Contents

	Prefc	ace	<i>page</i> vii	
1	Calculus in Locally Convex Spaces		1	
	1.1	Introduction	1	
	1.2	Curves in Locally Convex Spaces	3	
	1.3	Bastiani Calculus	7	
	1.4	Bastiani versus Fréchet Calculus on Banach Spaces	13	
	1.5	Infinite-Dimensional Manifolds	16	
	1.6	Tangent Spaces and the Tangent Bundle	19	
	1.7	Elements of Differential Geometry: Submersions		
		and Immersions	23	
2	Spac	es and Manifolds of Smooth Maps	30	
	2.1	Topological Structure of Spaces of Differentiable		
		Mappings	30	
	2.2	The Exponential Law and Its Consequences	37	
	2.3	Manifolds of Mappings	41	
3	Lifting Geometry to Mapping Spaces I: Lie Groups			
	3.1	(Infinite-Dimensional) Lie Groups	48	
	3.2	The Lie Algebra of a Lie Group	55	
	3.3	Regular Lie Groups and the Exponential Map	62	
	3.4	The Current Groups	69	
4	Lifting Geometry to Mapping Spaces II:			
	(Weak) Riemannian Metrics		80	
	4.1	Weak and Strong Riemannian Metrics	80	
	4.2	The Geodesic Distance on a Riemannian Manifold	84	
	4.3	Geodesics, Sprays and Covariant Derivatives	90	
	4.4	Geodesic Completeness and the Hopf-Rinow Theorem	n 102	

vi		Contents	
5	Weak	Riemannian Metrics with Applications	
	in Sha	ape Analysis	106
	5.1	The L^2 -metric and Its Cousins	107
	5.2	Shape Analysis via the Square Root Velocity Transform	114
6	Conn	ecting Finite-Dimensional, Infinite-Dimensional	
	and H	ligher Geometry	120
	6.1	Diffeomorphism Groups Determine Their Manifolds	120
	6.2	Lie Groupoids and Their Bisections	123
	6.3	(Re-)construction of a Lie Groupoid from Its Bisections	133
7	Euler	-Arnold Theory: PDEs via Geometry	138
	7.1	Introduction	138
	7.2	The Euler Equation for an Ideal Fluid	143
	7.3	Euler-Poincaré Equations on a Lie Group	146
	7.4	An Outlook on Euler-Arnold Theory	150
8	The G	Geometry of Rough Paths	157
	8.1	Introduction	157
	8.2	Iterated Integrals and the Tensor Algebra	159
	8.3	A Rough Introduction to Rough Paths	168
	8.4	Rough Paths and the Shuffle Algebra	176
	8.5	The Grand Geometric Picture (Rough Paths and Beyond)	181
Appe	ndix A	A Primer on Topological Vector Spaces and Locally	
		Convex Spaces	186
Appe	ndix B	Basic Ideas from Topology	206
Appendix C		Canonical Manifold of Mappings	213
Appendix D		Vector Fields and Their Lie Bracket	225
Appendix E		Differential Forms on Infinite-Dimensional	
		Manifolds	231
Appendix F		Solutions to Selected Exercises	244
	Refere	ences	256
Index			264