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CORRIGENDUM

Autonomous propulsion of nanorods trapped in an acoustic field – CORRIGENDUM

Jesse F. Collis[1], **Debadi Chakraborty**[1] **and John E. Sader**[1,†]

[1]ARC Centre of Excellence in Exciton Science, School of Mathematics and Statistics, The University of Melbourne, VIC 3010, Australia

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In Collis, Chakraborty & Sader (2017), the effect of Lagrangian motion of the particle on its propulsion velocity, U_{prop}, was ignored and is now included. Qualitative features of the propulsion remain unchanged, while some quantitative changes arise. All symbols are as defined in Collis *et al.* (2017), unless otherwise specified.

(i) We rederive the key results of § 2, which replaces the derivation in Appendix A. Consider a general rigid-body particle motion where the time-dependent position vector of a material point on the particle surface (in an inertial frame), $\boldsymbol{r}_p(t) \in S_p(t)$, is

$$\boldsymbol{r}_p(t) = \boldsymbol{X}(t) + \boldsymbol{A}(t) \cdot \boldsymbol{r}_0, \tag{1.1}$$

where $\boldsymbol{X}(t)$ is the displacement of the particle's geometric centre, $\boldsymbol{A}(t)$ is the rotation tensor and $\boldsymbol{r}_0 \in S_0$ is the initial position of the same material point. The velocity of this material point is therefore

$$\boldsymbol{U}_p(t) = \boldsymbol{U}(t) + \boldsymbol{\Omega}(t) \times (\boldsymbol{A}(t) \cdot \boldsymbol{r}_0), \tag{1.2}$$

where $\boldsymbol{U}$ and $\boldsymbol{\Omega}$ are the particle's linear and angular velocities, respectively.

Axisymmetry of the particle ensures rotation can only occur about the y-axis and small-amplitude motion in the far field constrains the angular displacement to be

$$\theta(t) = \epsilon\,\mathrm{Re}[-\mathrm{i}\bar{\Omega}_p \mathrm{e}^{-\mathrm{i}t}] + o(\epsilon), \tag{1.3}$$

and hence

$$\boldsymbol{\Omega}(t) = \mathrm{Re}[-\bar{\Omega}_p \mathrm{e}^{-\mathrm{i}t}]\boldsymbol{j} + o(\epsilon), \tag{1.4}$$

where $\bar{\Omega}_p$ is defined in (2.6). Note that the $O(\epsilon)$ term in (1.4), i.e. for steady and twice-frequency rotations, is zero. Similarly,

$$\boldsymbol{X}(t) = \epsilon\,\mathrm{Re}[\mathrm{i}\bar{W}_p \mathrm{e}^{-\mathrm{i}t}]\boldsymbol{k} + o(\epsilon), \quad \boldsymbol{U}(t) = \mathrm{Re}[\bar{W}_p \mathrm{e}^{-\mathrm{i}t}]\boldsymbol{k} + \epsilon \boldsymbol{U}_p^{(1)} + o(\epsilon), \tag{1.5a,b}$$

with $\bar{W}_p$ specified in (2.6). Substituting (1.3) and (1.5a) into (1.1), and Taylor expanding in ϵ, gives

$$\boldsymbol{r}_p(t) = \boldsymbol{r}_0 + \epsilon\,\mathrm{Re}[\bar{\boldsymbol{r}}_1 \mathrm{e}^{-\mathrm{i}t}] + o(\epsilon), \tag{1.6}$$

† Email address for correspondence: jsader@unimelb.edu.au

where $\bar{\boldsymbol{r}}_1 = \mathrm{i}(\bar{W}_p \boldsymbol{k} + \bar{\Omega}_p \boldsymbol{r}_0 \times \boldsymbol{j})$. Taylor expanding the fluid velocity field at the particle surface yields

$$\boldsymbol{u}|_{\boldsymbol{r}_p} = \boldsymbol{u}|_{\boldsymbol{r}_0} + \epsilon\, \mathrm{Re}[\bar{\boldsymbol{r}}_1 \mathrm{e}^{-\mathrm{i}t}] \boldsymbol{\cdot} \nabla \boldsymbol{u}|_{\boldsymbol{r}_0} + o(\epsilon). \tag{1.7}$$

