

CORRIGENDUM

Autonomous propulsion of nanorods trapped in an acoustic field – CORRIGENDUM

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In Collis, Chakraborty & Sader (2017), the effect of Lagrangian motion of the particle on its propulsion velocity, U_{prop} , was ignored and is now included. Qualitative features of the propulsion remain unchanged, while some quantitative changes arise. All symbols are as defined in Collis *et al.* (2017), unless otherwise specified.

(i) We rederive the key results of §2, which replaces the derivation in Appendix A. Consider a general rigid-body particle motion where the time-dependent position vector of a material point on the particle surface (in an inertial frame), $\mathbf{r}_p(t) \in S_p(t)$, is

$$\mathbf{r}_p(t) = \mathbf{X}(t) + \mathbf{A}(t) \cdot \mathbf{r}_0, \quad (1.1)$$

where $\mathbf{X}(t)$ is the displacement of the particle's geometric centre, $\mathbf{A}(t)$ is the rotation tensor and $\mathbf{r}_0 \in S_0$ is the initial position of the same material point. The velocity of this material point is therefore

$$\mathbf{U}_p(t) = \mathbf{U}(t) + \boldsymbol{\Omega}(t) \times (\mathbf{A}(t) \cdot \mathbf{r}_0), \quad (1.2)$$

where \mathbf{U} and $\boldsymbol{\Omega}$ are the particle's linear and angular velocities, respectively.

Axisymmetry of the particle ensures rotation can only occur about the y -axis and small-amplitude motion in the far field constrains the angular displacement to be

$$\theta(t) = \epsilon \operatorname{Re}[-i\bar{\Omega}_p e^{-it}] + o(\epsilon), \quad (1.3)$$

and hence

$$\boldsymbol{\Omega}(t) = \operatorname{Re}[-\bar{\Omega}_p e^{-it}] \mathbf{j} + o(\epsilon), \quad (1.4)$$

where $\bar{\Omega}_p$ is defined in (2.6). Note that the $O(\epsilon)$ term in (1.4), i.e. for steady and twice-frequency rotations, is zero. Similarly,

$$\mathbf{X}(t) = \epsilon \operatorname{Re}[i\bar{W}_p e^{-it}] \mathbf{k} + o(\epsilon), \quad \mathbf{U}(t) = \operatorname{Re}[\bar{W}_p e^{-it}] \mathbf{k} + \epsilon U_p^{(1)} + o(\epsilon), \quad (1.5a,b)$$

with \bar{W}_p specified in (2.6). Substituting (1.3) and (1.5a) into (1.1), and Taylor expanding in ϵ , gives

$$\mathbf{r}_p(t) = \mathbf{r}_0 + \epsilon \operatorname{Re}[\bar{\mathbf{r}}_1 e^{-it}] + o(\epsilon), \quad (1.6)$$

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where $\bar{\mathbf{r}}_1 = i(\bar{W}_p \mathbf{k} + \bar{\Delta}_p \mathbf{r}_0 \times \mathbf{j})$. Taylor expanding the fluid velocity field at the particle surface yields

$$\mathbf{u}|_{r_p} = \mathbf{u}|_{r_0} + \epsilon \operatorname{Re}[\bar{\mathbf{r}}_1 e^{-it}] \cdot \nabla \mathbf{u}|_{r_0} + o(\epsilon). \tag{1.7}$$

Substituting (1.7) into the asymptotic expansion for the fluid velocity field, (2.4a), gives the boundary conditions for the leading-order flow, (2.6), and the first-order flow,

$$\bar{\mathbf{u}}^{(1)}|_{r_0} = U_{prop} \mathbf{i} - \frac{1}{4}(\bar{\mathbf{r}}_1 \cdot \nabla \bar{\mathbf{u}}^{(0)*})|_{r_0} + \bar{\mathbf{r}}_1^* \cdot \nabla \bar{\mathbf{u}}^{(0)}|_{r_0}, \tag{1.8}$$

where $\bar{\mathbf{u}}^{(1)}$ and U_{prop} are the steady components of $\mathbf{u}^{(1)}$ and $\mathbf{i} \cdot \mathbf{U}_p^{(1)}$, respectively.

