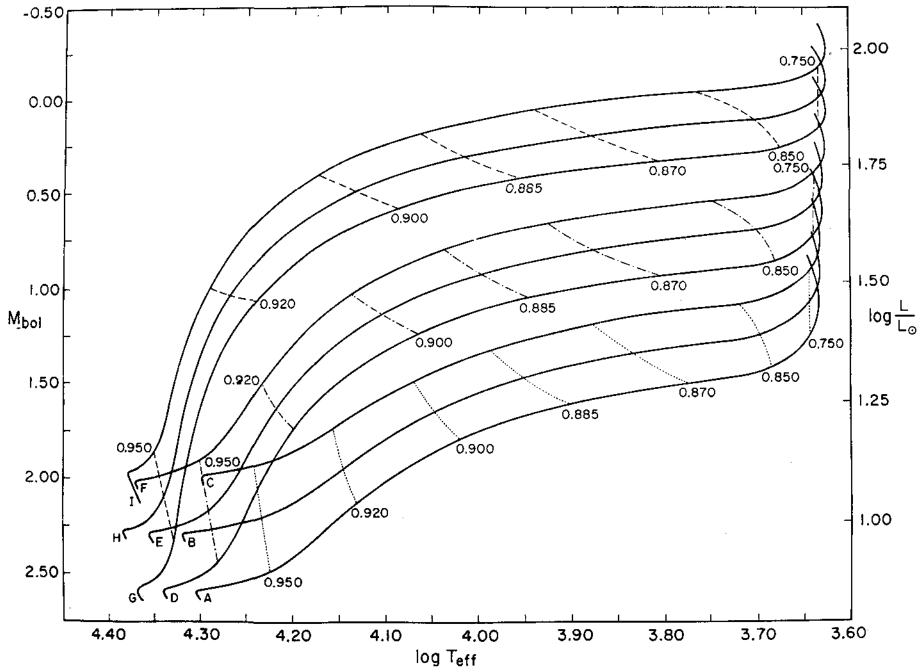

Fig. 2. Position of the ZAHB in the HR diagram as a function of the mass in the helium core and of the initial helium abundance, found in the envelope. If each ZAHB is defined by the parameters ($M_c/M_\odot$, Y), A corresponds to (0.425, 0.25), B to (0.450, 0.25), C to (0.475, 0.25), D to (0.425, 0.35), E to (0.450, 0.35), F to (0.475, 0.35), G to (0.425, 0.45), H to (0.450, 0.45) and I to (0.475, 0.45). $Z=0.01$ for all models shown in the figure. Lines of constant q are marked.

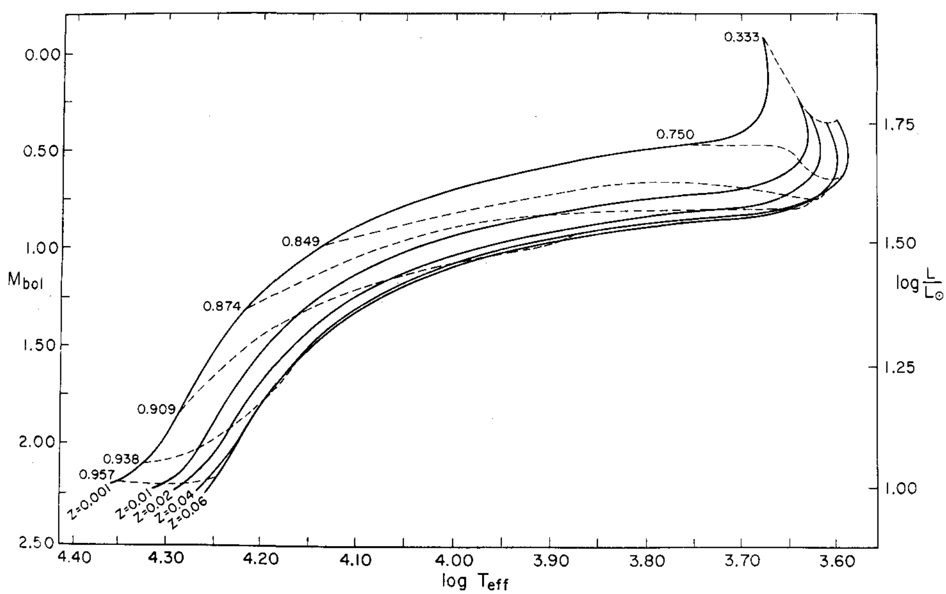

Fig. 3. Position of the ZAHB as a function of Z for ($M_c/M_\odot$, Y) = (0.450, 0.25). Lines of constant q are marked.