Substituting (1.7) into the asymptotic expansion for the fluid velocity field, (2.4*a*), gives the boundary conditions for the leading-order flow, (2.6), and the first-order flow,

$$\bar{\boldsymbol{u}}^{(1)}|_{\boldsymbol{r}_0} = U_{prop} \boldsymbol{i} - \tfrac{1}{4}(\bar{\boldsymbol{r}}_1 \boldsymbol{\cdot} \nabla \bar{\boldsymbol{u}}^{(0)*}|_{\boldsymbol{r}_0} + \bar{\boldsymbol{r}}_1^* \boldsymbol{\cdot} \nabla \bar{\boldsymbol{u}}^{(0)}|_{\boldsymbol{r}_0}), \tag{1.8}$$

where $\bar{\boldsymbol{u}}^{(1)}$ and U_{prop} are the steady components of $\boldsymbol{u}^{(1)}$ and $\boldsymbol{i} \boldsymbol{\cdot} \boldsymbol{U}_p^{(1)}$, respectively.

To determine the propulsion velocity, U_{prop}, we first calculate the force acting on the moving particle, i.e. $\boldsymbol{F} \equiv \int_{S_p} \boldsymbol{n} \boldsymbol{\cdot} \boldsymbol{\sigma}\, \mathrm{d}S$. Expanding the normal vector, $\boldsymbol{n}$, gives

$$\boldsymbol{n} = \boldsymbol{n}_0 + \epsilon\, \mathrm{Re}[\bar{\boldsymbol{n}}_1 \mathrm{e}^{-\mathrm{i}t}] + o(\epsilon), \tag{1.9}$$

where $\bar{\boldsymbol{n}}_1 = \mathrm{i}\bar{\Omega}_p \boldsymbol{n}_0 \times \boldsymbol{j}$ and $\boldsymbol{n}_0$ is the normal vector to the surface, S_0, defined in the text immediately following (1.1). Substituting the asymptotic expansion for the fluid variables, (2.4*a*,*b*), into the resulting expression for the force, and then Taylor expanding the surface integral, gives $\boldsymbol{F} = \boldsymbol{F}^{(0)} + \epsilon \boldsymbol{F}^{(1)} + o(\epsilon)$. The Fourier component of $\boldsymbol{F}^{(0)}$ is non-zero in the z-direction only, which is given by the left-hand side of (2.7*a*). Moreover, the steady component of $\boldsymbol{F}^{(1)}$ is

$$\bar{\boldsymbol{F}}^{(1)} = \int_{S_0} \boldsymbol{n}_0 \boldsymbol{\cdot} \bar{\boldsymbol{\sigma}}^{(1)} + \tfrac{1}{4}[\bar{\boldsymbol{n}}_1 \boldsymbol{\cdot} \bar{\boldsymbol{\sigma}}^{(0)*} + \bar{\boldsymbol{n}}_1^* \boldsymbol{\cdot} \bar{\boldsymbol{\sigma}}^{(0)} + \boldsymbol{n}_0 \boldsymbol{\cdot} (\bar{\boldsymbol{r}}_1 \boldsymbol{\cdot} \nabla \bar{\boldsymbol{\sigma}}^{(0)*} + \bar{\boldsymbol{r}}_1^* \boldsymbol{\cdot} \nabla \bar{\boldsymbol{\sigma}}^{(0)})]\, \mathrm{d}S, \tag{1.10}$$

which is non-zero in the x-direction only.

To evaluate $\boldsymbol{i} \boldsymbol{\cdot} \int_{S_0} \boldsymbol{n}_0 \boldsymbol{\cdot} \bar{\boldsymbol{\sigma}}^{(1)}\, \mathrm{d}S$, we use the generalised Lorentz reciprocal theorem on the fluid domain, V_0, which is specified as the exterior of the surface defined by the particle's initial position, i.e. S_0; not the true (time-dependent) fluid domain. The two flows used in the reciprocal theorem are (*a*) the first-order steady Stokes flow with body force as in (2.9*b*), but with boundary condition (1.8), and (*b*) an auxiliary (body-force-free) Stokes flow, $(\boldsymbol{u}', \boldsymbol{\sigma}')$, specified by unitary translation of the particle in the x-direction. This gives