To determine the propulsion velocity, U_{prop} , we first calculate the force acting on the moving particle, i.e. $\mathbf{F} \equiv \int_{S_p} \mathbf{n} \cdot \boldsymbol{\sigma} \, dS$. Expanding the normal vector, \mathbf{n} , gives

$$\mathbf{n} = \mathbf{n}_0 + \epsilon \operatorname{Re}[\bar{\mathbf{n}}_1 e^{-it}] + o(\epsilon), \tag{1.9}$$

where $\bar{\mathbf{n}}_1 = i\bar{\Delta}_p \mathbf{n}_0 \times \mathbf{j}$ and \mathbf{n}_0 is the normal vector to the surface, S_0 , defined in the text immediately following (1.1). Substituting the asymptotic expansion for the fluid variables, (2.4a,b), into the resulting expression for the force, and then Taylor expanding the surface integral, gives $\mathbf{F} = \mathbf{F}^{(0)} + \epsilon \mathbf{F}^{(1)} + o(\epsilon)$. The Fourier component of $\mathbf{F}^{(0)}$ is non-zero in the z -direction only, which is given by the left-hand side of (2.7a). Moreover, the steady component of $\mathbf{F}^{(1)}$ is

$$\bar{\mathbf{F}}^{(1)} = \int_{S_0} \mathbf{n}_0 \cdot \bar{\boldsymbol{\sigma}}^{(1)} + \frac{1}{4}[\bar{\mathbf{n}}_1 \cdot \bar{\boldsymbol{\sigma}}^{(0)*} + \bar{\mathbf{n}}_1^* \cdot \bar{\boldsymbol{\sigma}}^{(0)} + \mathbf{n}_0 \cdot (\bar{\mathbf{r}}_1 \cdot \nabla \bar{\boldsymbol{\sigma}}^{(0)*} + \bar{\mathbf{r}}_1^* \cdot \nabla \bar{\boldsymbol{\sigma}}^{(0)})] \, dS, \tag{1.10}$$

which is non-zero in the x -direction only.

To evaluate $\mathbf{i} \cdot \int_{S_0} \mathbf{n}_0 \cdot \bar{\boldsymbol{\sigma}}^{(1)} \, dS$, we use the generalised Lorentz reciprocal theorem on the fluid domain, V_0 , which is specified as the exterior of the surface defined by the particle's initial position, i.e. S_0 ; not the true (time-dependent) fluid domain. The two flows used in the reciprocal theorem are (a) the first-order steady Stokes flow with body force as in (2.9b), but with boundary condition (1.8), and (b) an auxiliary (body-force-free) Stokes flow, $(\mathbf{u}', \boldsymbol{\sigma}')$, specified by unitary translation of the particle in the x -direction. This gives

$$\int_{S_0} \mathbf{n}_0 \cdot (\mathbf{i} \cdot \bar{\boldsymbol{\sigma}}^{(1)} - \bar{\mathbf{u}}^{(1)} \cdot \boldsymbol{\sigma}') \, dS = -\frac{\beta}{4} \int_{V_0} \mathbf{u}' \cdot (\bar{\mathbf{u}}^{(0)} \cdot \nabla \bar{\mathbf{u}}^{(0)*} + \bar{\mathbf{u}}^{(0)*} \cdot \nabla \bar{\mathbf{u}}^{(0)}) \, dV. \tag{1.11}$$

Conservation of the particle's linear momentum (to first order in ϵ) requires $\bar{\mathbf{F}}^{(1)} = 0$. Substituting (1.8) into (1.11) and its result into (1.10) gives the corrected expression for (2.10):

$$\begin{aligned} U_{prop} &= \frac{\beta}{4F_p} \int_{V_0} \mathbf{u}' \cdot (\bar{\mathbf{u}}^{(0)} \cdot \nabla \bar{\mathbf{u}}^{(0)*} + \bar{\mathbf{u}}^{(0)*} \cdot \nabla \bar{\mathbf{u}}^{(0)}) \, dV \\ &+ \frac{1}{4F_p} \int_{S_0} (\mathbf{n}_0 \cdot \boldsymbol{\sigma}') \cdot (\bar{\mathbf{r}}_1 \cdot \nabla \bar{\mathbf{u}}^{(0)*} + \bar{\mathbf{r}}_1^* \cdot \nabla \bar{\mathbf{u}}^{(0)}) \\ &- \mathbf{i} \cdot [\mathbf{n}_0 \cdot (\bar{\mathbf{r}}_1 \cdot \nabla \bar{\boldsymbol{\sigma}}^{(0)*} + \bar{\mathbf{r}}_1^* \cdot \nabla \bar{\boldsymbol{\sigma}}^{(0)}) + \bar{\mathbf{n}}_1 \cdot \bar{\boldsymbol{\sigma}}^{(0)*} + \bar{\mathbf{n}}_1^* \cdot \bar{\boldsymbol{\sigma}}^{(0)}] \, dS; \end{aligned} \tag{1.12}$$

the surface integral in (1.12) must also be added to (2.11).