https://doi.org/10.1017/S0252921100033510 Published online by Cambridge University Press

seen that as q decreases from unity ($q=1$ on the helium main sequence) the sequence of models describes the horizontal branch to point C where it comes closest to the Hayashi line. As $q \to 0$, i.e. as the total mass tends to infinity, the sequence tends to the hydrogen main sequence.

One can also investigate the effects of varying the helium core masses of different chemical compositions. Figure 2 illustrates two effects on the position of the ZAHB, that of the choice of core mass and of helium abundance. Here the total mass increases from essentially the core mass to higher masses as one moves to the red along each ZAHB curve. Each triplet corresponds to a different helium content in the envelope, i.e. $Y=0.45$, 0.35, and 0.25 from high to low luminosities ($Z=0.01$ for all the models). For each constant-helium triplet the core masses used were $M_c=0.475$, 0.450, and 0.425 $M_\odot$ respectively, also in the direction of decreasing luminosities. Lines of constant q values are indicated. In Figure 3, $Y=0.35$ and $M_\odot=0.450$ $M_\odot$. Only the metal abundance is varied from $Z=0.001$ to $Z=0.06$. Lines of constant q, which are in this case also lines of constant mass, are marked. Finally Figure 4 shows the effect of varying the core mass while keeping the total mass constant. The points along the curves correspond to $M_c=0.425$, 0.450, 0.475 $M_\odot$ from right to left respectively ($Z=0.01$ for all models).

The above discussion shows that from a consideration of ZAHB models alone, it is possible to draw some conclusions about the differences between globular clusters and to attempt classification schemes based on chemical composition characteristics

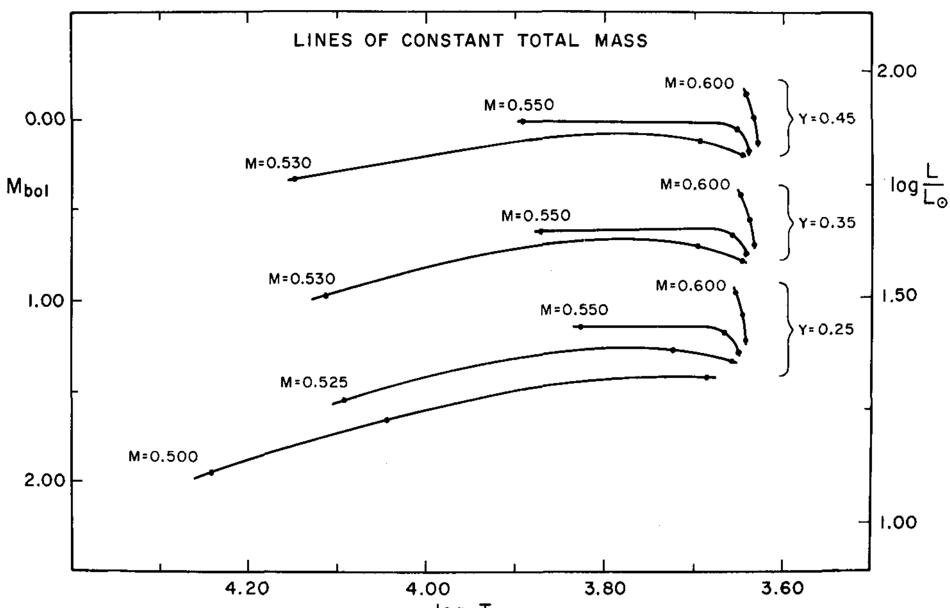

Fig. 4. Position of the ZAHB under the assumption of constant total mass and variable core mass. Each curve is defined by three models corresponding to $M_c=0.425$, 0.450 and 0.475 $M_\odot$ respectively from right to left. $Z=0.01$ for all models.

