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Alfvén resonance absorption in electron-positron plasmas

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Abstract. Waves propagating obliquely in a magnetized cold pair plasma experience an approximate resonance in the wavevector component perpendicular to the magnetic field, which is the analogue of the Alfvén resonance in normal electron-ion plasmas. Wave absorption at the resonance can take place via mode conversion to the analogue of the short wavelength inertial Alfvén wave. The Alfvén resonance could play a role in wave propagation in the pulsar magnetosphere leading to pulsar radio emission. Ducting of waves in strong plasma gradients may occur in the pulsar magnetosphere, which leads to the consideration of Alfvén surface waves, whose energy is concentrated in the region of strong gradients.

Keywords. plasmas, waves, pulsars

1. Introduction

Because the time and spatial scales associated with nonrelativistic electrons-positron pair plasmas are the same in each species, such pair plasmas have dispersive properties quite different to electron-ion plasmas. A distinctive feature of linear wave propagation in magnetized plasmas with equal numbers of same-mass pair species is that there is no Faraday rotation, so that waves propagating parallel to the magnetic field are linearly polarized, rather than circularly polarized, and that there is no analogue of the ion-acoustic wave (Stewart & Laing, 1992; Iwamoto, 1993 and Zank & Greaves, 1995).

If a cold plasma of equal-mass species is considered, but with different number densities of the species (uncompensated), the wave properties are altered, in that circularly polarized waves propagate along the magnetic field. Such an overall charge neutral plasma can be achieved if some of the charge resides on another species, for example consisting of ions or relatively massive dust particles. If the third species is effectively immobile, it can be thought of a charge sink, and the wave properties are determined by the dynamics of the (relatively) light pair species. Another circumstance of an uncompensated pair plasma occurs in the rotating pulsar magnetosphere, where different overall charges exist in different regions of the magnetospheric plasma in pulsar models.

An important feature of real astrophysical plasmas is that they are in general spatially nonuniform on the large scale and inhomogeneous on shorter scales. Waves in such plasmas experience reflection, transmission and absorption processes related to the local spatial variation of the plasma parameters such as the plasma frequencies and cyclotron frequencies. A particular process of interest in this paper is the Alfvén resonance absorption of wave energy, where, in the context of a normal electron-ion plasma, a fast magnetoacoustic wave propagates obliquely to the magnetic field in a density gradient, and at some point the local value of the Alfvén speed equals the wave phase velocity, or in other words the magnetoacoustic wave couples to the Alfvén wave. Energy absorption, via collisional or mode-conversion processes, occurs at this resonance point where the local value of the wavenumber perpendicular to the magnetic field becomes large (Cramer, 2001), and this process in normal plasmas has been invoked as a mechanism for the heating of the solar corona.

The refraction and ducting of waves, launched by emission processes in the nonuniform pulsar magnetosphere, have been invoked as important processes influencing the properties of the emergent pulsar radio waves escaping the magnetosphere, such as pulse profiles (Arons & Barnard, 1986 and Weltevrede *et al.*, 2003). Analyses of refraction of waves in a nonuniform plasma do not usually take into account resonance absorption processes such as the Alfvén resonance, which might play a role in wave propagation in the pulsar magnetosphere.

2. The Dispersion Equation

We consider the case of a cold magnetized plasma, composed of electrons and positrons, plus an effectively immobile species that retains part of the charge, positive or negative. The magnetic field is in the z-direction, and the dielectric tensor components are then:

$$K_{11} = K_{22} = 1 - \frac{\omega_{p+}^2 + \omega_{p-}^2}{\omega^2 - \Omega^2} = 1 - \frac{\omega_p^2}{\omega^2 - \Omega^2}, \tag{2.1}$$

$$\begin{aligned} K_{12} = -K_{21} &= -i\frac{(\omega_{p+}^2 - \omega_{p-}^2)\Omega}{\omega(\omega^2 - \Omega^2)} \\ &= -i\eta\frac{\omega_p^2\Omega}{\omega(\omega^2 - \Omega^2)}, \end{aligned} \tag{2.2}$$