$$\int_{S_0} \boldsymbol{n}_0 \boldsymbol{\cdot} (\boldsymbol{i} \boldsymbol{\cdot} \bar{\boldsymbol{\sigma}}^{(1)} - \bar{\boldsymbol{u}}^{(1)} \boldsymbol{\cdot} \boldsymbol{\sigma}')\, \mathrm{d}S = -\frac{\beta}{4} \int_{V_0} \boldsymbol{u}' \boldsymbol{\cdot} (\bar{\boldsymbol{u}}^{(0)} \boldsymbol{\cdot} \nabla \bar{\boldsymbol{u}}^{(0)*} + \bar{\boldsymbol{u}}^{(0)*} \boldsymbol{\cdot} \nabla \bar{\boldsymbol{u}}^{(0)})\, \mathrm{d}V. \tag{1.11}$$

Conservation of the particle's linear momentum (to first order in ϵ) requires $\bar{\boldsymbol{F}}^{(1)} = 0$. Substituting (1.8) into (1.11) and its result into (1.10) gives the corrected expression for (2.10):

$$\begin{aligned} U_{prop} = {} & \frac{\beta}{4F_p} \int_{V_0} \boldsymbol{u}' \boldsymbol{\cdot} (\bar{\boldsymbol{u}}^{(0)} \boldsymbol{\cdot} \nabla \bar{\boldsymbol{u}}^{(0)*} + \bar{\boldsymbol{u}}^{(0)*} \boldsymbol{\cdot} \nabla \bar{\boldsymbol{u}}^{(0)})\, \mathrm{d}V \\ & + \frac{1}{4F_p} \int_{S_0} (\boldsymbol{n}_0 \boldsymbol{\cdot} \boldsymbol{\sigma}') \boldsymbol{\cdot} (\bar{\boldsymbol{r}}_1 \boldsymbol{\cdot} \nabla \bar{\boldsymbol{u}}^{(0)*} + \bar{\boldsymbol{r}}_1^* \boldsymbol{\cdot} \nabla \bar{\boldsymbol{u}}^{(0)}) \\ & - \boldsymbol{i} \boldsymbol{\cdot} [\boldsymbol{n}_0 \boldsymbol{\cdot} (\bar{\boldsymbol{r}}_1 \boldsymbol{\cdot} \nabla \bar{\boldsymbol{\sigma}}^{(0)*} + \bar{\boldsymbol{r}}_1^* \boldsymbol{\cdot} \nabla \bar{\boldsymbol{\sigma}}^{(0)}) + \bar{\boldsymbol{n}}_1 \boldsymbol{\cdot} \bar{\boldsymbol{\sigma}}^{(0)*} + \bar{\boldsymbol{n}}_1^* \boldsymbol{\cdot} \bar{\boldsymbol{\sigma}}^{(0)}]\, \mathrm{d}S; \end{aligned} \tag{1.12}$$

the surface integral in (1.12) must also be added to (2.11).

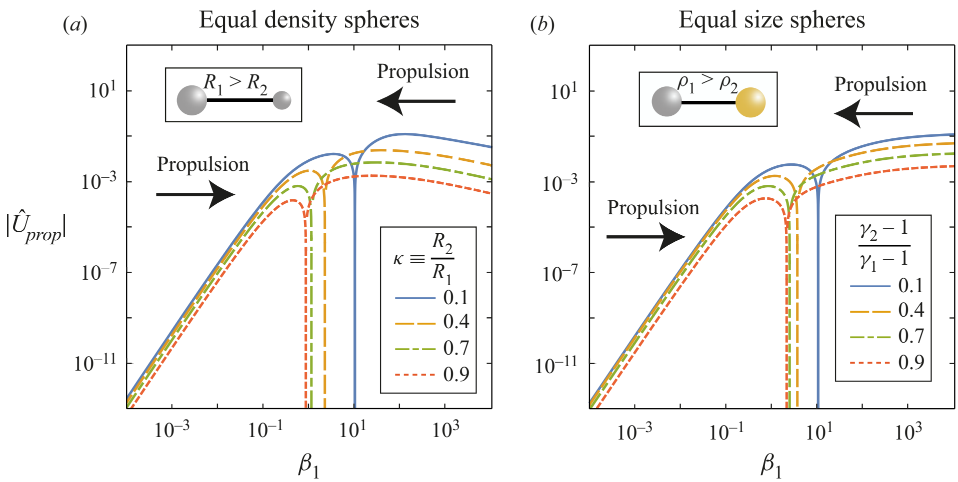

Figure 2. Corrected: axis scales are identical to Collis *et al.* (2017).