(e.g. Hartwick, 1968). For instance, the contraction of the horizontal branch into a clump of stars on the giant branch of old galactic clusters discussed by Cannon (1970) is easily explained in terms of high metallicity. Another important result first discussed by Faulkner (1966), is that unless horizontal branch stars describe long loops from the red to the blue as they evolve from the ZAHB, the masses of the bluest stars must be low ($\sim 0.60\ M_\odot$ or less) as compared to the masses of stars at the main sequence turnoff ($\sim 0.80\ M_\odot$). This problem has led to much discussion of mass loss on the giant branch (Iben and Rood, 1970).

B. EVOLUTIONARY CONSTRAINTS

But before too many conclusions are drawn concerning the nature of observed horizontal branches from considerations of ZAHB models alone, one must remember two important constraints:

(1) the phase of evolution which immediately precedes the ZAHB, i.e. the red giant stage and the core helium flash.

(2) the post-ZAHB evolution in which the bulk of the observed horizontal branch stars are found.

Let us consider (1) and (2) in turn.

(1) Implied in the above discussion of the horizontal branch is the notion now generally accepted that stars do not mix at the time of the core helium flash, but rather settle rapidly on the ZAHB following the flash (Schwarzschild and Härm, 1962). All computations indicate that the critical core mass M_c at the onset of the helium flash is independent of the total mass of the star in the range of interest. It depends only on chemical composition, being a weak function of Z, and a stronger function of Y (Demarque and Mengel, 1971, 1973). However, if rotation plays a role in the interiors of red giants, then M_c will depend on the angular momentum of individual stars (Kippenhahn, 1970), and a spread in M_c will be expected on the horizontal branch (van Albada and Baker, 1971; Demarque *et al.*, 1972).

(2) Equally critical is the shape of evolutionary tracks from the ZAHB. Early work by Hartwick, Härm and Schwarzschild (1968) gave long blueward loops for red horizontal branch stars. It soon became clear, however, that with the use of better opacities (Cox and Stewart, 1970) the blueward loops became much shorter (Faulkner and Iben, 1966; Iben and Rood, 1970; van Albada and Baker, 1971). Although this question is far from settled as evidenced by the discussion of the effects of semi-convection in the next section, at this moment it seems that we are forced to draw the following conclusions:

(A) Mass loss must occur in the evolution of halo stars between the main sequence turnoff and the horizontal branch, in all likelihood at the bright end of the red giant branch.

(B) The observed range in color of the horizontal branch is the result of one of two possible occurrences: (a) there is a spread in total masses on the horizontal branch, presumably the result of differential mass loss on the giant branch. (b) there is a spread in core masses, perhaps the result of internal rotation in red giants.

https://doi.org/10.1017/S0252921100033510 Published online by Cambridge University Press

Interpretation (a) was favored by Iben and Rood (1970) in their extensive paper on horizontal branch evolution. They concluded that the observed color spread of horizontal branch stars required a mass range of the order of 0.1–0.2 $M_{\odot}$ within a given cluster. They further discussed the rates of period changes predicted by theory and found them to be small, at least one order of magnitude less than observed rates quoted in the literature. The subsequent section shows that several important aspects of the Iben-Rood picture must be revised. However, on a number of grounds, it seems that interpretation (a) which they proposed is more likely to be correct than (b).

C. ROLE OF SEMI-CONVECTION

But an important point had escaped most investigators, i.e. the role of semi-convection. Schwarzschild (1970) had already emphasized the occurrence of semi-convection in horizontal branch models. Castellani *et al.* (1971a, b) considered in detail the effects of overshooting from the convective core and the subsequent formation of a semi-convective zone outside the convective core substantially modifying the internal structure of these stars. Evolutionary tracks recently constructed at Yale show that semi-convection plays a significant role in the evolution of horizontal branch stars (Demarque and Mengel, 1972; Sweigart and Demarque, 1972). Ignoring its effects leads to:

(a) too short blue loops in the HR diagram. Recent results indicate that the mass spread needed is less than 0.05 $M_{\odot}$.