$$K_{33} = 1 - \frac{\omega_p^2}{\omega^2}, \tag{2.3}$$

where ω_{p+} and ω_{p-} are the positron and electron plasma frequencies, and ω_p is the plasma frequency of the combined positron-electron fluid. Ω is the common cyclotron frequency, and η is a measure of the charge imbalance of the two light species:

$$\eta = (n_+ - n_-)/(n_+ + n_-).$$

All the other dielectric tensor components are zero. We define a function

$$A = K_{11} - \frac{c^2}{\omega^2}k_z^2. \tag{2.4}$$

The vanishing of this function for normal electron-ion plasmas defines the position of the Alfvén resonance, because for low frequency it corresponds to the point where the phase velocity along the magnetic field equals the Alfvén speed, where a resonance in the perpendicular wavenumber occurs (Cramer, 2001). A plays a similar role, under certain conditions, for a pair plasma.

In uniform plasma, the wave has wavenumber $\mathbf{k}$. If $k_y = 0$, the dispersion equation is:

$$k_x^4(c^2/\omega^2)^2K_{11} + k_x^2(c^2/\omega^2)(-A(K_{33} + K_{11}) - K_{12}^2) + (A^2 + K_{12}^2)K_{33} = 0. \tag{2.5}$$

3. Alfvén Resonance

We now fix ω and k_z, and vary the plasma density in the x-direction. There are 2 distinct modes. The local k_x^2 may approach zero, i.e. cutoff or strong reflection, or it may become very large, i.e. resonance or strong absorption, as shown in Figure 1.

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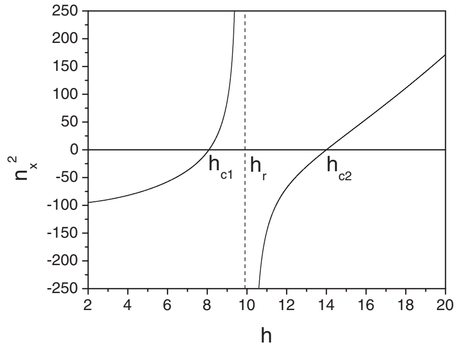
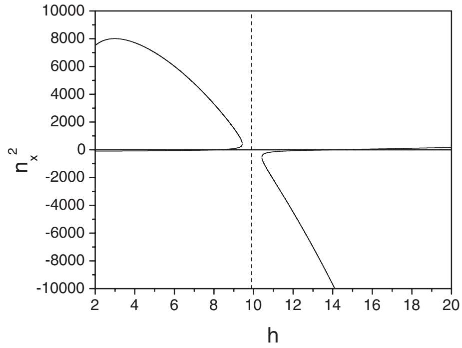


(a)



(b)

Figure 1. (a) Square of the perpendicular refractive index $n_x^2 = c^2 k_x^2/\Omega^2$ plotted against $h = \omega_p/\Omega$, for case $f \ll h$. Here $f = 0.1$, $n_z = 10$ and $\eta = 0.05$. The Alfvén resonance at h_r is shown by the dashed line, and the two cutoffs $h_{c1,2}$ are indicated. (b) As for (a), but the scale of n_x^2 is expanded, to reveal the second high $|n_x|$ mode on each side of the Alfvén resonance

If $\omega_p \gg \omega$, $|K_{33}| \gg 1$, there are two approximate solutions:

$$c^2 k_{x1}^2/\omega^2 = A + K_{12}^2/A, \tag{3.1}$$

$$c^2 k_{x2}^2/\omega^2 = AK_{33}/K_{11}. \tag{3.2}$$

If $A \approx 0$, there is no longer a true Alfvén resonance where $|n_x^2| \to \infty$, but $|n_x^2|$ acquires large values

$$n_{x1,2}^2 \approx \pm K_{12}(-K_{33}/K_{11})^{1/2} \approx \pm i\eta \frac{h^3}{f^2(1+h^2)^{1/2}} \tag{3.3}$$

If $\omega_p \lesssim \omega$, then $|K_{33}|$ is no longer much greater than unity, and there is no longer a pronounced resonance: the moduli of the refractive indices of the two modes are of comparable size in the vicinity of the resonance point.