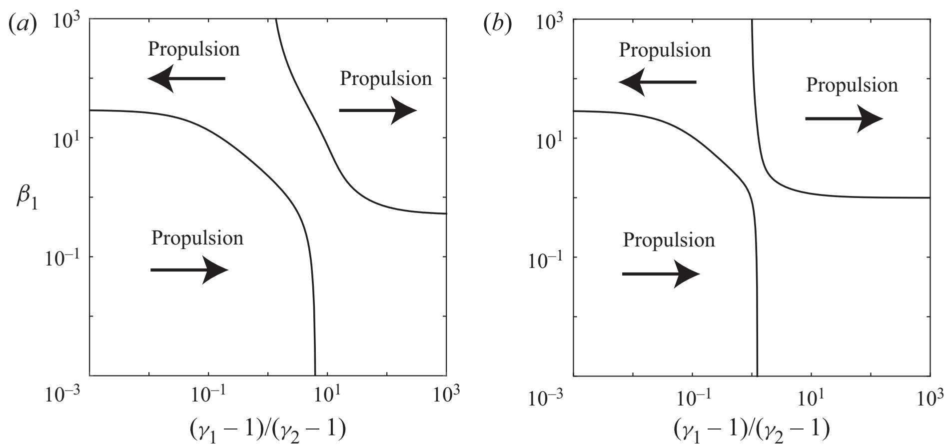

Figure 4. Corrected: axis scales are identical to Collis *et al.* (2017).

(ii) Corrected asymptotic forms for the propulsion velocity of the dumbbell, (3.6) and (3.7), are, respectively,

$$U_{prop} = \frac{(\kappa - 1)(\gamma_1 - 1)^2}{A} \begin{cases} -\dfrac{1+\kappa^4}{243}\beta_1^3, & \beta_1 \ll 1, \\ \dfrac{45\sqrt{2}}{(1+2\gamma_1)^3\kappa(1+\kappa)}\beta_1^{-1/2}, & \beta_1 \gg 1, \end{cases} \tag{1.13}$$

and

$$U_{prop} = \frac{(\gamma_2 - \gamma_1)}{A} \begin{cases} -\dfrac{\gamma_1 + \gamma_2 - 2}{486}\beta_1^3, & \beta_1 \ll 1, \\ \dfrac{3(4\gamma_1\gamma_2 - \gamma_1 - \gamma_2 - 2)}{(1+2\gamma_1)^2(1+2\gamma_2)^2}, & \beta_1 \gg 1. \end{cases} \tag{1.14}$$

End shape	Material	γ_1	γ_2	U_{prop} ($\times 10^{-4}$)
Hemi-spherical	Au	19.3	19.3	−6.2
	RuAu	12.5	19.3	−26.0
	AuRu	19.3	12.5	14.4
Hemi-spheroidal	Au	19.3	19.3	−32.8
	RuAu	12.5	19.3	−38.5
	AuRu	19.3	12.5	−15.9

Table 1. Corrected.

(iii) Corrected figures 2 and 4 are similar to Collis *et al.* (2017), with quantitative changes to the zero propulsion point. Corrected table 1 shows closer agreement with experiments, which indicate that the concave and light end lead the motion; similar changes to figure 5 (not shown).

(iv) Changes to the caption of figure 3: (*c*) Non-zero contribution from the body force is cancelled by that of the surface integrals in (2.10), giving $U_{prop} = 0$. (*f*) Combining these two flows produces a jet that propels the sphere in the negative x-direction, with $U_{prop} = -0.039$.

Declaration of interests. The authors report no conflict of interest.

Author ORCIDs.
Jesse F. Collis https://orcid.org/0000-0003-0992-101X;
John E. Sader https://orcid.org/0000-0002-7096-0627.

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