(b) too short horizontal branch lifetimes. The new lifetimes are nearly twice as long as those of Iben and Rood (1970).

The detailed analysis of the effect of overshooting from the convective core into the semi-convective zone further led to the discovery of what appears to be a new 'composition instability', which is described in a subsequent paper (Sweigart and Demarque, 1973). This instability may be of relevance to the study of RR Lyrae stars since it predicts the existence of relatively rapid period changes.

3. Long Period Variables

A. W VIRGINIS STARS

All horizontal branch stars with masses above a certain limiting mass become red giants for a second time, evolving along the asymptotic branch. The internal structure of the star consists of an inert core of carbon and oxygen surrounded by a helium burning shell. A hydrogen burning shell still continues to provide the major part of the luminosity. A major feature of this phase of evolution is the repeated occurrence of a thermal instability in the thin helium burning shell of the star. The history of the star then consists of a series of relaxation cycles including alternating quiescent phases and active phases. The active phases are characterized by one or several brief periods of rapid helium burning. During these shell flashes, peak helium burning rates may exceed 10^7 solar luminosities, while the hydrogen burning shall is practically extinguished (Schwarzschild and Härm, 1965, 1967; Sweigart, 1971; Mengel, 1972). This evolution continues along the asymptotic branch until one of the following two events may occur:

(1) the star reaches such a high luminosity that the ionization instability discussed by Lucy (1967) and by Paczynski and Ziolkowski (1968) takes place in the envelope, probably leading to the loss of the envelope, with or without the help of a shell flash. The star eventually becomes a planetary nebula.

(2) due to its low mass, the star never reaches such a high luminosity. When the envelope mass is of the order of 0.01 $M_{\odot}$, it starts on its blueward journey across the HR diagram without any appreciable mass loss, at a luminosity level which depends on its total mass and chemical composition.

Both possibilities lead to a single crossing of the W Virginis instability strip from red to blue as the star leaves the asymptotic branch and evolves toward the white dwarf stage.

Further crossings of the W Virginis region may occur during the relaxation cycles (Schwarzschild and Härm, 1970; Mengel, 1972). In most cases, as the star undergoes large changes at the base of the envelope as a result of a helium shell flash, it moves up and down the asymptotic branch. However, in some instances, Schwarzschild and Härm (1970) found that their models left the asymptotic branch and described loops in the HR diagram extending into the W Virginis instability strip. Unfortunately, the original theory predicts period changes which are much more rapid than the available observations would indicate. But slower loops can occur at higher luminosities. A paper by Mengel (1972) at this colloquium gives a discussion of this problem in terms of improved interior models.

It may become possible to test the hypothesis of a spread in total mass on the horizontal branch by studying the W Virginis stars and associated bright non-variables which are found in clusters above the horizontal branch (Zinn *et al.*, 1972). If the mass spread hypothesis is correct, then one would expect a variety of stars to traverse the W Virginis instability strip, principally stars describing slow loops (Mengel, 1972), but also others evolving from the red to the blue on their way to the white dwarf stage. One would then expect a wide spread in luminosities among W Virginis variables and non-variables at the same phase of evolution. If on the other hand all horizontal branch stars have essentially the same mass, then a narrower range of luminosities for the W Virginis variables as well as stars to the blue of the instability strip should be observed within any one cluster.

B. RED VARIABLES

The place of the long-period red variables in the overall evolutionary picture is still unclear. It is possible that at least some of these objects are red giants, i.e. in the pre-horizontal-branch phase of evolution. It seems more likely, however, that they belong to the advanced stages of the asymptotic branch evolution. At any rate, it is tempting to relate these stars to the phase immediately preceding the mass loss process through ionization instability mentioned in the previous section. Theoretical research in this area is presently being conducted at the Princeton Observatory.

Already Smith and Rose (1972) have made hydrodynamic calculations of relaxation oscillations in the envelopes of luminous red giants. They suggest that such oscillations would be driven by high radiation pressure gradients at the base of the envelope, or by instability to non-adiabatic pulsations as recently studied by Keeley (1970).