The larger the value of $|K_{33}|$, the narrower the stop-band about the resonance point, and the sharper the resonance. Equation (3.1), representing a (single) mode with a true resonance at $A = 0$ enclosed by two cutoffs, becomes a better approximation for the actual dispersion relation. This mode is equivalent to the fast compressional wave in cold normal ion-electron plasmas with the electron inertia neglected, which also experiences a cutoff-resonance-cutoff triplet (Cramer, 2001), with the role of the finite ion-cyclotron frequency due to Hall currents in the case of the normal plasma now being played by the imbalance in the pair plasma densities.

Equation (3.1) can be written for $f \ll 1$ in terms of the wavenumbers as

$$v_A^2 k_x^2 = \omega^2 - v_A^2 k_z^2 - \frac{\eta^2 \Omega^2 \omega^2}{\omega^2 - v_A^2 k_z^2}, \tag{3.4}$$

which shows explicitly the Alfvén resonance ($k_x^2 \to \infty$) occurring where the parallel wave phase velocity equals the Alfvén velocity, with the dispersive term depending on the pair species imbalance factor η.

The large $|n_x|$ mode has the approximate dispersion relation

$$\omega^2 = \frac{v_A^2 k_z^2}{1 + \delta_e^2 k_x^2} \tag{3.5}$$

where $\delta_e = c/\omega_p =$ electron inertial length. This is the analogue of the Inertial Alfvén

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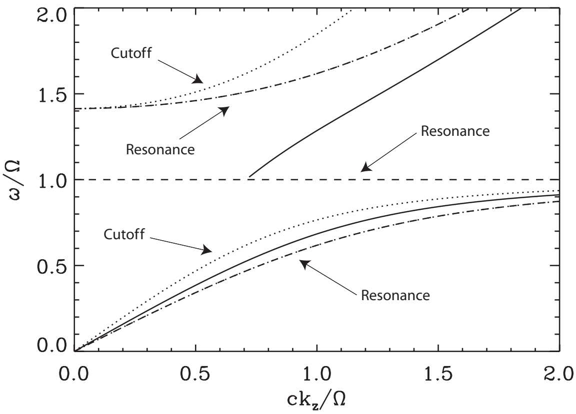

Figure 2. Surface wave dispersion relation (solid curves). Here $h = 1$, $k_y = k_z$ and $\eta = 0$.

wave that occurs in normal plasmas. Surface waves exist on a pair-plasma/vacuum interface for $k_y \neq 0$ (Figure 2). The surface waves may be damped due to Alfvén resonance damping, or coupling into the Inertial Alfvén wave.

4. Conclusion

It has been shown that the Alfvén resonance absorption process occurs in a pair plasma. Analogies are drawn between the resonance-cutoff structures of noncompensated pair plasmas and those of normal electron-ion plasmas with finite ion-cyclotron frequency, with the role of the imbalanced Hall currents in the latter case being played by the imbalanced species densities in the pair plasma.

A true Alfvén resonance can be identified only in the limit of wave frequency much less than the plasma frequency. A long wavelength wave propagating into the "Alfvén resonance" point will partially mode convert into a short wavelength mode which is the pair-plasma version of the "Inertial Alfvén Wave" which occurs in the Alfvén resonance heating of fusion plasmas, and in the form of solitary waves in the Earth's magnetosphere. On a sharp interface between plasmas of differing densities, such as a magnetic flux tube or density duct in the pulsar magnetosphere, a localized surface wave exists. The analogues of Alfvén surface waves on an electron-positron plasma/vacuum interface have been demonstrated. Future work could allow for thermal effects (Stewart and Laing, 1992) and relativistic effects (Luo, Melrose & Fussel, 2002) on the pair plasma dielectric tensor.

References

Arons, J. & Barnard, J. J. 1986, *ApJ*, 302, 120
Cramer, N. F. 2001, *The Physics of Alfvén Waves* (Berlin: Wiley-VCH)
Iwamoto, N. 1993, *Phys. Rev. E*, 47, 604
Luo, Q., Melrose, D. B., & Fussell, D. 2002, *Phys. Rev. E*, 66, 026405.
Stewart, G. A. & Laing, E. W. 1992, *J. Plasma Phys.*, 47, 295
Weltevrede, P., Stappers, B. W., van den Horn, L. J., & Edwards, R.T. 2003, *A&A*, 412, 473
Zank, G. P. & Greaves, R. G. 1995, *Phys. Rev. E*, 51, 6079

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