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Jun Le Goh, ***Measuring the Relative Complexity of Mathematical Constructions and Theorems***, Cornell University, USA, 2019. Supervised by Richard A. Shore. MSC: 03B30, 03D30. Keywords: computable reducibilities, reverse mathematics.

Abstract

We investigate the relative complexity of mathematical constructions and theorems using the frameworks of computable reducibilities and reverse mathematics.

First, we study the computational content of various theorems with reverse mathematical strength around Arithmetical Transfinite Recursion (ATR_0) from the point of view of computable reducibilities, in particular Weihrauch reducibility. We show that it is equally hard to construct an embedding between two given well-orderings, as it is to construct a Turing jump hierarchy on a given well-ordering. We obtain a similar result for Fraïssé's conjecture restricted to well-orderings.

We then turn our attention to König's duality theorem, which generalizes König's theorem about matchings and covers to infinite bipartite graphs. We show that the problem of constructing a König cover of a given bipartite graph is roughly as hard as the following "two-sided" version of the aforementioned jump hierarchy problem: given a *linear* ordering L, construct either a jump hierarchy on L (which may be a pseudohierarchy), or an infinite L-descending sequence. We also obtain several results relating the above problems with choice on Baire space (choosing a path on a given ill-founded tree) and unique choice on Baire space (given a tree with a unique path, produce said path).

Next, we investigate three known ways to formalize the notion of solving a problem by applying other problems in series: the compositional product, the reduction game, and the step product. We clarify the relationships between them by giving sufficient conditions for them to be equivalent. We also show that they are not equivalent in general.

Next, we turn our attention to the parallel product. In joint work with Dzhafarov, Hirschfeldt, Patey, and Pauly, we investigate the infinite pigeonhole principle for different numbers of colors and how these problems behave under Weihrauch reducibility with respect to parallel products.

Finally, we leave the setting of computable reducibilities for the setting of reverse mathematics. First, we define a Σ^1_1 axiom of finite choice and investigate its relationships with other theorems of hyperarithmetic analysis. For one, we show that it follows from Arithmetic Bolzano-Weierstrass. On the other hand, using an elaboration of Steel's forcing with tagged trees, we show that it does not follow from Δ^1_1 comprehension. Second, in joint work with James Barnes and Richard A. Shore, we analyze a theorem of Halin about disjoint rays in graphs. Our main result shows that Halin's theorem is a theorem of hyperarithmetic analysis, making it only the second "natural" (i.e., not formulated using concepts from logic) theorem with this property.

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Andrea Vaccaro, C*-***algebras and the Uncountable*****: A Systematic Study of the Combinatorics of the Uncountable in the Noncommutative Framework***, Università di Pisa, Italy – York University, USA, 2019. Supervised by Ilijas Farah and Alessandro Berarducci (cosupervisor). MSC: 03E75 (primary), 46L05, 03E35. Keywords: C*-algebras, Naimark's problem, Jensen's diamond, Calkin algebra, forcing, liftings.

Abstract

This dissertation investigates nonseparable C*-algebras using methods coming from set theory [2]. C*-algebras are objects usually studied in the framework of functional analysis, and they are defined as self-adjoint, norm-closed subalgebras of $\mathcal{B}(H)$ (the algebra of bounded linear operators on a complex Hilbert space H). The Gelfand transform establishes an equivalence between the category of abelian C*-algebras and the category of locally compact, Hausdorff spaces, motivating the idea that C*-algebras are the noncommutative analogues of topological spaces. The thesis consists of three independent chapters.

The first chapter concerns Naimark's problem, an old open question about irreducible representations of C*-algebras. A representation of a C*-algebra $\mathcal{A}$ is a $*$-homomorphism $\pi : \mathcal{A} \to \mathcal{B}(H)$ for some Hilbert space H, and it is irreducible if it cannot be decomposed as direct sum of two nontrivial subrepresentations. The representation theory of separable C*-algebras has been widely investigated by operator algebraists, and its study culminated with Glimm's seminal work on type I C*-algebras. On the other hand, the representation theory of nonseparable C*-algebras is characterized by more erratic and extreme behaviors, which do not allow a complete generalization of the results holding in the separable setting. Naimark's problem is an example of this discrepancy. It is well known that the C*-algebra of the compact operators on a Hilbert space has a unique irreducible representation up to spatial equivalence, the identity. Naimark's problem asks whether this property characterizes the algebras of compact operators up to isomorphism. A *counterexample* (*to Naimark's problem*) is C*-algebra that is not isomorphic to the algebra of compact operators on some Hilbert space, yet still has only one irreducible representation up to spatial equivalence. Such algebras have to be nonseparable, and in 2004 Akemann and Weaver [1] used Jensen's diamond $\diamondsuit$ in combination with some deep results on the representation theory of separable C*-algebras to build the first counterexamples. It is not known whether a positive answer to Naimark's problem is consistent with ZFC. This chapter focuses on the problem of finding more characterizing properties for the counterexamples. Some elementary observations on the structural properties of these C*-algebras seem to suggest that the trace space (a classical C*-algebraic invariant) of a counterexample is trivial. We prove that this is not the case and we show that, assuming $\diamondsuit$, almost every trace space of a separable C*-algebra also occurs as the trace space of a counterexample (see also [4]).

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