Acknowledgements

The author's research is supported by Grant GP 21345 from the National Science Foundation. He wishes to express his special thanks to Dr J. G. Mengel and Dr A. V. Sweigart for many discussions of the points covered in this paper.

References

Albada, T. S. van and Baker, N.: 1971, *Astrophys. J.* **169**, 311.
Albada, T. S. van and Baker, N.: 1972, in A. G. Davis Philip (ed.), *The Evolution of Population II Stars*, Dudley Observatory Report No. 4, p. 193.
Cannon, R. D.: 1970, *Monthly Notices Roy. Astron. Soc.* **150**, 111.
Castellani, V., Giannone, P., and Renzini, A.: 1971a, *Astrophys. Space Sci.* **10**, 340.
Castellani, V., Giannone, P., and Renzini, A.: 1971b, *Astrophys. Space Sci.* **10**, 355.
Christy, R. F.: 1966, *Astrophys. J.* **144**, 108.
Cox, A. N. and Stewart, J.: 1970, *Astrophys. J. Suppl.* **19**, 261.
Demarque, P. and Mengel, J. G.: 1971, *Astrophys. J.* **164**, 317.
Demarque, P. and Mengel, J. G.: 1972, *Astrophys. J.* **171**, 583.
Demarque, P. and Mengel, J. G.: 1973, *Astron. Astrophys.*, **22**, 121.
Demarque, P., Mengel, J. G., and Sweigart, A. V.: 1972, *Astrophys. J. Letters* **173**, L27.
Faulkner, J.: 1966, *Astrophys. J.* **144**, 978.
Faulkner, J. and Iben, I., Jr.: 1966, *Astrophys. J.* **144**, 995.
Giannone, P.: 1967, *Z. Astrophys.* **65**, 226.
Gross, P. G.: 1972, Ph.D. dissertation, Yale University.
Hartwick, F. D. A.: 1968, *Astrophys. J.* **154**, 475.
Hartwick, F. D. A., Härm, R., and Schwarzschild, M.: 1968, *Astrophys. J.* **151**, 389.
Hayashi, C., Hoshi, R., and Sugimoto, D.: 1962, *Prog. Theor. Phys. Suppl.*, No. 22, p.1.
Hoyle, F. and Schwarzschild, M.: 1955, *Astrophys. J. Suppl.* **2**, 1.
Iben, I., Jr. and Rood, R. T.: 1970, *Astrophys. J.* **161**, 587.
Keeley, D. A.: 1970, *Astrophys. J.* **161**, 643.
Kippenhahn, R.: 1970, *Astron. Astrophys.* **8**, 50.
Kraft, R. P.: 1972, in A. G. Davis Philip (ed.), *The Evolution of Population II Stars*, Dudley Observatory Report No. 4, p. 69.
Larson, R. B.: 1965, *Publ. Astron. Soc. Pacific* **77**, 452.
Laskarides, P. G.: 1972, Ph.D. dissertation, University of Victoria.
Lucy, L. B.: 1967, *Astron. J.* **72**, 813.
Mengel, J. G.: 1972, Ph.D. dissertation, Yale University (see also this volume, p. 214).
Oosterhoff, P. Th.: 1939, *Observatory* **62**, 104.
Paczynski, B. and Ziółkowski, J.: 1968, *Proc. IAU Symposium No. 34 on Planetary Nebulae.*
Petersen, J. O.: 1972, *Astron. Astrophys.* **19**, 197.
Rood, R. T.: 1970, *Astrophys. J.* **161**, 145.
Schwarzschild, M.: 1940, *Harvard Circ. No. 437.*
Schwarzschild, M.: 1970, *Quart. J. Roy. Astron. Soc.* **11**, 12.
Schwarzschild, M. and Härm, R.: 1962, *Astrophys. J.* **136**, 158.
Schwarzschild, M. anb Härm, R.: 1965, *Astrophys. J.* **142**, 855.
Schwarzschild, M. anb Härm, R.: 1967, *Astrophys. J.* **150**, 961.
Schwarzschild, M. and Härm, R.: 1970, *Astrophys. J.* **160**, 341.
Smith, R. L. and Rose, W. K.: 1972, *Astrophys. J.* **176**, 395.

https://doi.org/10.1017/S0252921100033510 Published online by Cambridge University Press

Sweigart, A. V.: 1971, *Astrophys. J.* **168**, 79.
Sweigart, A. V. and Demarque, P.: 1972, *Astron. Astrophys.*, **20**, 445.
Sweigart, A. V. and Demarque, P.: 1973, this volume, p. 221.
Zinn, R. J., Newell, E. B., and Gibson, J.: 1972 *Astron. Astrophys.* **18**, 390.