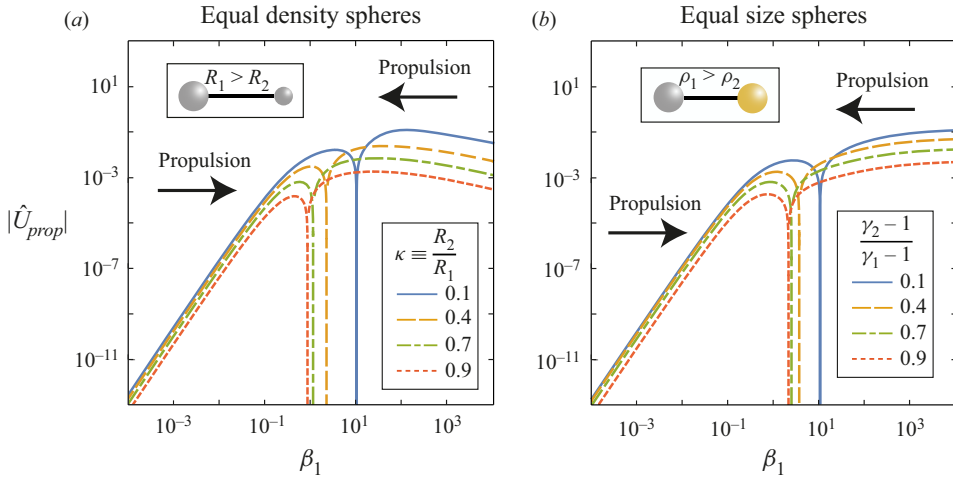


Figure 2. Corrected: axis scales are identical to Collis *et al.* (2017).

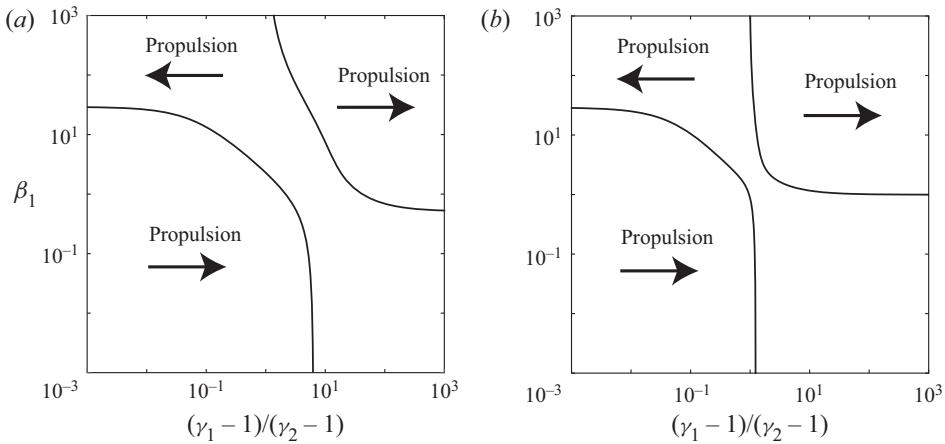


Figure 4. Corrected: axis scales are identical to Collis *et al.* (2017).

(ii) Corrected asymptotic forms for the propulsion velocity of the dumbbell, (3.6) and (3.7), are, respectively,

$$U_{prop} = \frac{(\kappa - 1)(\gamma_1 - 1)^2}{A} \begin{cases} -\frac{1 + \kappa^4}{243} \beta_1^3, & \beta_1 \ll 1, \\ \frac{45\sqrt{2}}{(1 + 2\gamma_1)^3 \kappa (1 + \kappa)} \beta_1^{-1/2}, & \beta_1 \gg 1, \end{cases} \quad (1.13)$$

and

$$U_{prop} = \frac{(\gamma_2 - \gamma_1)}{A} \begin{cases} -\frac{\gamma_1 + \gamma_2 - 2}{486} \beta_1^3, & \beta_1 \ll 1, \\ \frac{3(4\gamma_1\gamma_2 - \gamma_1 - \gamma_2 - 2)}{(1 + 2\gamma_1)^2 (1 + 2\gamma_2)^2}, & \beta_1 \gg 1. \end{cases} \quad (1.14)$$

End shape	Material	γ_1	γ_2	$U_{prop} (\times 10^{-4})$
Hemi-spherical	Au	19.3	19.3	-6.2
	RuAu	12.5	19.3	-26.0
	AuRu	19.3	12.5	14.4
Hemi-spheroidal	Au	19.3	19.3	-32.8
	RuAu	12.5	19.3	-38.5
	AuRu	19.3	12.5	-15.9

Table 1. Corrected.

(iii) Corrected figures 2 and 4 are similar to Collis *et al.* (2017), with quantitative changes to the zero propulsion point. Corrected table 1 shows closer agreement with experiments, which indicate that the concave and light end lead the motion; similar changes to figure 5 (not shown).

(iv) Changes to the caption of figure 3: (c) Non-zero contribution from the body force is cancelled by that of the surface integrals in (2.10), giving $U_{prop} = 0$. (f) Combining these two flows produces a jet that propels the sphere in the negative x -direction, with $U_{prop} = -0.039$.

Declaration of interests. The authors report no conflict of interest.

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