DISCUSSION

Belserene: About the time scale for evolution through the instability strip, you said that the theorists need longer times than the ones we find. Are you referring to blueward evolution? Can you accept 2 to 10 million years as reasonable in a cluster with a very blue horizontal branch, where the only RR Lyr stars are the ones we catch on their way to the asymptotic branch.

Demarque: Yes, it is conceivable, depending on how blue the star was originally.

Walborn: If the horizontal branch is interpreted as a mass sequence, are the RR Lyrae stars then stars of a certain mass rather than stars at a certain evolutionary stage?

Demarque: Neither. Stars of different evolutionary stages will be found in the instability strip at any given time.

van den Bergh: Recent work by Peimbert seems to indicate that the oxygen-to-iron ratio might differ from cluster to cluster. Could you tell us how this might affect the structure of the horizontal branch? Could this be the 'second parameter' that seems to be required to account for the differences in the observed population gradients along the horizontal branch of clusters with the same [Fe/H]?

Demarque: Yes, the colours of horizontal branch stars depend most critically on the strength of the hydrogen burning shell, which depends directly on the abundance of CNO elements. Opacity effects are less important.

Lloyd Evans: (1) If the flash occurs at a luminosity below the observed tip of the giant branch shouldn't we see a discontinuity in luminosity function at the position of the flash? (2) The suggestion that some red variables, hopefully losing mass, are at the flash stage of evolution, does not seem to be much help in explaining the low mass of blue HB stars since the most pronounced red variables (Miras especially) are found in just those (metal rich) clusters with only the red stub horizontal branch.

Demarque: (1) The rate of evolution on the asymptotic branch is roughly one third that on the giant branch. In view of the small numbers of stars at this luminosity level, it would be difficult to observe this effect. (2) Little can be said about the masses of the stars on the red stub if the metal abundance is high. It is possible that such stars may have lost mass on the giant branch.

Schwarzschild: Could Dr Baker give us his view regarding the plausible mass or mass-range for the horizontal branch? I would feel that spectroscopic determinations do not yet reach the relatively high accuracy here required. Next, I still feel rather doubtful that finite-amplitude pulsation theory – as much as I admire its successes – can yet decisively contribute to this question. There remains then the general position of the horizontal branch in the HR diagram; can it give fairly definite masses for the horizontal branch?

Demarque: Yes, interior models give masses in the range 0.55–0.65 $\mathfrak{M}_\odot$. Recent computations by Sweigart and myself indicate that on the basis of the observed spread in colour on horizontal branches, there is a mass spread of about 0.05 $\mathfrak{M}_\odot$ in a given cluster.

Cox: In connection with this question of whether the horizontal branch can be explained without any mass loss, could you again review what would happen to a star undergoing a helium flash in the case of no (or very little) mass loss? Is this an acceptable situation?

Demarque: If we adopt a mass of 0.8 $\mathfrak{M}_\odot$ for the main sequence turnoff, and there is no mass loss on the giant branch, present models predict no horizontal branch, but rather a red clump similar to that found in old clusters of the disk population.

Feast: If I understood you correctly, you said that it may be necessary to consider that the various red variables, W Virginis, and RV Tauri stars in clusters can only be understood if there is a range of masses. Might one then expect that a cluster which contains a good selection of these variables would also show a larger than normal scatter of non-variables around the red giant branch? This appears to be the case, for instance, in ω Cen.

Demarque: I think personally that recent evolutionary calculations, in particular the work of Mengel, favor the hypothesis of a mass spread. But the situation is far from settled. I agree with your remark on ω Cen.